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Membership of Academic Societies:
MSJ (The Mathematical Society of Japan)

Research Interest:

- Commutative Algebra
- Representation Theory of Algebras

Research Summary:

Commutative algebra is the theory of commutative rings. This theory has close relationships with many areas of mathematics, including algebraic geometry, number theory, representation theory, noncommutative algebra, algebraic topology, algebraic combinatorics, computational algebra, and recently, even physics and algebraic statistics. I have mainly been working in the boundary between commutative algebra and representation theory of algebras.

The subject of representation theory of algebras is to understand the structure of the module category, i.e., the category of finitely generated modules, over a given noetherian algebra. The structure of the module category will be clarified if we can classify all the indecomposable finitely generated modules, but this is regarded as "impossible" in general. (Most algebras have wild representation type, and it is known that over such an algebra it is hopeless to classify the indecomposable finitely generated modules.) Thus, in modern representation theory of algebras, the main approach to try to understand the structure of the module category is to investigate subcategories of the module category having good properties and triangulated categories associated to the module category, like derived categories, stable categories and singularity categories.

My research area is "representation theory of commutative rings." Namely, the purpose of my study is to understand the structure of the module category of a given commutative noetherian ring. Representation theory of Cohen–Macaulay rings was born in the 1970s as a higher dimensional version of representation theory of finite dimensional algebras, which explores the subcategory of the module category of a Cohen–Macaulay ring consisting of (maximal) Cohen–Macaulay modules. The meaning of Cohen–Macaulay rings has initially been found in the ideal theory as a local theory of algebraic geometry. These rings are important from the viewpoints of both homological algebra and algebraic combinatorics, and have been playing a crucial role in modern commutative algebra. I have been studying module categories of commutative rings and their subcategories and associated triangulated categories, always having in mind Cohen–Macaulay rings, especially Gorenstein rings, which possess plenty of dualities and symmetries. My current biggest interests are in classifying resolving subcategories of module categories and thick subcategories of derived categories.

Major Publications:

[1] R. Takahashi, Classifying thick subcategories of the stable category of Cohen–Macaulay modules, *Adv. Math.* **225** (2010), no. 4, 2076–2116.

[2] R. Takahashi, Contravariantly finite resolving subcategories over commutative rings, *Amer. J. Math.* **133** (2011), no. 2, 417–436.

[3] S. B. Iyengar; R. Takahashi, Annihilation of cohomology and strong generation of module categories, *Int. Math. Res. Not. IMRN* (2016), no. 2, 499–535.

[4] H. Dao; T. Kobayashi; R. Takahashi, Burch ideals and Burch rings, *Algebra Number Theory* **14** (2020), no. 8, 2121–2150.

[5] R. Takahashi, Dominant local rings and subcategory classification, *Int. Math. Res. Not. IMRN* (2023), no. 9, 7259–7318.

Awards and Prizes:

- 2004, MSJ Takebe Prize, "Homological studies of Cohen–Macaulay rings"
- 2020, MSJ Algebra Prize, "Subcategories of module categories of commutative rings"

Education and Appointments:

2000	BhSc at Kyoto University
2004	Ph.D. at Okayama University
2006	Assistant Professor, Shinshu University
2009	Associate Professor, Shinshu University
2012	Associate Professor, Nagoya University
2022	Professor, Nagoya University

Message to Prospective Students:

"Commutative algebra is a beautiful and deep theory in its own right" — This sentence appears at the beginning of the introduction of [3]. When I was a third-year undergraduate, I met a mathematician, who became my Ph.D. advisor later, and he gave me motivations to study commutative algebra. As soon as I started studying commutative algebra, its systematic theory very much attracted me. Commutative algebra is an area whose entry level is low; one can start studying it only by basic knowledge on rings given in the undergraduate course. If you have not yet studied commutative algebra itself, go to a bookstore or a library to get and study [3]. Then, even if you face a place which you cannot understand (i.e., which you cannot explain in the case where someone asks you), do not skip it. Consider each sentence until you understand it.

The book [3] has been thought of as the most excellent book in commutative algebra all over the world, and probably all of those who deal with commutative algebra possess this book. Basically one can understand it by undergraduate algebra, that is, linear algebra, group theory, ring theory and general topology. If [3] turns out to be too difficult for you, you can begin with [1] alternatively.

After learning [3], you should move to [2]. This book contains a lot of basic facts which are usually assumed as preliminary knowledge in papers on commutative algebra. Thus classical commutative algebra can be enjoyed enough by studying [3], but to understand recent results in commutative algebra and to get your own results, you need knowledge given in [2]. The book [4] handles representation theory of Cohen–Macaulay rings, and contains classification of Gorenstein rings of finite Cohen–Macaulay representation type and Cohen–Macaulay modules over them, which has been completed in the 1980s. One can understand this book after learning [2]. Since this is deeply interesting, I recommend you to study it.

[1] M. F. Atiyah; I. G. MacDonald, *Introduction to commutative Algebra*, Westview Press, 1994.

[2] W. Bruns; J. Herzog, *Cohen-Macaulay rings*, Cambridge University Press, 1998.

[3] H. Matsumura, *Commutative ring theory*, Cambridge University Press, 1989.

[4] Y. Yoshino, *Cohen-Macaulay modules over Cohen-Macaulay rings*, Cambridge University Press, 1990.