Abstract
Knörrer periodicity produces an equivalence between the singularity categories of Gorenstein cyclic quotient surface singularities and commutative finite dimensional algebras $C[x]/(x^n)$.

We produce an equivalence between the singularity categories of arbitrary cyclic quotient surface singularities and certain (not necessarily commutative) finite dimensional algebras.

Knörrer Periodicity
Buchweitz introduced the singularity category in [2].

Definition. Suppose $R$ is a Noetherian C-algebra.

- The singularity category is the triangulated category defined by the Verdier quotient $D_{sg}(R) = D^b(R)/Perf(R)$.
He showed the singularity category generalises:
- The category of matrix factorisations for hypersurface rings.
- The stably category of maximal Cohen-Macaulay modules (MCMs) for Gorenstein rings.

Buchweitz also restated Knörrer periodicity.

Theorem ([3], [2], Knörrer Periodicity). Suppose $K = \kappa[z_1, \ldots, z_k]/(f)$, $R = K[z_1, \ldots, z_k]/(z_1^2 + y^2 + f)$.

Then there is an equivalence $D_{sg}(R) \cong D_{sg}(K)$ of singularity categories.

Related equivalences for NC resolutions have been studied using relative singularity categories.

Definition. Suppose $\Lambda$ is a Noetherian C-algebra and $P$ is a projective $X$-module.

- The relative singularity category is the triangulated category defined by the (idempotent completion of the) Verdier quotient $\Delta_{sg}(\Lambda) = D^b(\Lambda)/Pf(\Lambda)$.

Theorem ([4]). Suppose that $R, K$ are $\text{MCM}$-representation finite, complete, Gorenstein, C-algebras. Then:

- There is an equivalence $D_{sg}(R) \cong D_{sg}(K)$ of singularity categories.

- There is an equivalence $\Delta_{sg}(\Lambda) \cong \Delta_{sg}(\Lambda)$ of relative singularity categories.

Cyclic Quotient Surface Singularities
For $0 < a < r$ coprime integers define:

- The finite group $\mathbb{Z}/r \cong \langle \gamma \rangle \langle X \rangle < GL_2(\mathbb{C})$ where $\gamma$ is a primitive $r^a$-th root of unity.

- The invariant algebra $R_{a,r} = C[Z]/(y-g^{r^a})$.

- The Hirzebruch-Jung continued fraction expansion $r = a_0 - \frac{1}{a_1 - \frac{1}{\cdots - \frac{1}{a_{n-1} - \frac{1}{a_n}}}} = \lfloor a_{n-1}/a_n \rfloor$.

Cyclic quotient surface singularities include non-Gorenstein examples, and we will consider equivalences analogous to Knörrer periodicity between general cyclic quotient surface singularities and finite dimensional algebras.

Knörrer Invariant Algebras
The algebra $K_{r,a}$ can be presented as the monomial algebra

$$K_{r,a} := \frac{C[Z_1, Z_2]}{(Z_2, \frac{Z_1}{Z_2} \left(\sum_{i= \lfloor r/k \rfloor}^{\lfloor r/2 \rfloor} (Z_2)^{2i-2}\right) Z_2^{i} \mid 1 < k \leq l}$$

Further:

- The dimension of $K_{r,a}$ is $r$.

- As right $K_{r,a}$-modules there are $n + 1$ isomorphism classes of indecomposable ideals $I < K_{r,a}$.

- The largest of these indecomposable ideals has dimension $a$.

- There is an isomorphism $\Delta_{r,a} \cong \text{End}_{K_{r,a}}(\text{F}_{0}^{\infty}(I))$.

Examples:

- $K_{3,1,1} = \frac{C[Z_1]}{(Z_1 Z_1 Z_1)}$ which occurs in Knörrer periodicity.

- $K_{3,1,1} = \frac{C[Z_1]}{(Z_1 Z_1 Z_1)}$ is the radical square zero algebra.

References


