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**Membership of Academic Societies:**

MSJ (The Mathematical Society of Japan)

**Research Interest:**

- Probability
- Large scale interacting systems

**Research Summary:**

Probability theory means “measure theoretic probability theory” which is completely different from combinatorial probability and statistics. Lectures in measure theory are usually given to junior students majoring in mathematics. In measure theory, we define “areas” of measurable sets in a set  $S$  and then we can construct Lebesgue integral of a measurable function. In measure theoretical probability, measurable sets in sample space  $\Omega$  is called events and their “areas” are identified as probabilities. Measure theoretic probability allows us to consider “infinitely many trials” mathematically and analyze them.

The large scale interacting systems are closely related to physics. We shall explain it through polymers which is one of my research interests.

*Polymers* are large molecules created via polymerization of small molecules, *monomers*, e.g.  $(-\text{CH}_2-)$ . Each bond between  $-\text{C}-\text{s}$  has a freedom of rotation, i.e. is random. We will simplify the models by ignoring exclusivity between monomers. We consider a polymer chain with length  $n + 1$  and we set the position of one endpoint (monomer 0) of chain as 0 and denote by  $S_i$  the position of  $i$ th-monomers. Moreover, we assume that the displacements  $\{X_i := S_i - S_{i-1}\}_{i=1}^n$  are independent and identically distributed. Then, this is a random walk that is well-studied. Now, we study the shape of polymer chains. Investigating polymers macroscopically can be regarded as the scaling limit of random walk. When  $\{X_i\}_{i=0}^n$  satisfies a proper assumption, the scaling limit of  $\{S_i\}_{i=0}^n$  is the Brownian motion (**invariance principle**). Thus, the shape of polymers in an ideal medium can be regarded as the trace of Brownian motion.

How about the case for the polymers in a medium with impurities? We may believe that there exists some interaction between impurities and monomers. This interaction is described via a new probability measure called Gibbs measure. This measure is generated by assigning a weight representing an interaction to each path  $\{S_i\}_{i=0}^n$ . Changing a parameter (e.g. concentration of impurities), the shape of polymers under Gibbs measure is changed completely (**phase transition**). Thus, interaction between impurities and monomers (large scale interaction) yields a new phenomenon.

There are other physical models classified as large scale interacting systems and many branches.

Usually, the phase transition is characterized in terms of the quantity “free energy”. Recently, I have studied the asymptotics of the free energy and found the universality structure behind the physical models.

## Major Publications:

- [1] C. Cosco, S. Nakajima, M. Nakashima: Law of large numbers and fluctuations in the sub-critical and  $L^2$  regions for SHE and KPZ equation in dimension  $d \geq 3$ . *Stochastic Process. Appl.* **151** (2022), 127–173.
- [2] M. Nakashima: Free energy of directed polymers in random environment in 1+1-dimension at high temperature. *Electron. J. Probab.* **24** (2019), No. 50.
- [3] M. Nakashima: Branching random walks in random environment and super-Brownian motion in random environment. *Ann. Inst. Henri Poincaré Probab. Stat.* **51** (2015), no. 4, 1251–1289.

## Awards and Prizes:

- 2014, MSJ Takebe Katahiro Prize for Encouragement of Young Researchers “Study of branching random walks in random environment”

## Education and Appointments:

- 2012 Assistant Professor, University of Tsukuba
- 2015 Associate Professor, Nagoya University

## Message to Prospective Students:

When you learn measure theoretic probability, calculus and linear algebra are needed. Moreover, it is better to master the measure theory. Before entering graduate school, you have to learn the measure theoretic probability ( $\sigma$ -algebra, independence, law of large numbers, central limit theorem, conditional expectation). The following is a standard textbook of measure theoretic probability.

- [1] Williams, “Probability with martingales”, Cambridge Mathematical Textbooks, 1991.