

**FORMAL POWER SERIES SOLUTIONS OF DIFFERENTIAL  
EQUATIONS  
– GEVREY PROPERTY AND SUMMABILITY –**

MASAFUMI YOSHINO

1. INTRODUCTION

The object of the talk is to state recent developments and motivations of the study on (divergent) formal power series solutions of differential equations. We will consider so-called singular ordinary and partial differential equations, and non-Kowalevskian equations. These equations play an important role in actual applications. The typical examples are: irregular singular ODE, transforming equations of a singular vector field to its normal form and the heat equation. We are mainly interested in the so-called asymptotic analysis of the equations, namely construction and estimates of formal solutions, realization and summability and global behavior of solutions. We begin with the study of Gevrey property of formal power series solutions by M. Miyake, and then we explain subsequent works by his students Shirai, Hibino and Ichinobe, and by Miyake-Yoshino. We then talk on the summability of formal solutions. Beginning with the well-known results by M. Miyake on the heat equation, we introduce the ones by Balser, Schäfke and so on. Finally, we explain recent developments of the moment summability method and related new topics, which are closely related with the above work by Miyake for the heat equation.

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DEPARTMENT OF MATHEMATICS, GRADUATE SCHOOL OF SCIENCE, HIROSHIMA UNIVERSITY,  
1-3-1 KAGAMIYAMA, HIGASHI-HIROSHIMA, HIROSHIMA 739-8526, JAPAN  
*E-mail address:* yoshino@math.sci.hiroshima-u.ac.jp