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Membership of Academic Societies:
(MSJ) The Mathematical Society of Japan

Research Interest:

- Mathematical Analysis of Fundamantal Equations for Fluid Mechanics
- Fourier Analysis

Research Summary:

My main research interest is in mathematical analysis of fundamental equations for fluid mechanics. Recently, I study motion of non-Newtonian fluids (fluids of macromolecule such as gel and ketchup). Navier-Stokes equations which describes the motion of incompressible Newtonian fluids is the following:

$\partial_t u - \mathrm{div}(\nu Du) + u \cdot \nabla u + \nabla p = f$, div $u = 0$

Here, u is a velocity vector field of the fluid, $Du = \frac{1}{2}\left(\nabla u + (\nabla u)^T\right)$, p is a pressure scalar field and ν is a viscosity of the fluid which is a positive constant and f is an external force. u and p are unknown functions and a problem of solvability of the system under suitable initial and boundary data is initial-boundary-value problem of Navier-Stokes equations. Main characteristics of the equations is non-linearity, having several unknowns and incompressibility condition div$u = 0$. The analysis of the equations are difficult because one couldn't use a maximum principle for the solutions which holds for certain non-linear heat equations. In fact, global in time existence of the classical solutions for the Navier-Stokes equations in 3-dim. space is a famous open problem and is one of seven millennium problems posed by Clay Mathematics Institute. There have been many studies about local in time existence of classical solutions and global in time existence of classical solutions concerning the equation.

My recent research interest is in non-Newtonian fluids which is described by the equations whose difference with the Navier-Stokes equations is that the viscosity is a non-constant function which depends on the largeness of Du. Among the equations which describes various non-Newtonian fluids is power-law fluid equations in which the Laplacian part of the Navier-Stokes equations are replaced by the p-Laplacian type operator. The solvability of the equations depends on the value of p. Maximal monotone operator theory is needed to show the existence of global weak solution and in 60's Ladyzhenskaya and Lions show the unique existence of global weak solutions for large p. To show the existence of weak solutions for smaller p, Fourier analytic method called "Lipschitz truncation method" is needed and it is studied well by a group around Ruzicka after '00. For an analysis of two-phase Newtonian fluids, such as oil and water, L^p estimates (or, L^pmaximal regularity) of its linearized equation around initial data is needed. A similar method can be employed for single power-law fluids and the local existence of its classical solutions were shown by Bothe-Pr"uss('07).

I am studying problems related with global existence of weak solution for two-phase power-law fluid and local existence of its classical solution. I am also interested in a study of deeper property of a linearized equation of a single power-law fluid equation and related Probability Theory and Fourier Analysis.

Major Publications:

[1] H. Abels and Y. Terasawa, On Stokes operators with variable viscosity in bounded and unbounded domains, Math. Ann. **344** (2009), 381–429.

[2] H. Abels and Y. Terasawa, Non-homogeneous Navier-Stokes systems with order-parameter-dependent stresses, Math. Methods Appl. Sci. **33** (2010), 1532–1544.

[3] H. Abels, L. Diening and Y. Terasawa, Existence of weak solutions for a diffuse interface model of non-Newtonian two-phase flows, Nonlinear Anal. Real World Appl. **15** (2014), 149–157.

Education and Appointments:

2007	Graduated from Gradate school of Mathematics at Hokkaido University
2009	Research Assistant, Tohoku University
2010	Project Researcher, The University of Tokyo
2011	JSPS Fellow, The University of Tokyo
2012	Project Research Associate, The University of Tokyo
2014	Associate Professor, Nagoya University

Message to Prospective Students:

To investigate a motion of incompressible viscous fluids, one needs to master basic theories on Partial Differential Equations, Functional Analysis and Fourier Analysis etc. In the seminar, it is preferable that students first master one of these topics well and then later study students study more advanced topics such as solvability theory of incompressible viscous fluid equations based on it. It is also preferable that students expand their knowledge of other fields in the course of their study of advanced topics. If students hope, both basic course and advanced course could be held.

I list a candidate for texts used in the course, but students could choose other books.

1. S. Krantz, A Panorama of Harmonic Analysis, The Mathematical Association of America.
2. T. Hytönen, Weighted Norm Inequalities, Lecture Note available on Web.
3. H. Tanabe, Functional Analytic Methods for Partial Differential Eqautions, CRC Press.
4. H. Brezis, Functional Analysis, Sobolev Spaces and Partial Differential Equations, Springer.
5. M. Giaquinta, L. Martinazzi, An introduction to the regularity theory for elliptic system, harmonic maps and minimal graphs, Edizioni Della Normale.
6. A. McIntosh, Operator Theory - Spectra and Functional Calculi, Lecture Note available on Web.

Students who would like to take our master course small class are required to have sound knowledge of basic facts in Calculus, Ordinary Differential Equation, Complex Analysis, Lebesgue Integration Theory and Functional Analysis. If a student doesn't have enough knowledge to study in the course, he or she needs to study it in case it is needed. After these studies, one studies more specialized topics such as analysis of motion of incompressible fluids. One could also choose another subject related to Fourier analysis or PDE.

In their Doctor Course, I advise students on more advanced topics. In Doctor Course, students' hope is valued concerning a choice of their research topics.