ダークエネルギーの理論模型と観測からの選別

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Dark energy candidates

● The simplest candidate: Cosmological constant

Equation of state
$w_{\rm DE} = -1$

→ If the cosmological constant originates from the vacuum energy, its energy scale is enormously larger than the dark energy scale.

● Dynamical dark energy models

Quintessence, k-essence, chaplygin gas, coupled dark energy, f (R) gravity, scalar-tensor theories, Braneworld, Galileon, …

● Apparent acceleration

Inhomogeneous models, metric backreaction

Dynamical dark energy models

1. Modified matter models

- Quintessence: Acceleration driven by the potential energy $V(\phi)$ of a field ϕ

$$\mathcal{L} = X - V(\phi) \qquad X = -g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi/2$$

- K-essence: Acceleration driven by the kinetic energy X of a field ϕ

$$\mathcal{L} = K(\phi, X)$$

e.g. Dilatonic ghost condensate:

$$K = -X + ce^{\lambda\phi}X^2$$

2. Modified gravity models

- $f(R)$ gravity: The Lagrangian is the function of a Ricci scalar R.
- Scalar-tensor gravity: $\mathcal{L} = F(\phi)R + K(\phi, X)$
- DGP model: Acceleration by the gravitational leakage to extra dimensions.
- Galileon gravity: The Lagrangian is constructed to satisfy the Galilean symmetry $\partial_\mu\phi \to \partial_\mu\phi + b_\mu$ in the flat spacetime.

Such as $X\Box\phi$

Most general single-field scalar-tensor theories with second-order equations of motion (4 dimensions)

$$S = \int d^4x\sqrt{-g}\left[K(\phi, X) - G_3(\phi, X)\Box\phi + \mathcal{L}_4 + \mathcal{L}_5\right]$$

Horndeski (1974)
Deffayet et al (2011)
Charmousis et al (2011)

$$\mathcal{L}_4 = G_4(\phi, X)\, R + G_{4,X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)\,(\nabla^\mu\nabla^\nu\phi)\right]$$

$$\mathcal{L}_5 = G_5(\phi, X)\, G_{\mu\nu}\,(\nabla^\mu\nabla^\nu\phi) - \frac{1}{6}G_{5,X}[(\Box\phi)^3 - 3(\Box\phi)\,(\nabla_\mu\nabla_\nu\phi)\,(\nabla^\mu\nabla^\nu\phi) + 2(\nabla^\mu\nabla_\alpha\phi)\,(\nabla^\alpha\nabla_\beta\phi)\,(\nabla^\beta\nabla_\mu\phi)]$$

This action covers most of the dark energy models proposed in literature.

- LCDM: $K = -\Lambda, \quad G_3 = 0, \quad G_4 = M_{\rm pl}^2/2, \quad G_5 = 0$
- Quintessence and K-essence: $K = K(\phi, X), \quad G_3 = 0, \quad G_4 = M_{\rm pl}^2/2, \quad G_5 = 0$
- f(R) gravity and scalar-tensor gravity: $G_4 = F(\phi)$

$$\mathcal{L} = \frac{M_{\rm pl}^2}{2} f(R) \quad \Rightarrow \quad K = -\frac{M_{\rm pl}^2}{2}\left(Rf_{,R} - f\right), \quad G_3 = 0, \quad G_4 = \frac{1}{2}M_{\rm pl}\phi, \quad G_5 = 0$$

$\phi = M_{\rm pl} f_{,R}$ is the scalar degree of freedom.

- Galileon

$$K = -c_2 X, \qquad G_3 = \frac{c_3}{M^3}X, \qquad G_4 = \frac{1}{2}M_{\rm pl}^2 - \frac{c_4}{M^6}X^2, \qquad G_5 = \frac{3c_5}{M^9}X^2$$

Conditions for the avoidance of ghosts and Laplacian instabilities

In the presence of two fluids (radiation and matter) we expand the Horndenki's action up to second-order with the perturbed metric

$$ds^2 = -(1+2\Psi)dt^2 + a^2(t)(1+2\Phi+2h_{ij})\delta_{ij}dx^i dx^j$$

The second-order action for tensor perturbations is given by,

$$S_T^{(2)} = \sum_\lambda \int dt d^3x\, a^3 Q_T \left[\dot{h}_\lambda^2 - \frac{c_T^2}{a^2}(\partial h_\lambda)^2\right]$$

Kobayashi, Yamaguchi, Yokoyama (2011)
Gao and Steer (2011)
De Felice and S.T. (2011)

$$\Rightarrow \quad Q_T \equiv w_1/4 \geq 0\,, \qquad c_T^2 \equiv w_4/w_1 \geq 0$$

where $w_1 = 2(G_4 - 2XG_{4,X}) - 2X(G_{5,X}\dot{\phi}H - G_{5,\phi})\,, \quad w_4 = 2G_4 - 2XG_{5,\phi} - 2XG_{5,X}\ddot{\phi}$

The conditions for the avoidance of scalar ghosts and Laplacian instabilities are

$$Q_S \equiv \frac{w_1(4w_1w_3 + 9w_2^2)}{3w_2^2} \geq 0\,,$$

ρ_A, ρ_B: densities of fluids
w_A, w_B: Equations of state

$$c_S^2 \equiv \frac{3(2w_1^2w_2H - w_2^2w_4 + 4w_1w_2\dot{w}_1 - 2w_1^2\dot{w}_2) - 6w_1^2[(1+w_A)\rho_A + (1+w_B)\rho_B]}{w_1(4w_1w_3 + 9w_2^2)} \geq 0\,,$$

De Felice and S.T., (2011)

where

$$w_2 = -2G_{3,X}X\dot{\phi} + 4G_4H - 16X^2G_{4,XX}H + 4(\dot{\phi}G_{4,\phi X} - 4G_{4,X})X + 2G_{4,\phi}\dot{\phi}$$
$$+8X^2HG_{5,\phi X} + 2H(6G_{5,\phi} - 5G_{5,X}\dot{\phi}H) - G_{5,XX}\dot{\phi}^5H^2$$

$$w_3 = 3X(K_{,X} + 2XK_{,XX}) + 6X(3X\dot{\phi}HG_{3,XX} - G_{3,\phi X}X - G_{3,\phi} + 6H\dot{\phi}G_{3,X})$$
$$+18H(4HX^3G_{4,XXX} - HG_4 - 5X\dot{\phi}G_{4,\phi X} - G_{4,\phi}\dot{\phi} + 7HG_{4,X}X + 16HX^2G_{4,XX} - 2X^2\dot{\phi}G_{4,\phi XX})$$
$$+6H^2X(2H\dot{\phi}G_{5,XXX}X^2 - 6X^2G_{5,\phi XX} + 13XH\dot{\phi}G_{5,XX} - 27G_{5,\phi X}X + 15H\dot{\phi}G_{5,X} - 18G_{5,\phi})$$

Particular theories

- **K-essence (including Quintessence)**

$$Q_T = M_{\rm pl}^2/4 > 0\,, \qquad c_T^2 = 1 > 0$$ → Automatically satisfied

$$Q_S = \frac{X}{H^2}(K_{,X} + 2XK_{,XX}) > 0\,, \qquad c_S^2 = \frac{K_{,X}}{K_{,X} + 2XK_{,XX}} > 0$$

→ This requires that $K_{,X} > 0$, and $K_{,X} + 2XK_{,XX} > 0$

- **f(R) theories**

The Lagrangian $\mathcal{L} = (M_{\rm pl}^2/2)f(R)$ can be recovered for

$$K = -(M_{\rm pl}^2/2)(Rf_{,R} - f)\,, \quad G_3 = G_5 = 0\,, \quad G_4 = M_{\rm pl}\phi/2 \qquad \text{where} \qquad \phi = M_{\rm pl} f_{,R}$$

$$Q_T = \frac{1}{4}M_{\rm pl}\phi > 0, \qquad Q_S = \frac{3M_{\rm pl}\dot{\phi}^2}{(2H\phi + \dot{\phi})^2}\phi > 0$$ → Ghosts are absent for $f_{,R} > 0$.

$$c_T^2 = 1 > 0, \qquad c_S^2 = 1 > 0$$ → Automatically satisfied

The mass squared of the scalar degree of freedom is

$$M^2 \equiv -K_{,\phi\phi} = 1/(2f_{,RR})$$ → To avoid tachyonic instability we require $f_{,RR} > 0$

Viable f(R) models satisfy these conditions, e.g., $$f(R) = R - \lambda R_c \frac{(R/R_0)^{2n}}{(R/R_0)^{2n} + 1} \qquad (n > 0)$$

(Hu and Sawicki, 2007)

Friedmann equations in the flat FLRW background

$$S = \int d^4x\sqrt{-g}\,[K(\phi,X) - G_3(\phi,X)\Box\phi + \mathcal{L}_4 + \mathcal{L}_5] + \underline{S_m} + \underline{S_r}$$

Non-relativistic matter　　Radiation

The background equations of motion are

$$3M_{\rm pl}^2 H^2 = \rho_{\rm DE} + \rho_m + \rho_r$$

$$-2M_{\rm pl}^2 \dot{H} = \rho_{\rm DE} + P_{\rm DE} + \rho_m + 4\rho_r/3$$

$\rho_{\rm DE}$ and $P_{\rm DE}$ are the density and pressure of the "dark" component.

$$\rho_{\rm DE} = 2XK_{,X} - K - 2XG_{3,\phi} + 6X\dot{\phi}HG_{3,X} - 6H^2G_4 + 3M_{\rm pl}^2H^2 + 24H^2X(G_{4,X} + XG_{4,XX})$$
$$-12HX\dot{\phi}G_{4,\phi X} - 6H\dot{\phi}G_{4,\phi} - 6H^2X(3G_{5,\phi} + 2XG_{5,\phi X}) + 2H^3X\dot{\phi}(5G_{5,X} + 2XG_{5,XX})$$

The equation of state of dark energy is given by

$$w_{\rm DE} = \frac{P_{\rm DE}}{\rho_{\rm DE}}$$

→ The evolution of $w_{\rm DE}$ is different depending on dark energy models.

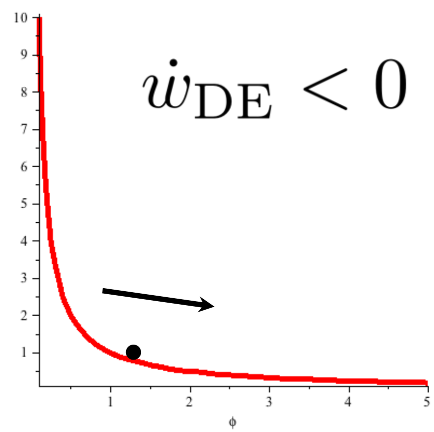
Dark energy equation of state: modified matter models

(1) LCDM → $w_{\rm DE} = -1$

(2) Quintessence → $w_{\rm DE}$ depends on the potential.

(a) Freezing models

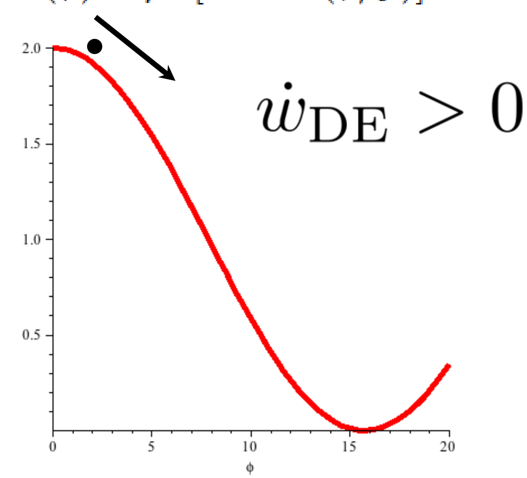
$$V(\phi) = M^{4+n}\phi^{-n}\,, \quad (n>0)$$



$w_{\rm DE}$ decreases toward -1.

(b) Thawing models e.g. axions

$$V(\phi) = \mu^4\left[1+\cos(\phi/f)\right]$$



$w_{\rm DE}$ increases from -1.

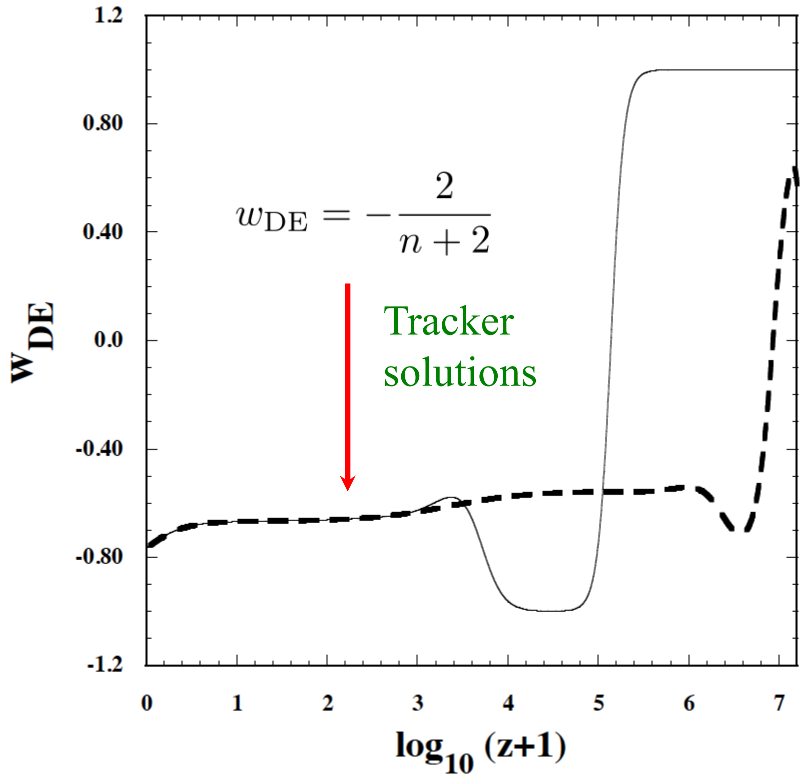
(3) k-essence

$w_{\rm DE}$ depends on $K(\phi, X)$, but typically the evolution of $w_{\rm DE}$ is similar to that in thawing models (initially $w_{\rm DE}$ is close to -1 and starts to grow).

Quintessence

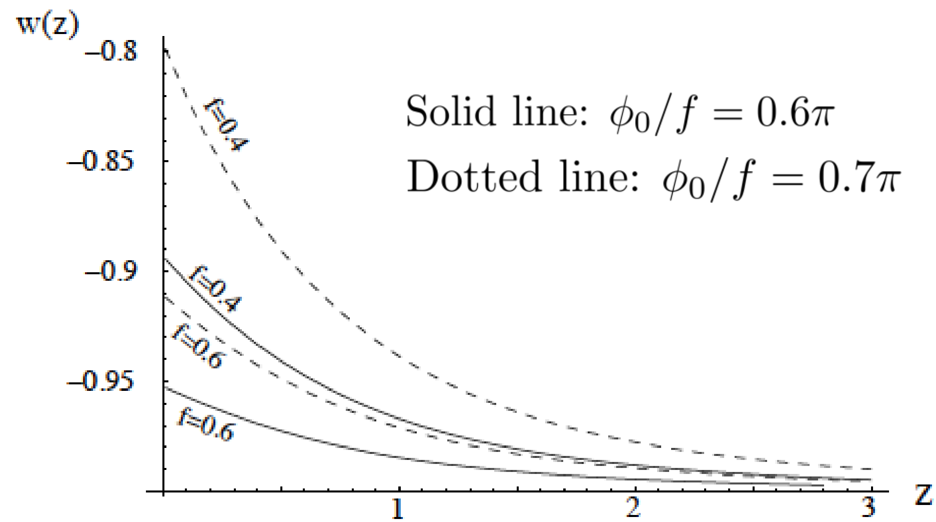
(a) Freezing models

$$V(\phi) = M^{4+n}\phi^{-n} \quad (n=1)$$



(B) Thawing models

$$V(\phi) = \mu^4 [1+\cos(\phi/f)]$$



In general thawing models can satisfy observational constraints easier than freezing models.

Observational constraints on dark energy at the background level

One can constrain the dark energy equation of state from a number of independent observations.

1. Supernovae (type Ia)

Luminosity distance: $d_L(z) = (1+z)\int_0^z \frac{d\tilde{z}}{H(\tilde{z})}$ (for the flat Universe)

2. CMB shift parameter

$$\mathcal{R} = \sqrt{\Omega_m^{(0)}}\int_0^{z_{\rm dec}} \frac{dz}{H(z)/H_0}$$

$z_{\rm dec} \simeq 1090$

$\mathcal{R} = 1.725 \pm 0.018$

(Komatsu et al, 2010)

3. BAO distance

$$r_{\rm BAO}(z) = r_s(z_{\rm drag})/D_V(z)$$

with $D_V(z) = [(1+z)^2 d_A^2(z) z/H(z)]^{1/3}$

$z_{\rm drag} \simeq 1020$

$r_{\rm BAO}(z=0.2) = 0.1905 \pm 0.0061$

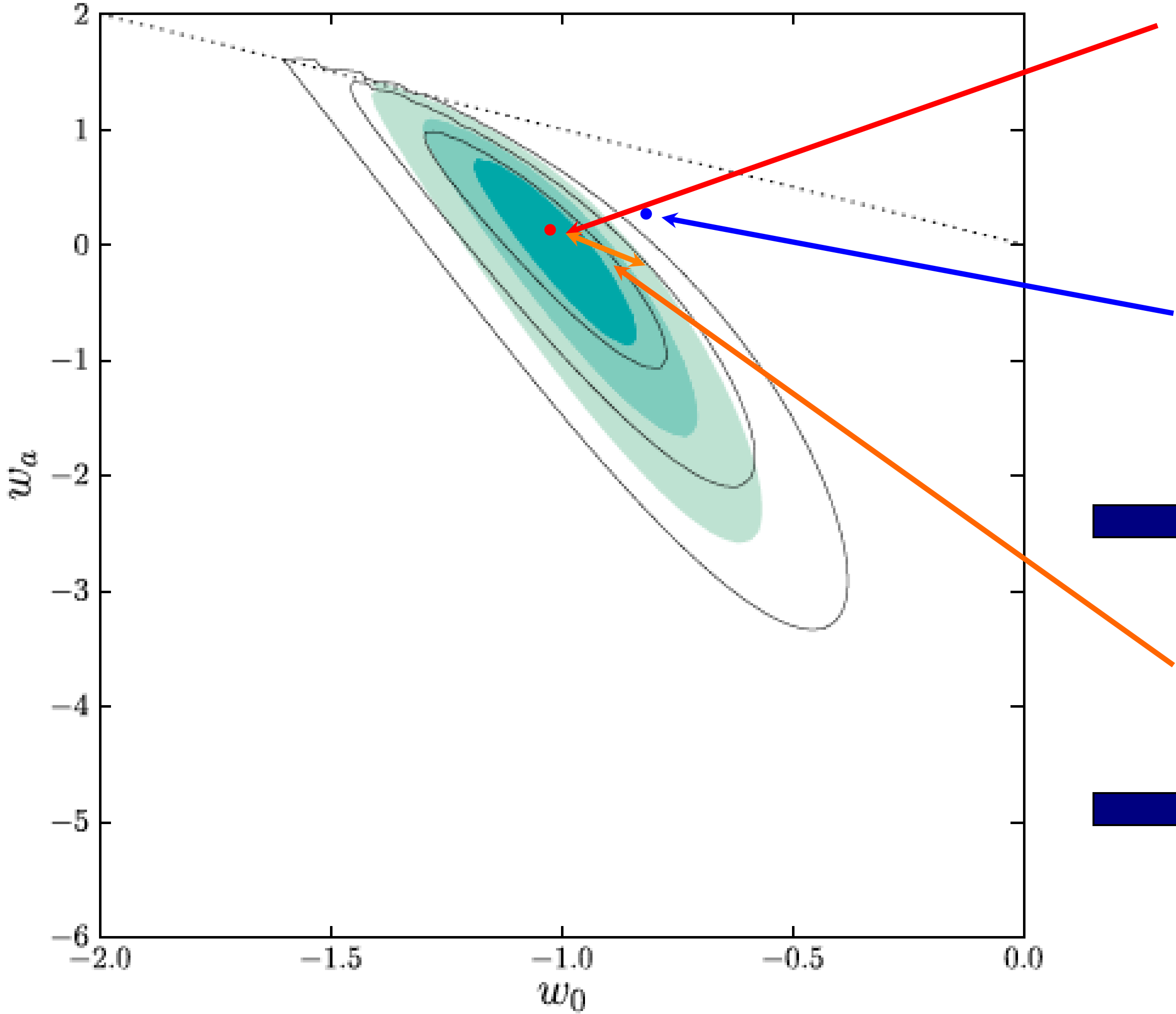
$r_{\rm BAO}(z=0.35) = 0.1097 \pm 0.0036$

(Percival et al, 2010)

Observational constraints (SNIa, CMB, BAO, HST)

Suzuki et al (2011)

Parametrization: $w(a) = w_0 + w_a(1-a)$



LCDM model (within 1 sigma)

Freezing model with

$$V(\phi) = M^5 \phi^{-1}$$

Under observational pressure!

Thawing models

Allowed in current observations

Dark energy equation of state: modified gravity models

(1) f(R) gravity

Viable f(R) dark energy models are constructed to satisfy local gravity constraints in the region of high density.

$$f(R) = R - \lambda R_c \frac{(R/R_0)^{2n}}{(R/R_0)^{2n}+1} \quad (n>0)$$

$$f(R) \simeq R - \lambda R_0[1-(R/R_0)^{-2n}]$$

for $R \gg R_0$

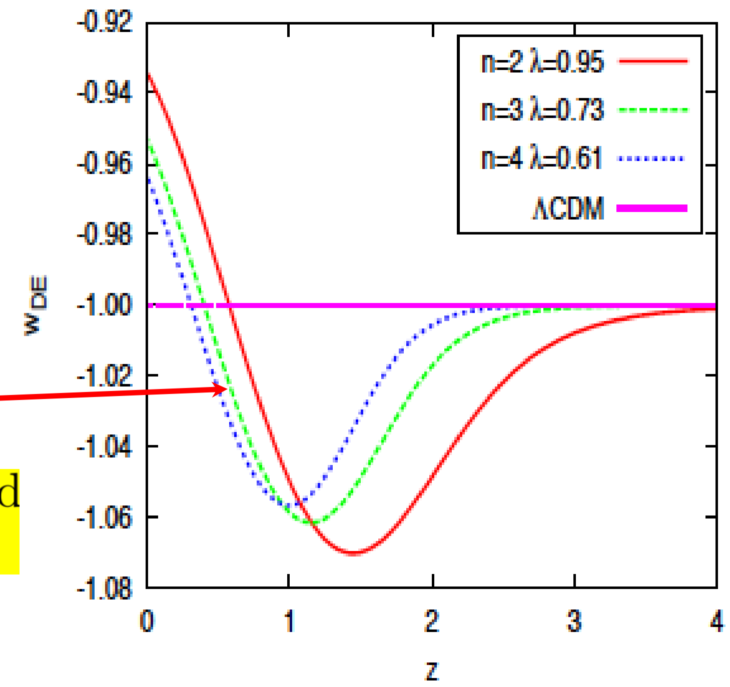
Dark energy equation of state

$$w_{\rm DE} = \frac{w_{\rm eff}}{1-f_{,R}\Omega_m}$$

where

$$w_{\rm eff} = -1 - 2\dot{H}/(3H^2), \quad \Omega_m = \rho_m/(3f_{,R}H^2)$$

$w_{\rm DE} < -1$ without ghosts



Since $w_{\rm DE}$ is close to -1, $f(R)$ gravity is allowed from observations at the background level.

(2) Galileons

In the DGP braneworld model a brane-bending mode ϕ gives rise to a field self-interaction of the form $\Box\phi(\partial^\mu\phi\partial_\mu\phi)$

- But the DGP model is plagued by the ghost problem.
- This problem can be evaded by considering more general field self interactions respecting the Galilean symmetry: $\partial_\mu\phi \to \partial_\mu\phi + b_\mu$

$$K = -c_2 X, \qquad G_3 = \frac{c_3}{M^3}X, \qquad G_4 = \frac{1}{2}M_{\rm pl}^2 - \frac{c_4}{M^6}X^2, \qquad G_5 = \frac{3c_5}{M^9}X^2$$

(in the Horndeski's action)

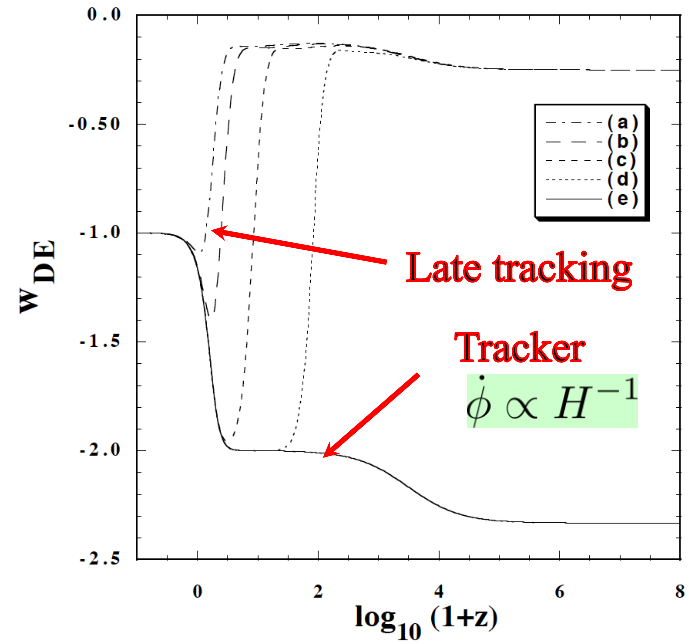
There is a tracker solution with

$$w_{\rm DE} = -2 \ \text{(matter era)}$$

However the tracker is disfavored from the joint data analysis of SNIa, CMB, BAO.

Only the late-time tracking solution is allowed observationally.

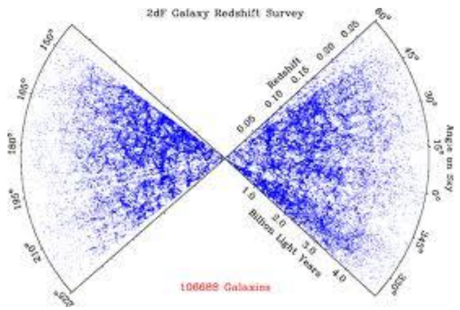
(Nesseris, De Felice, S.T., 2010)



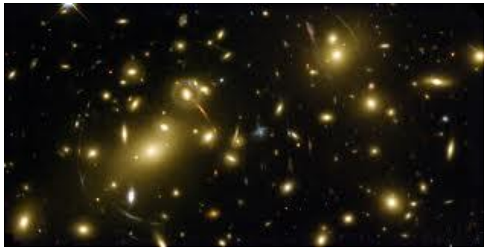
Discrimination of models from density perturbations

In order to place constraints on dark energy models from the observations of large-scale structure, weak lensing, CMB (ISW effect) etc, we need to study the evolution of density perturbations.

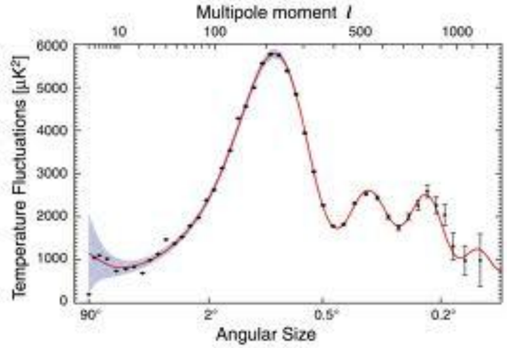
LSS



Weak lensing



CMB



Perturbed metric: $ds^2 = -(1+2\Psi)dt^2 + a^2(t)(1+2\Phi)\delta_{ij}dx^i dx^j$

Non-relativistic matter: $\rho_m = \rho_m(t) + \delta\rho_m(t, \mathbf{x})$

with the four velocity $u^\mu = (1-\Psi, \nabla^i v)$

v is the rotational-free velocity potential.

Density perturbations in the Horndeski's theory

$\delta \equiv \delta\rho_m/\rho_m$ and $\theta \equiv \nabla^2 v$ obey

$$\dot{\delta} = -\theta/a - 3\dot{\Phi}$$

$$\dot{\theta} = -H\theta + (k^2/a)\Psi$$

The growth rate of matter perturbations is related with the peculiar velocity.

We introduce the gauge-invariant density contrast: $\delta_m \equiv \delta + \frac{3aH}{k^2}\theta$

$$\ddot{\delta}_m + 2H\dot{\delta}_m + \frac{k^2}{a^2}\Psi = 3\left(\ddot{I} + 2H\dot{I}\right)$$

where $I \equiv (aH/k^2)\theta - \Phi$

The two gravitational potentials Φ and $-\Psi$ are generally different:

$$B_6\Phi + B_8\Psi = -B_7\delta\phi$$

There are other perturbation equations. See De Felice, Kobayashi, S.T. (2011).

where

$$B_6 = 4[G_4 - X(\ddot{\phi}G_{5,X} + G_{5,\phi})]$$

$$B_8 = 4[G_4 - 2XG_{4,X} - X(H\dot{\phi}G_{5,X} - G_{5,\phi})]$$

$$B_7 = -4G_{4,X}H\dot{\phi} - 4(G_{4,X} + 2XG_{4,XX})\ddot{\phi} + 4G_{4,\phi} - 8XG_{4,\phi X} + 4(G_{5,\phi} + XG_{5,\phi X})\ddot{\phi}$$
$$-4H[(G_{5,X} + XG_{5,XX})\ddot{\phi} - G_{5,\phi} + XG_{5,\phi X}]\dot{\phi} + 4X[G_{5,\phi\phi} - (H^2 + \dot{H})G_{5,X}]$$

In GR ($G_4 = M_{\rm pl}^2/2$) one has $B_6 = B_8 = 2M_{\rm pl}^2$ and $B_7 = 0$. $\Rightarrow$ $\Phi = -\Psi$

Quasi-static approximation on sub-horizon scales

De Felice, Kobayashi, S.T. (2011).

For the modes deep inside the Hubble radius ($k \gg aH$) we can employ the quasi-static approximation under which the dominant terms are those including k^2/a^2, δ_m, and $M^2 \equiv -K_{,\phi\phi}$.

$$\ddot{\delta}_m + 2H\dot{\delta}_m + \frac{k^2}{a^2}\Psi \simeq 0 \quad \text{and} \quad \frac{k^2}{a^2}\Psi \simeq -4\pi G_{\rm eff}\rho_m\delta_m$$

where the effective gravitational coupling $G_{\rm eff}$ is

$$G_{\rm eff} = \frac{2M_{\rm pl}^2[(B_6D_9 - B_7^2)(k/a)^2 - B_6M^2]}{(A_6^2B_6 + B_8^2D_9 - 2A_6B_7B_8)(k/a)^2 - B_8^2M^2}G$$

where

$$A_6 = -2XG_{3,X} - 4H(G_{4,X} + 2XG_{4,XX})\dot{\phi} + 2G_{4,\phi} + 4XG_{4,\phi X} + 4H(G_{5,\phi} + XG_{5,\phi X})\dot{\phi} - 2H^2X(3G_{5,X} + 2XG_{5,XX})$$

$$D_9 = -K_{,X} + \text{derivative terms of } G_3, G_4, G_5$$

In GR, $G_4 = M_{\rm pl}^2/2$, $B_6 = B_8 = 2M_{\rm pl}^2$, $A_6 = B_7 = 0$, $D_9 = -K_{,X}$ $\Rightarrow$ $G_{\rm eff} = G$

In the massive limit ($M^2 \to \infty$) with $B_6 \simeq B_8 \simeq 2M_{\rm pl}^2$ we also have $G_{\rm eff} \simeq G$

In the massless limit $M^2 \to 0$ we have

$$G_{\rm eff} = \frac{2M_{\rm pl}^2(B_6D_9 - B_7^2)}{A_6^2B_6 + B_8^2D_9 - 2A_6B_7B_8}G$$

$\Rightarrow$ The effect of modified gravity manifests itself.

Gravitational potentials

We introduce the parameter $\eta \equiv -\Phi/\Psi$ between two gravitational potentials. On sub-horizon scales this ratio can be estimated as

$$\eta \simeq \frac{(B_8 D_9 - A_6 B_7)(k/a)^2 - B_8 M^2}{(B_6 D_9 - B_7^2)(k/a)^2 - B_6 M^2}$$

⟹ $\eta = 1$ for GR $(A_6 = 0)$.

The effective gravitational potential associated with the deviation of light rays in CMB and weak lensing observations is

$$\Phi_{\rm eff} \equiv (\Psi - \Phi)/2$$

On sub-horizon scales we have

$$\Phi_{\rm eff} \simeq -\frac{3}{2} \frac{G_{\rm eff}}{G} \frac{1+\eta}{2} \left(\frac{aH}{k}\right)^2 \Omega_m \delta_m$$

where $\Omega_m \equiv \frac{\rho_m}{3M_{\rm pl}^2 H^2}$

Modified ($\frac{G_{\rm eff}}{G} \frac{1+\eta}{2}$) Modified ($\delta_m$)

- In $f(R)$ gravity and Brans-Dicke theories $(G_{\rm eff}/G)(1+\eta)/2 = 1$

  ⟹ $\Phi_{\rm eff}$ is directly affected by the modified evolution of δ_m.

- In Galileon gravity $(G_{\rm eff}/G)(1+\eta)/2 \neq 1$.

  ⟹ δ_m is not the only source for the modified evolution of $\Phi_{\rm eff}$.

Brans-Dicke theories (including f(R) gravity)

$$K = M_{\rm pl}\omega_{\rm BD}X/\phi - V(\phi)\,, \qquad G_3 = G_5 = 0\,, \qquad G_4 = M_{\rm pl}\phi/2$$

where $\omega_{\rm BD}$ is the Brans-Dicke parameter.

On sub-horizon scales one has

$$G_{\rm eff} = \frac{M_{\rm pl}}{\phi}\frac{4+2\omega_{\rm BD}+2(\phi/M_{\rm pl})(Ma/k)^2}{3+2\omega_{\rm BD}+2(\phi/M_{\rm pl})(Ma/k)^2}G\,, \qquad \eta = \frac{1+\omega_{\rm BD}+(\phi/M_{\rm pl})(Ma/k)^2}{2+\omega_{\rm BD}+(\phi/M_{\rm pl})(Ma/k)^2}$$

- In the early cosmological epoch where $M^2 \gg k^2/a^2$ one has $G_{\rm eff} \simeq G$ and $\eta \simeq 1$.

  ⇒ $\delta_m \propto t^{2/3}\,, \quad \Phi_{\rm eff} = {\rm const.}$ during the matter era (as in GR).

- In the late cosmological epoch where $M^2 \ll k^2/a^2$, one has

$$G_{\rm eff} \simeq \frac{M_{\rm pl}}{\phi}\frac{4+2\omega_{\rm BD}}{3+2\omega_{\rm BD}}G, \qquad \eta \simeq \frac{1+\omega_{\rm BD}}{2+\omega_{\rm BD}}$$

In $f(R)$ gravity ($\omega_{\rm BD}=0$), $G_{\rm eff} \simeq 4G/(3f_{,R})$, $\eta \simeq 1/2$.

⇒ $\delta_m \propto t^{(\sqrt{33}-1)/6}, \quad \Phi_{\rm eff} \propto t^{(\sqrt{33}-5)/6}$

during the matter era (enhanced growth).

Constraints from large-scale structure

The galaxy perturbation δ_g is related with δ_m via the bias factor b, i.e., $\delta_g = b\delta_m$.

$\theta = \nabla^2 v$ is related with $f_m \equiv \dot{\delta_m}/(H\delta_m)$ via

$$\theta/(aH) \simeq -f_m \delta_m$$

The galaxy power spectrum in the redshift space can be modeled as

$$\mathcal{P}_g^s(\mathbf{k}) = \mathcal{P}_{gg}(\mathbf{k}) + 2\mu^2 \mathcal{P}_{g\theta}(\mathbf{k}) + \mu^4 \mathcal{P}_{\theta\theta}(\mathbf{k})$$

μ is the cosine of the angle of the $\boldsymbol{k}$ vector and along the line of sight.

The real space galaxy power spectrum: $(b\sigma_8)^2$

The cross power spectrum: $(b\sigma_8)(f_m\sigma_8)$

The real space velocity power spectrum: $(f_m\sigma_8)^2$

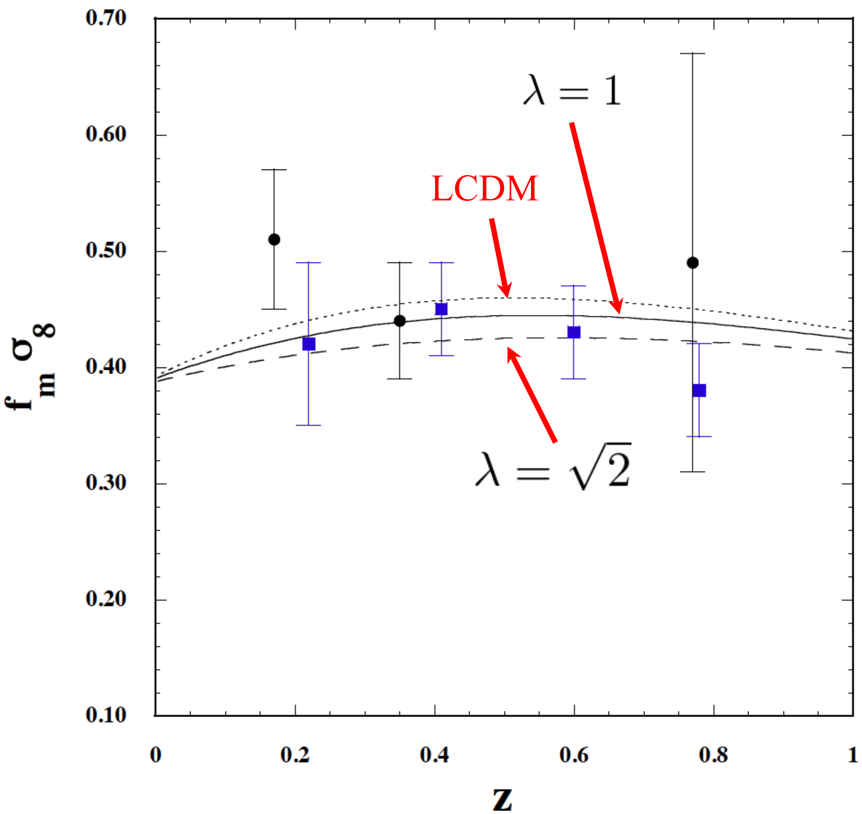
σ_8 is the rms mass fluctuations in spheres within the radius $8\,h^{-1}$ Mpc^{-1}.

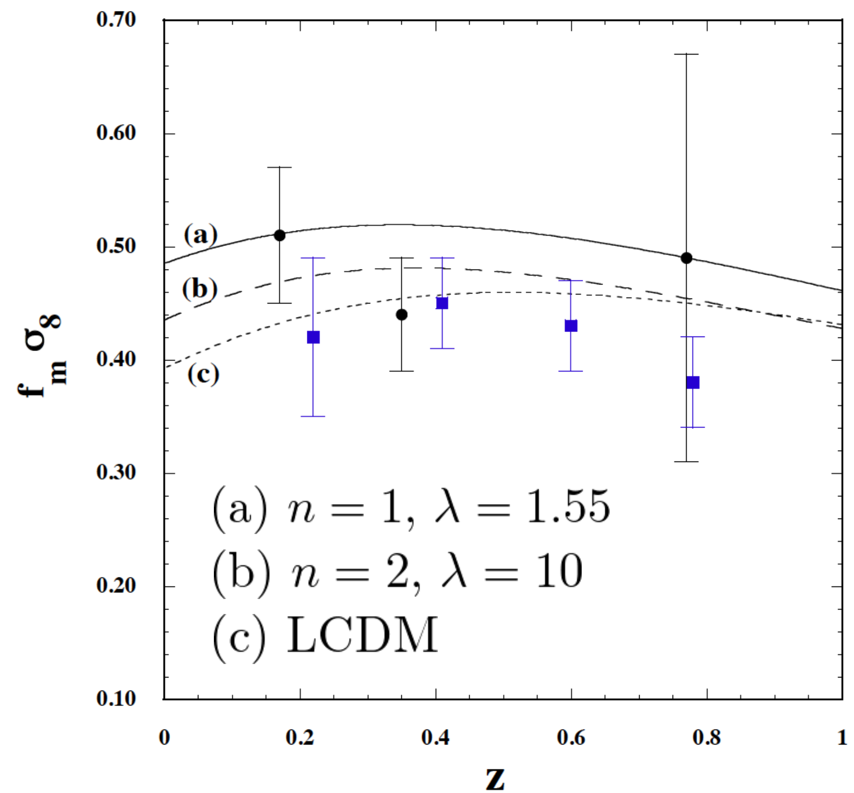
The redshift space distorsions are known as an additive component by observing $b\sigma_8$ and $f_m\sigma_8$.

Constraints from $f_m\sigma_8$

Quintessence with
$V(\phi) = V_0 e^{-\lambda\phi/M_{\rm pl}}$

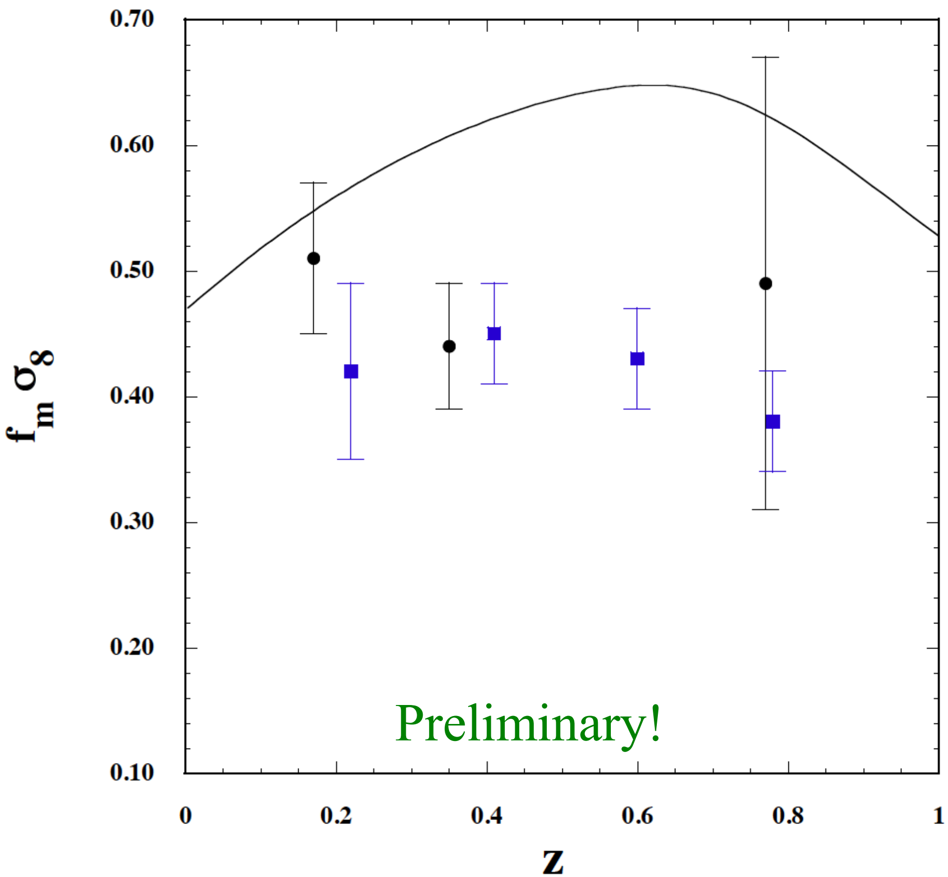
$$f(R) = R - \lambda R_c \frac{(R/R_0)^{2n}}{(R/R_0)^{2n}+1} \quad (n>0)$$





Blue data come from WiggleZ (Blake et al, 2011)

Covariant Galileon (late-time tracking solution)



Because of the large growth rate of δ_m as well as the large anisotropic parameter η the Galileon seems to be disfavored from the recent data.

Summary of current status of dark energy models

(1) Cosmological constant

Observationally favored, but theoretically further progress is required.

(2) Modified matter models

- Quintessence, k-essence: Thawing models are fine, but the freezing models tend to be in tension with observations.
- Chaplygin gas $(P = -A/\rho)$ ✕ Excluded from the observations of large-scale structure.

(3) Modified gravity models

- f(R) gravity, Brans-Dicke theories: The cases with largest deviation from GR are in tension with recent observations.
- DGP braneworld: ✕ Ruled out from the observations and the ghost problem.
- Covariant Galileon: It is likely that the future LSS observations can rule out this model.