HIRAI, Hiroshi Professor

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Membership of Academic Societies:
The Japan Society for Industrial and Applied Mathematics (JSIAM) ,
The Operations Research Society of Japan (ORSJ)

Research Interest:

- Optimization
- Algorithm
- Discrete Mathematics

Research Summary:

Discrete optimization (or combinatorial optimization) is a mathematical theory that aims at designing algorithms to find the most desirable one among a finite but huge number of combinations in a realistic time. The theory of discrete optimization began in the mid-1960s with a series of studies by J. Edmonds, in which he proposed the thesis of "*efficient algorithm ≒ polynomial-time algorithm*," introduced *polyhedral methods*, and showed the importance of *matroids* and *submodular functions* in algorithm design. The polyhedral method is one of the basic paradigms of algorithm design, which is "*to design an algorithm from the viewpoint of continuous optimization by embedding a discrete optimization problem into a convex optimization problem in Euclidean space.*" While many discrete optimization problems are NP-hard, understanding the class P of problems with polynomial-time algorithms remains an important challenge. Matroids and submodular functions, which are closely related to class P, are recognized as discrete convex sets and functions, and have evolved into the theory of convex analysis on integer lattice points—*discrete convex analysis*—.

I worked on multi-commodity extensions of Ford-Fulkerson's maximum flow minimum-cut theorem in network flow problem [1], and polynomial-time solvability classification of facility location problems that generalizes the minimum cut problem [2]. These studies deal with discrete convex optimization on special graph structures, and I found that they can be embedded as geodesically convex optimization on nonpositively curved metric spaces (CAT(0)-spaces) [3]. I then obtained the idea of "developing convex optimization/algorithm theory for nonpositively curved spaces, and applying it to discrete optimization", which updates the above paradigm. Since then, I have been conducting research based on this idea. Recently, I focus on the following two themes.

The first is *Edmonds problem*, that is the symbolic rank compution of matrices containing variables. Although it is a fundamental problem with many applications, the existence of deterministic polynomial-time algorithms is an important open problem in theoretical computer science. Also it may be called *algebraic combinatorial optimization*, since it generalizes a class of combinatorial optimization problems such as bipartite matching and linear matroid intersection. Recently, a noncommutative version of Edmonds problem was introduced, in which the variables are *noncommutative*. It was shown that the rank (*noncommutative rank*) in this setting can be computed in polynomial time. The computation of the noncommutative rank becomes a new type of optimization problem over the family of vector subspaces. I have developed a polynomial-time algorithm using convex optimization of a non-manifold CAT(0) space [8]. I further generalized it to a weighted version (degree computation of noncommutative determinants) [5].

The second is study of convex optimization on Hadamard manifolds and symmetric spaces. The noncommutative Edmonds problem over $\mathbb{C}$, which can also be formulated as a norm minimization problem over an orbit of group action, becomes convex optimization on symmetric spaces of nonpositive curvature. Various other applications have been found. While studying differential geometry, I am trying to develop convex analysis and interior point methods on such manifolds [9, 10].

I have also studied discrete structures such as lattices, matroids, buildings, and discrete metric spaces due to my interest as stages for discrete optimization [4, 6, 7].

For more information on my research, please visit my web page.

Major Publications:

[1] H. Hirai: The maximum multiflow problems with bounded fractionality, *Mathematics of Operations Research* **39** (2014), 60–104.

[2] H. Hirai, Discrete convexity and polynomial solvability in minimum 0-extension problems, *Mathematical Programming, Series A* **155**, (2016) 1–55.

[3] H. Hirai: L-convexity on graph structures, *Journal of the Operations Research Society of Japan* **61** (2018), 71–109.

[4] H. Hirai: Uniform semimodular lattices and valuated matroids, *Journal of Combinatorial Theory, Series A***165** (2019), 325–359.

[5] H. Hirai: Computing the degree of determinants via discrete convex optimization on Euclidean buildings, *SIAM Journal on Applied Algebra and Geometry* **3** (2019), 523–557.

[6] J. Chalopin, V. Chepoi, H. Hirai and D. Osajda: Weakly modular graphs and nonpositive curvature, *Memoirs of the AMS* **268**, no.1309, (2020).

[7] H. Hirai: A nonpositive curvature property of modular semilattices, *Geometriae Dedicata* **214** (2021), 427–463.

[8] M. Hamada and H. Hirai: Computing the nc-rank via discrete convex optimization on CAT(0) spaces, *SIAM Journal on Applied Geometry and Algebra* **5** (2021), 455–478.

[9] H. Hirai: Convex analysis on Hadamard spaces and scaling problems, arXiv, 2022.

[10] H. Hirai, H. Nieuwboer, and M. Walter: Interior-point methods on manifolds: theory and applications, arXiv, 2023.

Awards and Prizes:

- 2014, Research Award, The Operations Research Society of Japan.
- 2018, The Young Scientists' Award, The Commendation for Science and Technology by the Minister of Education, Culture, Sports, Science and Technology.

Education and Appointments:

2004 Master of Information Science and Technology, Graduate School of Information Science and Technology, The University of Tokyo
2004 Research Associate, Research Institute for Mathematical Sciences, Kyoto University
2007 Assistant Professor, Research Institute for Mathematical Sciences, Kyoto University
2010 Lecturer, Graduate School of Information Sciences and Technology, The University of Tokyo
2014 Associate Professor, Graduate School of Information Sciences and Technology, The University of Tokyo
2023 Professor, Graduate School of Mathematics, Nagoya University

Message to Prospective Students:

In the small group seminar, you will study the following basic textbooks on optimization and algorithms, and tackle specific problems according to your interests.

[1] S. Boyd and L. Vandenberghe: *Convex Optimization*, Cambridge University Press, 2004.

[2] J. Kleinberg and E. Tardos: *Algorithm Design*, Pearson, 2006.

I expect that you will find interesting research directions if you review various problems in mathematical science from the viewpoints of optimization, algorithms, and computational complexity.