

\mathcal{D} -modules in prime characteristic

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Abstract

The irreducible characters for a simple algebraic group G over a field of prime characteristic p are now known for large p after Lusztig's program; Kashiwara + Tanisaki obtained the irreducible characters for affine Lie algebras over \mathbb{C} , using the theory of \mathcal{D} -modules; from those Kazhdan + Lusztig deduce the irreducible characters for quantum algebras over \mathbb{C} at a p -th root of unity by establishing a categorical equivalence with the preceding; finally, Andersen + Jantzen + Soergel prove for all p outside an unknown finite set the equalities of certain irreducible characters for G and the corresponding quantum algebra. In the last step, because of the characteristic difference there cannot be a categorical equivalence between the two, and a difficulty persists in showing the equalities under a reasonable bound on p .

In the spring of last year, however, a breakthrough was brought by Bezrukavnikov + Mirkovic + Rumynin. Noting that the center of the sheaf \mathcal{D} of rings of crystalline differential operators on the flag variety \mathcal{B} of G may be thought of as the Frobenius twist of the cotangent bundle over \mathcal{B} , if χ is a nilpotent character of the Lie algebra \mathfrak{g} of G , they establish a derived equivalence between the category of finite dimensional modules over a central reduction U_χ^0 of the universal enveloping algebra of \mathfrak{g} and the category $\text{Coh}_\chi(\mathcal{D})$ of coherent \mathcal{D} -modules "supported" in the Springer fiber $\mathcal{B}_\chi^{(1)}$ of χ through the moment morphism; they find a categorical equivalence of the category $\text{Coh}(\mathcal{D}_\chi)$ of coherent modules over a central reduction \mathcal{D}_χ of \mathcal{D} and the category $\text{Coh}(\mathcal{B}_\chi^{(1)})$ of coherent modules over $\mathcal{B}_\chi^{(1)}$; finally they show the equality of the rank of the Grothendieck group of \mathcal{B}_χ to the dimension of the cohomology of the corresponding Springer fiber $\mathcal{B}_\chi^{\mathbb{C}}$ over \mathbb{C} with coefficient in \mathbb{Q} . As a corollary, the number of irreducibles for \mathfrak{g} with central character χ is obtained; in the case $\chi = 0$ the result may be obtained from the representation theory of G and has been known for 50 years, otherwise the result is new verifying a new conjecture of Lusztig.

A distinction of their new approach is, while importing the characteristic 0 flavour, to stay in prime characteristic till the very last. Their \mathcal{D} is Berthelot's ring of PD -differential operators or the 0-th term of his ring of arithmetic differential operators. We'd like to investigate their path, hoping to find further application of arithmetic differential operators in the representation theory of G .