Exotic Branes

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Kobayashi-Maskawa Institute
for the Origin of Particles and the Universe

Nagoya Univeristy Global COE Program

International Spring School 2011

Gauge Theory, Gravity, and String Theory

Date: March 21-24, 2011
Place: School of Science, Nagoya University
Web: site:http://www.gcoe.phys.nagoya-u.ac.jp/ggs2011/

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Feb 20

Oren Bergman:
On M2-branes

Sumit Das:
Time-dependent processes in AdS/CFT

Nadav Drukker:
Wilson loops in gauge and string theory

Romuald Janik:
QGP and AdS/CFT — hydrodynamics and beyond

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Introduction

Duality in string/M-theory

- Relates various objects in string / M-theory

$$T\colon\ \mathrm{D}p \leftrightarrow \mathrm{D}(p+1),\ \mathrm{F1} \leftrightarrow \mathrm{P},\ \mathrm{NS5} \leftrightarrow \mathrm{KKM},\ \dots$$

$$S\colon\ \mathrm{F1} \leftrightarrow \mathrm{D1},\ \mathrm{NS5} \leftrightarrow \mathrm{D5},\ \dots$$

U-duality

- Enhances in lower dims.
  - M-theory on T^k: $E_{k(k)}(\mathbb{Z})$

[Cremmer+Julia, & others] [Hull+Townsend]

k	D	$G(\mathbb{Z})$
1	10	1
2	9	$SL(2,\mathbb{Z}) \times \mathbb{Z}_2$
3	8	$SL(3,\mathbb{Z}) \times SL(2,\mathbb{Z})$
4	7	$O(5,5,\mathbb{Z})$
5	6	$SL(5,\mathbb{Z})$
6	5	$E_{6(6)}(\mathbb{Z})$
7	4	$E_{7(7)}(\mathbb{Z})$
8	3	$E_{8(8)}(\mathbb{Z})$

Codim-2 objects and U-duality

- U-duality on codim-2 objects produces exotic states

Known codim-2 object → (U-duality) → Exotic states

Known 10D/11D object wrapped on internal torus

10D/11D origin unclear!

They can have mass $\sim g_s^{-3}, g_s^{-4}$

[9707217 Elitzur+Giveon+Kutasov+Rabinovici]
[9712047 Blau+O'Loughlin]
[9809039 Obers+Pioline]

Codim-2 objects: example

- Example: Type II on T^2

	1	2	3	4	5	6	7	8	9
NS5	·	·	O	O	O	O	O	~	~

(T^2 along directions 8, 9)

↓ T-duality along x^8, x^9

"5^2_2"

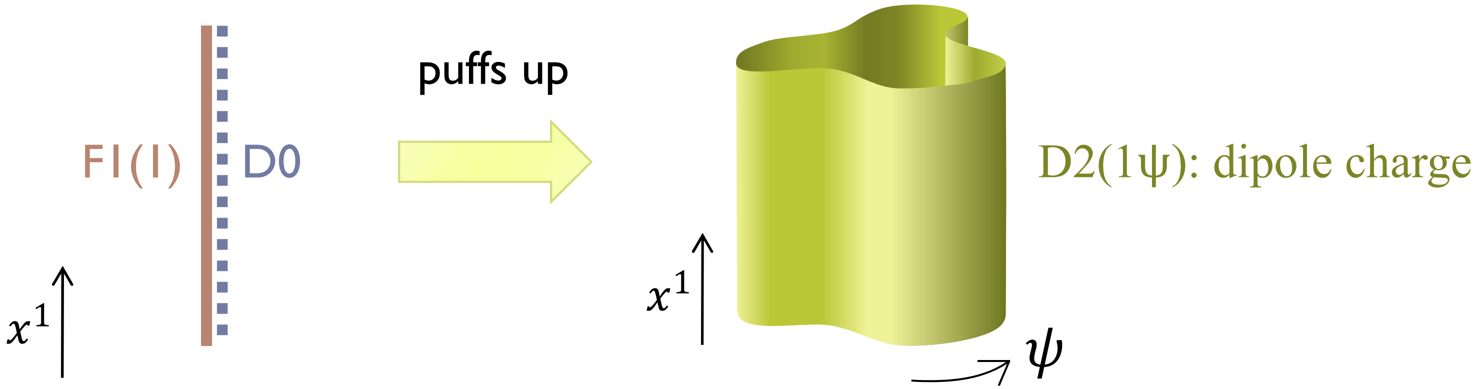
We will see that
this is a "T-fold".

Supertube effect

- Codim-2 object problematic
  - Log divergences

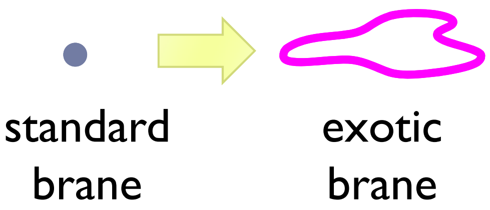
$$V \sim \frac{1}{r^{d-2}} \quad \xrightarrow{d=2} \quad V \sim \log\left(\frac{\mu}{r}\right)$$

- Are they relevant? Why care?
- Supertube effect = spontaneous polarization [Mateos+Townsend]



Importance of exotic branes

- Non-exotic branes can puff up to produce exotic *dipole* charges



  → No log divergence

  → Exotic branes are relevant to non-exotic physics;
  More common than previously thought!

- Black holes: bound states of branes

  → Generic microstates involve exotic charges

  → Microstate (non-)geometries?

Outline

- Introduction ✓
- Exotic states & their higher-D origin
- Sugra description
- Supertube effect
- Black hole microstates
- Conclusion

Exotic states and their higher-D origin

Compactification to 3D

- M on T^8 or Type II on T^7

    → 3D $\mathcal{N} = 16$ sugra

    → U-duality group $E_{8(8)}(\mathbb{Z})$: generated by T- and S-dualities

    → 128 moduli scalars (in 3D, scalar = vector) $\in SO(16)\backslash \mathrm{E}_{8(8)}(\mathbb{R})/E_{8(8)}(\mathbb{Z})$

- Particle multiplet:

    → Start from a point-like object

    e.g. D7(3456789) wrapped on T^7

    → Take T- and S-dualities to get other states

Exotic states in 3D

- Particle multiplet:

[9707217 Elitzur+Giveon+Kutasov+Rabinovici]
[9712047 Blau+O'Loughlin]
[9809039 Obers+Pioline]

Type IIA	P ($\mathbf{7}$), F1 ($\mathbf{7}$), D0 ($\mathbf{1}$), D2 ($\mathbf{21}$), D4 ($\mathbf{35}$), D6 ($\mathbf{7}$), NS5 ($\mathbf{21}$), KKM ($\mathbf{42}$), 5^2_2 ($\mathbf{21}$), 0^7_3 ($\mathbf{1}$), 2^5_3 ($\mathbf{21}$), 4^2_3 ($\mathbf{35}$), 6^1_3 ($\mathbf{7}$), $0^{(1,6)}_4$ ($\mathbf{7}$), 1^6_4 ($\mathbf{7}$)
Type IIB	P ($\mathbf{7}$), F1 ($\mathbf{7}$), D1 ($\mathbf{7}$), D3 ($\mathbf{35}$), D5 ($\mathbf{21}$), D7 ($\mathbf{1}$), NS5 ($\mathbf{21}$), KKM ($\mathbf{42}$) , 5^2_2 ($\mathbf{21}$), 1^6_3 ($\mathbf{7}$), 3^4_3 ($\mathbf{35}$), 5^2_3 ($\mathbf{21}$), 7_3 ($\mathbf{1}$), $0^{(1,6)}_4$ ($\mathbf{7}$), 1^6_4 ($\mathbf{7}$)
M-theory	P ($\mathbf{8}$), M2 ($\mathbf{28}$), M5 ($\mathbf{56}$), KKM ($\mathbf{56}$), 5^3 ($\mathbf{56}$), 2^6 ($\mathbf{28}$), $0^{(1,7)}$ ($\mathbf{8}$)

- Notation for exotic states

$$b^c_n : M = \frac{R^b (R^c)^2}{g_s^n} \qquad b^{(d,c)}_n : M = \frac{R^b (R^c)^2 \left(R^d\right)^3}{g_s^n}$$

<u>Example:</u> $5^2_2(34567,89)$: $M = \dfrac{R_3 \cdots R_7\,(R_8 R_9)^2}{g_s^2 l_s^8}$

Duality rules

- Duality rules can be read off from:

$$T_y: \quad R_y \to \frac{l_s^2}{R_y}, \quad g_s \to \frac{l_s}{R_y} g_s \qquad\qquad S: \quad g_s \to \frac{1}{g_s}, \quad l_s \to g_s^{1/2} l_s$$

- Example:

$$\mathrm{NS5}(34567) \xrightarrow{T_8} \mathrm{KKM}(34567,8) \xrightarrow{T_9} 5_2^2(34567,89)$$

$$M = \frac{R_3 \cdots R_7}{g_s^2 l_s^6} \xrightarrow{T_8} \frac{R_3 \cdots R_7}{(g_s l_s/R_8)^2 l_s^6} = \frac{R_3 \cdots R_7 R_8^2}{g_s^2 l_s^8} \quad : 5_2^1 = \mathrm{KKM}$$

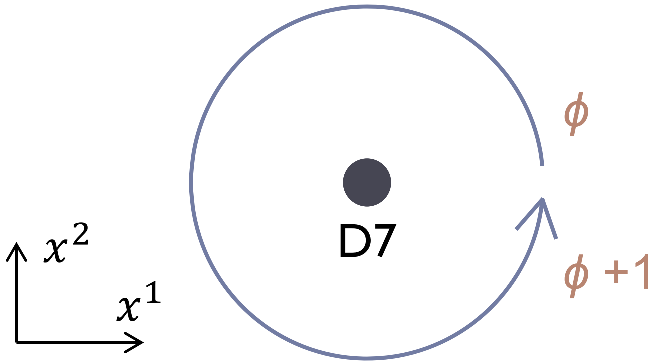
$$\xrightarrow{T_9} \frac{R_3 \cdots R_7 R_8^2}{(g_s l_s/R_9)^2 l_s^8} = \frac{R_3 \cdots R_7 (R_8 R_9)^2}{g_s^2 l_s^{10}} \quad : 5_2^2$$

Higher D origin = U-folds (1)

- Claim: higher D origin is
  U-fold = non-geometric background

E.g. D7 on T^7

- (Magnetically) coupled to RR 0-form C_0
- 3D scalar $\phi = C_0$
- Monodromy: $\phi \rightarrow \phi + 1$ shift (part of $SL(2,\mathbb{Z})$ duality of IIB)



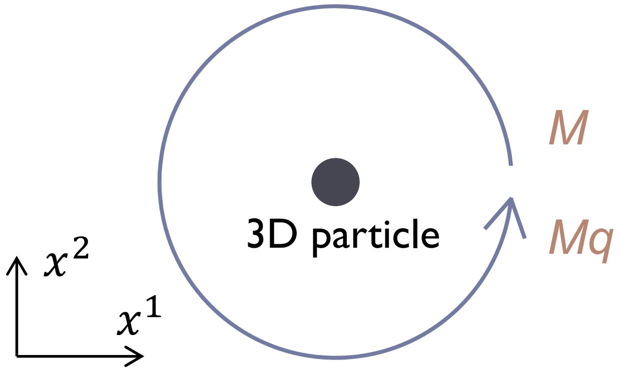
Higher D origin = U-folds (2)

- ϕ gets combined with other scalars to form moduli matrix

$$M \in \mathcal{M} = SO(16)\backslash E_{8(8)}(\mathbb{R})/E_{8(8)}(\mathbb{Z})$$

- Shifting symmetry + S, T-dualities ⟹ $E_{8(8)}(\mathbb{Z})$
- Can consider a particle with general U-duality monodromy

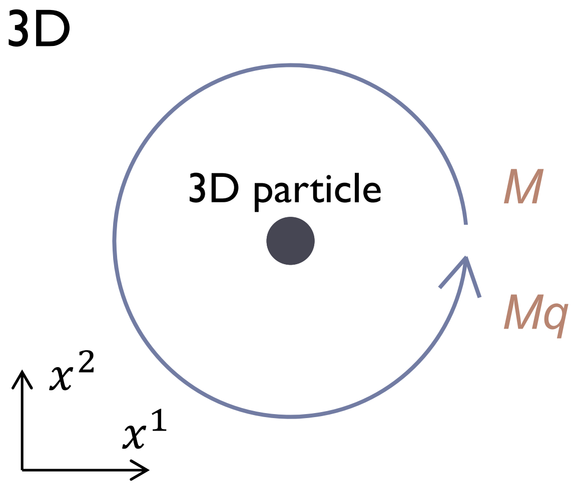
$$q \in E_{8(8)}(\mathbb{Z}) \equiv G(\mathbb{Z})$$

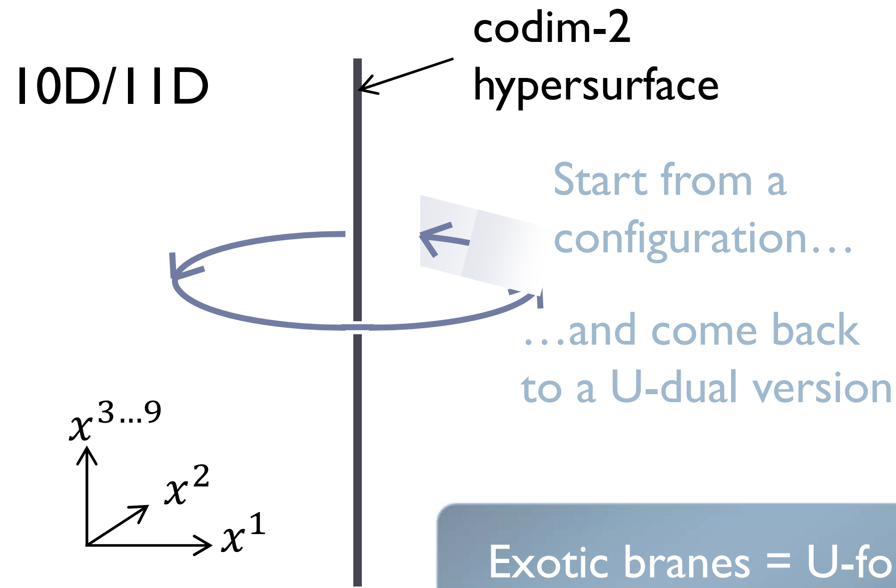


"Charge" of a 3D particle is U-duality monodromy around it!

Higher D origin = U-folds (3)

- In 10D/11D, we have a non-geometric U-fold





Exotic branes = U-folds

Cf. F-theory, U-branes & non-geom bg

[Greene+Shapere+Vafa+Yau] [Vafa] F-theory
[Kumar+Vafa] [Liu+Minasian] [Hellerman+McGreevy+Williams] contractible U-branes
[Dabholkar+Hull] [Flournoy+Wecht+Williams] … non-contractible U-branes & moduli stabl'n

Lesson 1:

Exotic branes

= Non-geometric U-folds

Sugra description of exotic states

Sugra solution for 5^2_2 (1)

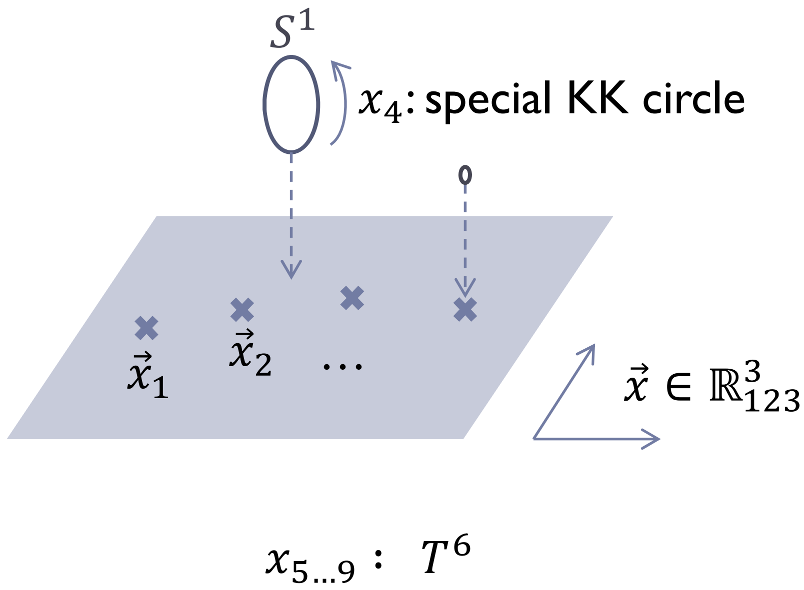
Want to make use of $KKM(56789,4) \xrightarrow[T_3]{} 5^2_2(56789,34)$

KKM(56789,4):

$$ds^2 = -dt^2 + Hd\vec{x}^2$$
$$+H^{-1}(dx_4 + \omega)^2 + dx^2_{56789}$$

$$e^{2\Phi} = 1, \qquad d\omega = *_3\, dH$$

$$H = 1 + \sum_p H_p\,, \qquad H_p = \frac{R_4}{2|\vec{x} - \vec{x}_p|}$$



Sugra solution for 5^2_2 (1)

compactify x_3 = array centers along x_3

$$H(r) = \sum_p \frac{R_4}{2|\vec{x} - \vec{x}_p|} \approx h + \sigma \log\left(\frac{\mu}{r}\right)$$

[Sen] [Blau+O'Loughlin]

T-dualize along x_3 (Buscher rule)

Sugra solution for 5^2_2 (2)

$5^2_2(56789,34)$ metric:

$$ds^2 = -dt^2 + H(dr^2 + r^2 d\theta^2) + HK^{-1} dx_{34}^2 + dx_{56789}^2$$

$$B_{34} = -K^{-1}\theta\sigma, \qquad e^{2\Phi} = HK^{-1}, \qquad K \equiv H^2 + \sigma^2\theta^2$$

r, θ: $\mathbb{R}^2$

$x^{3,4}$: T^2

$x^{5\ldots9}$: T^6

$$H(r) = h + \sigma \log\left(\frac{\mu}{r}\right) \qquad \sigma = \frac{R_3 R_4}{2\pi\alpha'}$$

Cf. [Blau+O'Loughlin]: 6^1_3

- T-fold structure:

$$\theta = 0: \quad G_{33} = G_{44} = H^{-1},$$

$$\theta = 2\pi: \quad G_{33} = G_{44} = \frac{H}{H^2 + (2\pi\sigma)^2}$$

→ x_3-x_4 torus size doesn't come back to itself!

T-fold structure of 5_2^2

- Package 3-4 part of G, B:

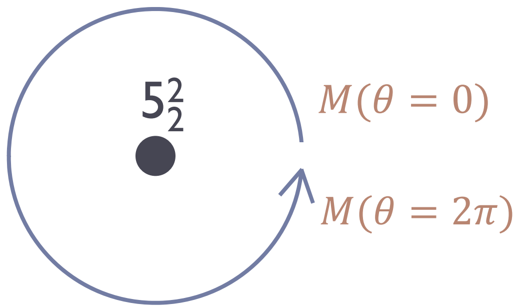
$$M = \begin{pmatrix} G^{-1} & G^{-1}B \\ -BG^{-1} & G - BG^{-1}B \end{pmatrix}$$

T-duality acts as

$$M \to M' = \Omega^t M \Omega, \qquad \Omega \in SO(2,2,\mathbb{R})$$

T-duality monodromy around 5_2^2:

$$\Omega = \begin{pmatrix} 1 & 0 \\ 2\pi\sigma & 1 \end{pmatrix}: \qquad M(\theta = 0) \to M(\theta = 2\pi)$$



Comments

- Not well-defined as stand-alone objects
  - Log divergence
    - → Superpositions (cf. F-theory 7-branes)
    - → Configs with higher codims. (next topic!)
- Easy to get sugra metric for other exotic branes
  - Questionable for states with $M \sim {g_s}^{-3}, {g_s}^{-4}$

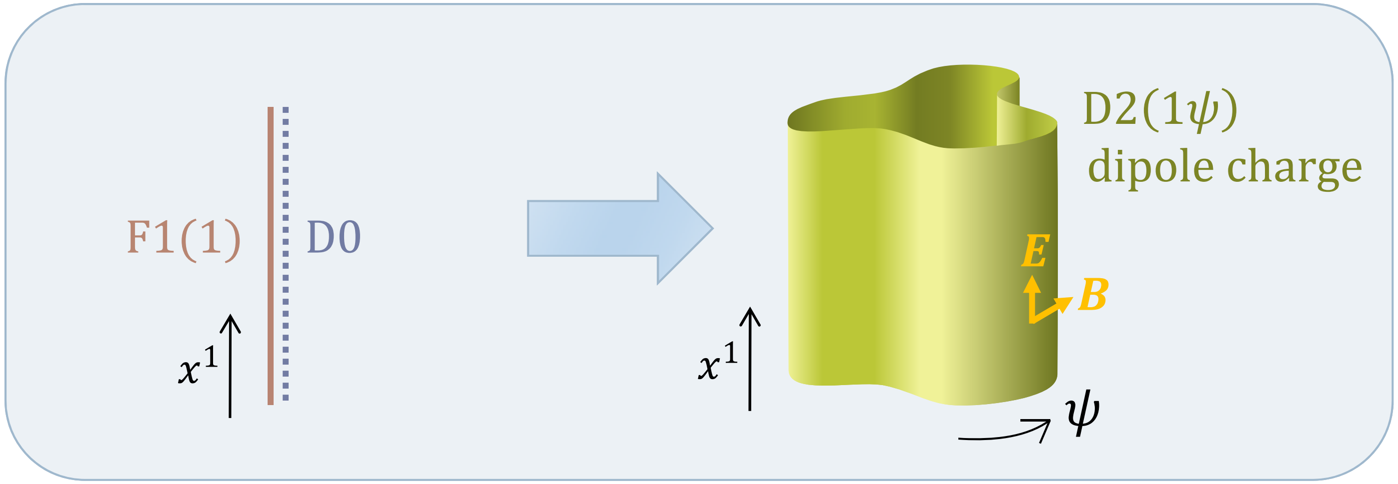
Supertube effect and exotic branes

Supertube effect

"puffs up"

$$\mathrm{D0} + \mathrm{F1}(1) \Rightarrow \mathrm{D2}(1\psi)$$

[Mateos+Townsend]



- Spontaneous polarization effect (cf. Myers effect)
- Produces new dipole charge
- Cross section = arbitrary curve
- Fluctuations of curve → large degeneracy $\sim e^{\#\sqrt{N_{F1}N_{D0}}}$

Dualizing supertube effect

Original supertube effect:

$$\mathrm{D0} + \mathrm{F1}(1) \rightarrow \mathrm{D2}(1\psi)$$

dualize!

Various other known puff-ups:

$$\mathrm{F1}(1) + \mathrm{P}(1) \rightarrow \mathrm{F1}(\psi)$$ FP sys

$$\mathrm{D1}(1) + \mathrm{D5}(12345) \rightarrow \mathrm{KKM}(2345\psi, 1)$$ LM geom

$$\mathrm{M2}(12) + \mathrm{M2}(34) \rightarrow \mathrm{M5}(1234\psi)$$ black ring

"Exotic" puff-ups

$$\mathrm{D0} + \mathrm{F1}(1) \to \mathrm{D2}(1\psi)$$

dualize

Exotic puff-up:

$$\mathrm{D4}(6789) + \mathrm{D4}(4589) \to 5_2^2(4567\psi, 89)$$

More exotic:

$$\mathrm{D3}(589) + \mathrm{NS5}(46789) \to 5_3^2(4567\psi, 89)$$

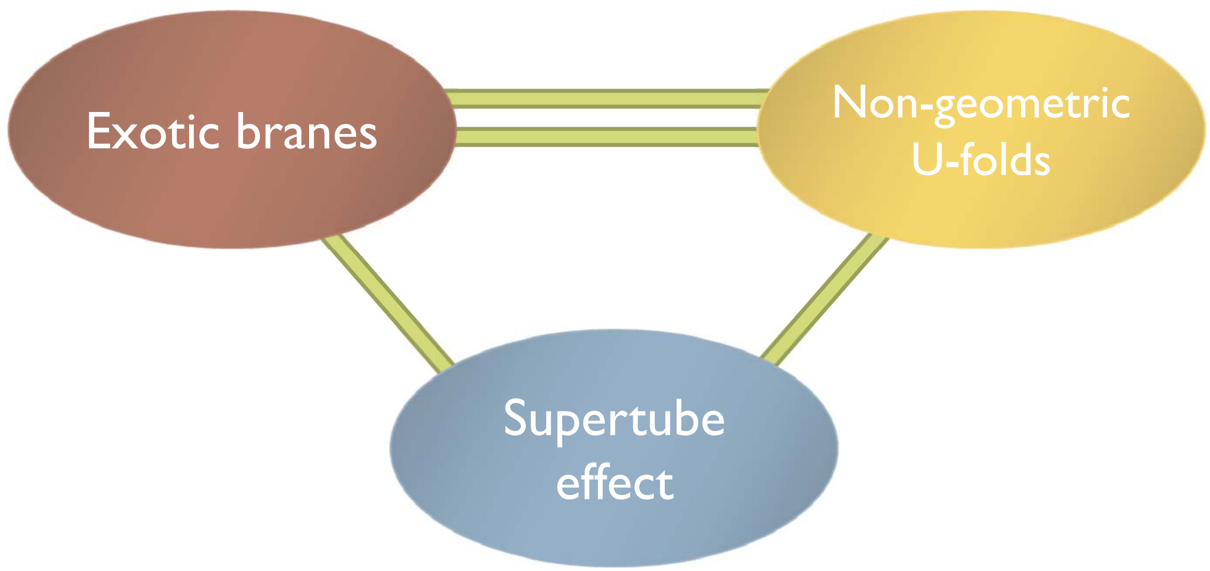
Still more exotic:

$$\mathrm{NS5}(46789) + \mathrm{KKM}(46789{,}5) \to 1_4^6(\psi, 456789)$$

- Standard branes can polarize into exotic branes
- Only dipoles → no log divergence

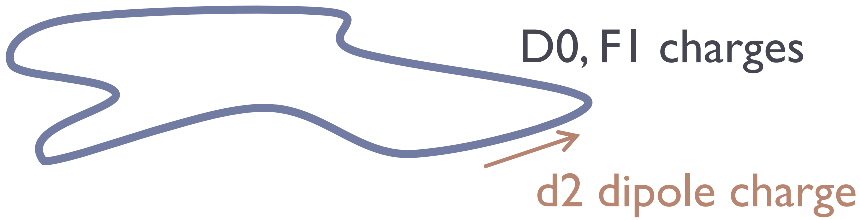
Lesson 2:

Exotic branes are important
for generic physics of string theory



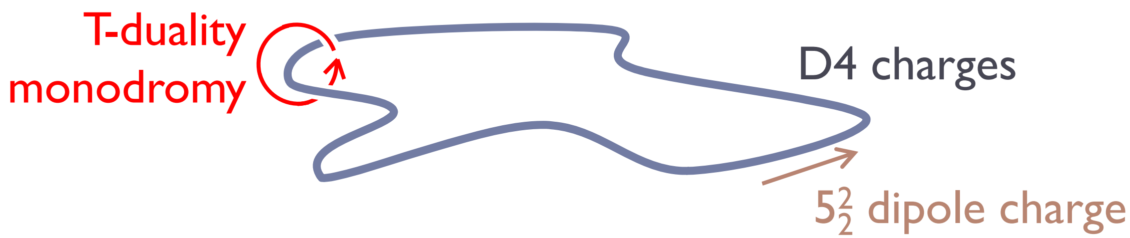
Example: D4+D4→5^2_2

- Basic sugra supertube



dualize

- Exotic 2-charge solution



Metric for D4+D4→5_2^2

D4(6789)+D4(4589)→5_2^2(4567ψ,89)

$$ds^2 = -\frac{1}{\sqrt{f_1 f_2}}(dt - A)^2 + \sqrt{f_1 f_5}\, dx_{123}^2 + \sqrt{\frac{f_1}{f_2}} dx_{45}^2 + \sqrt{\frac{f_2}{f_1}} dx_{67}^2 + \frac{\sqrt{f_1 f_2}}{f_1 f_2 + \gamma^2}\, dx_{89}^2,$$

f_i , A : sourced along curve

$$f_1 = 1 + \frac{Q_1}{L}\int_0^L \frac{dv}{|\vec{x} - \vec{F}(v)|}, \quad f_2 = 1 + \frac{Q_1}{L}\int_0^L \frac{\left|\dot{\vec{F}}(v)\right|^2}{|\vec{x} - \vec{F}(v)|} dv, \quad A_i = -\frac{Q_1}{L}\int_0^L \frac{\dot{F}_i(v)}{|\vec{x} - \vec{F}(v)|}$$

$$d\gamma = *_3\, dA, \quad d\beta_I = *_3\, df_I$$

- γ , β_i have monodromy around curve

$$\gamma \to \gamma - 2q, \quad \beta_I \to \beta_I - 2Q_I$$ → T-fold structure just as before

- Asymptotically flat 4D

Metric for D4+D4→5^2_2

Other fields:

$$e^{2\Phi} = \frac{\sqrt{f_1 f_2}}{f_1 f_2 + \gamma^2}, \qquad B^{(2)}_{89} = \frac{\gamma}{f_1 f_2 + \gamma^2}, \qquad C^{(3)} = -\gamma\rho + \sigma$$

$$\rho = (f_2^{-1} + dt - A) \wedge dx^4 \wedge dx^5 + (f_1^{-1} + dt - A) \wedge dx^6 \wedge dx^7$$

$$\sigma = (\beta_1 - \gamma dt) \wedge dx^4 \wedge dx^5 + (\beta_2 - \gamma dt) \wedge dx^6 \wedge dx^7$$

Circular D4+D4→5^2_2

For circular profile, all functions can be explicitly written down

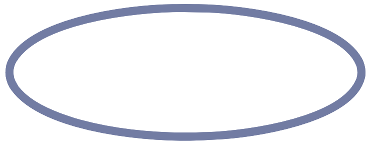
$$dx_{123}^2 = \frac{R^2}{(\cos\phi - y)^2}\left[\frac{dy^2}{y^2-1} + (y^2-1)d\psi^2 + d\phi^2\right]$$

$$f_I = 1 + \frac{Q_I}{R}\sqrt{\frac{cos\,\phi - y}{-2y}}\,F\left(\frac{1}{4},\frac{3}{4};1;z^2\right),\quad A_\psi = -\frac{qR}{2}\frac{y^2-1}{(cos\,\phi - y)^{1/2}(-2y)^{3/2}}\,F\left(\frac{3}{4},\frac{5}{4};2;z^2\right)$$

$$\gamma = -\frac{q\sqrt{1-y}}{4\sqrt{2}(-y)^{3/2}}\left\{(1+y)\mathbf{F}\left(\frac{\phi}{2}\middle|\frac{2}{1-y}\right)F\left(\frac{3}{4},\frac{5}{4};2;z^2\right)\right.$$
$$\left. + u\mathbf{E}\left(\frac{\phi}{2}\middle|\frac{2}{1-y}\right)\left[3F\left(\frac{3}{4},\frac{1}{4};2;z^2\right) + F\left(\frac{3}{4};\frac{5}{4};2;z^2\right)\right]\right\}$$

$$\beta_I = \cdots$$

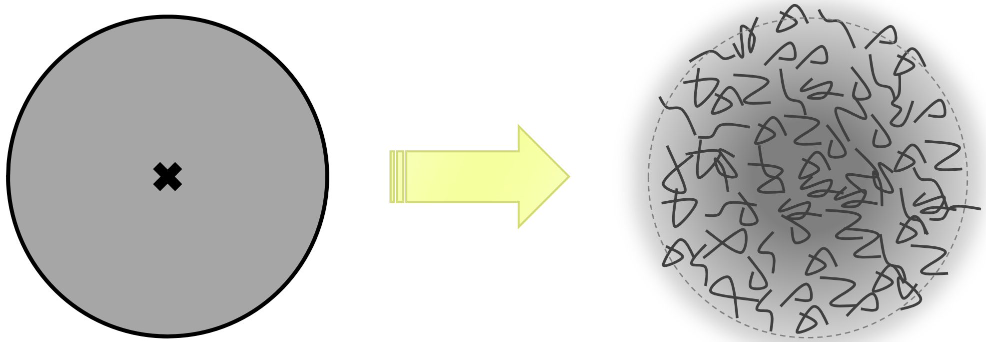
$$z = 1 - y^{-2}$$



Exotic branes and black hole microstates

Mathur's fuzzball proposal

- BHs are filled with stringy fuzz



"Conventional" picture of BH

"Fuzzball" picture of BH

No horizon
No singularity

- BH microstates = different configurations of the fuzz
- BH entropy = stat. mech. entropy of the fuzz

$$S_{\mathrm{BH}} = \frac{A}{4G_N} \stackrel{?}{=} S_{\mathrm{fuzz}}$$

Sugra microstates

Supersym. BHs ⇨ Microstates describable within sugra as *smooth, horizonless* solutions?

- 2-charge system: Successful [Lunin+Mathur] [Lunin+Maldacena+Maoz] [Rychkov]
  - All microstates have been constructed within sugra
  - Counting sugra sol'ns reproduces micro entropy: $S_{\text{sugra}} = S_{\text{micro}}$
  - But horizon vanishes classically
- 3-charge system: Not so successful
  - Many sugra microstates have been constructed [Bena+Warner] [Berglund+Gimon+Levi]
  - Evidence that they are *not* enough [de Boer+El-Showk+Messamah+Van de Bleeken]

They looked for *geometric* solutions in sugra
and didn't find enough microstates.

Proposal:

Generic BH microstates involve
exotic charges and are non-geometric!

Puff-ups and BH microstates (2)

- Standard 4D BH system

$$\mathrm{D0},\ \ \mathrm{D4(6789)},\ \ \mathrm{D4(4589)},\ \ \mathrm{D4(4567)}$$

: Well studied for microstate counting

[Maladacena+Strominger+Witten]

- Possible puff-ups:

	$\mathrm{D4(6789)}$	puff up	$\mathrm{NS5}(6789\psi)$	$5^2_2(6789,45\psi)$
$\mathrm{D0}$	$\mathrm{D4(4589)}$	⟹	$\mathrm{NS5}(4589\psi)$	$5^2_2(4589,67\psi)$
	$\mathrm{D4(4567)}$		$\mathrm{NS5}(4567\psi)$	$5^2_2(4567,89\psi)$

non-geometric

second puff up ⟹ More exotic charges?

Puff-ups and BH microstates (1)

- 5D BH system

$$\mathrm{M2}(56),\ \ \mathrm{M2}(78),\ \ \mathrm{M2}(9\mathrm{A})$$

: Well studied for microstate geometry

[Mathur] [Bena+Warner] [Berglund+Gimon+Levi]
[de Boer+El-Showk+Messamah+Van de Bleeken]

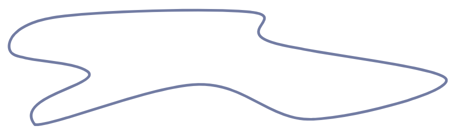
- Possible puff-ups:

			second puff up	
$\mathrm{M2}(56)$	puff up →	$\mathrm{M5}(789\mathrm{A}\psi)$	→	$5^3(789\mathrm{A}\phi, 56\psi)$
$\mathrm{M2}(78)$		$\mathrm{M5}(569\mathrm{A}\psi)$		$5^3(569\mathrm{A}\phi, 78\psi)$
$\mathrm{M2}(9\mathrm{A})$		$\mathrm{M5}(5678\psi)$		$5^3(5678\phi, 9A\psi)$
		cf. black ring		non-geometric

Puff-ups and BH microstates (3)

2-charge system

- Worldvolume theory:
  - Higgs branch coming from intersection of two stacks
- Gravity:
  - Fluctuation of 1-dimensional object

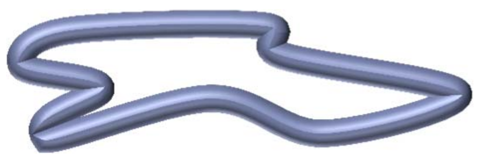


$$S_{\text{brane}} = S_{\text{gravity}}$$

Puff-ups and BH microstates (4)

3-charge system

- Worldvolume theory:
  - More complicated Higgs branch from triple intersection
- Gravity:
  - Fluctuation of 2-dimensional object?



    — Exotic branes has just the right dimensionality

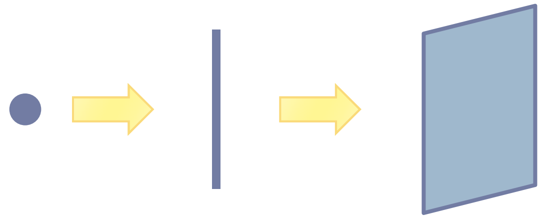
  - Explains missing entropy in sugra microstate geometries?

$S_{\text{brane}} = S_{\text{gravity}}$??

[Bena+Warner] [Berglund+Gimon+Levi]
[de Boer+El-Showk+Messamah+Van den Bleeken]

In progress…

- Double puff-up possible if straight
  - Supersymmetry analysis works
  - Generic rule for puffed-up charges



- Not (yet) clear if we can bend it / curl it up
  - Need to extend the existing ansatz in sugra
  - Not globally geometric; only locally

Conclusions

Conclusions

- Exotic branes = non-geometries (U-folds)
- Exotic charges = U-duality monodromies
- Relevant even for non-exotic physics
  by supertube effect

⇒ **Exotic branes are not at all exotic;**
They are everywhere!

Conclusions

- Unexplored exotic land out there awaiting us!
  - Classification of exotic branes (bound states, etc.)
  - AdS/CFT
  - Double bubbles
  - Microstate non-geometries
  - 4D black ring??

Thanks!

Sugra solution for 5^2_2 (1)

Want to make use of $KKM(56789,4) \xrightarrow{T_3} 5^2_2(56789,34)$

KKM(56789,4):

$$ds^2 = -dt^2 + Hd\vec{x}^2 + H^{-1}(dx^4 + \omega)^2 + dx^2_{56789}$$

$$e^{2\Phi} = 1, \quad d\omega = *_3 dH,$$

$$H = 1 + \sum_p H_p, \quad H_p = \frac{R_4}{2|\vec{x} - \vec{x}_p|}$$

$\vec{x} \in \mathbb{R}^3_{123}$: noncompact

x^4: special KK circle

$x^{5\ldots9}$: T^6

$\vec{x}_p$: positions of centers in $\mathbb{R}^3_{123}$

⇓ compactify x^3 = array centers along x^3

$$H(r) = h + \sigma \log\left(\frac{\mu}{r}\right)$$

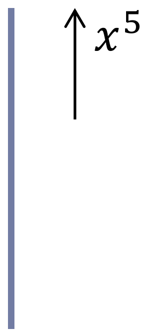
[Sen] [Blau+O'Loughlin]

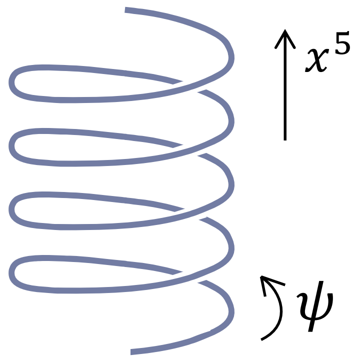
⇓ T-dualize along x^3 (Buscher rule)

Double puff-up

- D1-D5-P system and 3→2→1 puff-up

D1(5) D5(56789) P(5)	first puff up →	$d5(\psi 6789)$ $d1(\psi)$ ~~$kkm(\psi 6789,5)$~~ $p(\psi)$	second puff up →	?





?

Straight tube

- D1-D5-P system and 3→2→1 puff-up

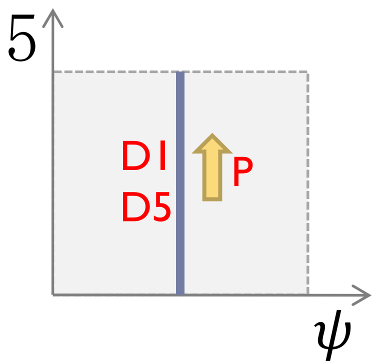
D1(5)
D5(56789)
P(5)

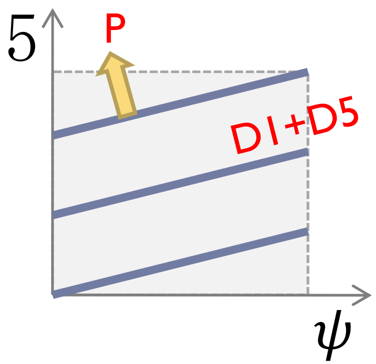
first puff up →

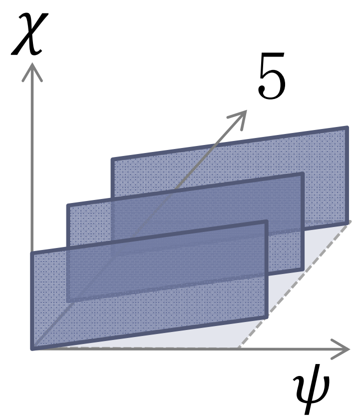
d5(ψ6789)
d1(ψ)
p(ψ)

second puff up →

kkm(χ6789,5)
kkm(χ6789, ψ)
p(χ)







Susy projector analysis (1)

- Original D1-D5-P preserves supercharge $Q = \begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix}$

D1(5)
D5(56789)
P(5)

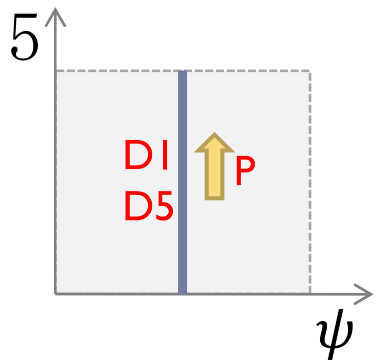
$$\Pi_1 Q = \Pi_2 Q = \Pi_3 Q = 0$$

$$\Pi_i = \frac{1}{2}(1 + P_i), \quad i = 1,2,3$$

$$P_1 = \Gamma^{05}\sigma^1,$$

$$P_2 = \Gamma^{056789}\sigma^1,$$

$$P_3 = \Gamma^{05}$$



Susy projector analysis (2)

- First puff-up:

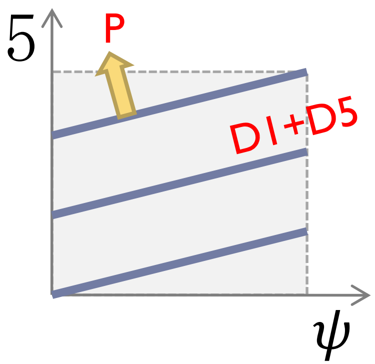
$$\hat{\Pi}_i = \frac{1}{2}(1+\hat{P}_i), \quad i = 1,2$$

d5(ψ6789)
d1(ψ)
p(ψ)

$$\hat{P}_1 = c_1 s_2(-s_3\Gamma^{05} + c_3\Gamma^{0\psi}) + c_1 c_2(c_3\Gamma^{05} + s_3\Gamma^{0\psi})\sigma^1 - s_1 c_2(-s_3\Gamma^{056789} + c_3\Gamma^{0\psi 6789}) + s_1 s_2(c_3\Gamma^{0\psi 6789} + s_3\Gamma^{0\psi 6789})\sigma^1$$

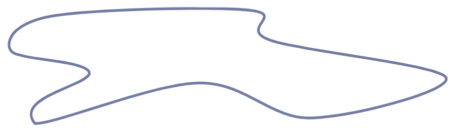
$$\hat{P}_2 = -s_1 c_2(-s_3\Gamma^{05} + c_3\Gamma^{0\psi}) + s_1 s_2(c_3\Gamma^{05} + s_3\Gamma^{0\psi})\sigma^1 - c_1 s_2(-s_3\Gamma^{056789} + c_3\Gamma^{0\psi 6789}) + c_1 c_2(c_3\Gamma^{0\psi 6789} + s_3\Gamma^{0\psi 6789})\sigma^1$$

$$c_a = \cos\theta_a, \quad s_a = \sin\theta_a, \quad a = 1,2,3$$



- ψ can be any direction
- θ are constrained but there is one free parameter.
- $\hat{\Pi}_1 Q = \hat{\Pi}_2 Q = 0$ for any ψ, θ

→ Can puff up into any curve with any density

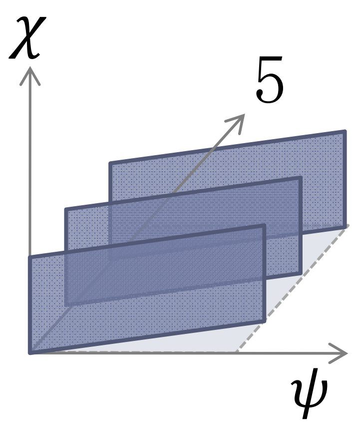


Susy projector analysis (3)

- Second puff-up:

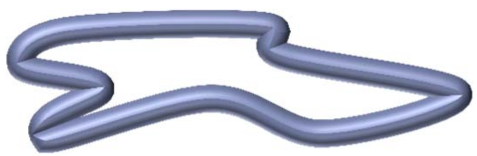
$$\hat{\hat{\Pi}} = \frac{1}{2}\left(1 + \hat{\hat{P}}\right)$$

kkm(χ6789,5)
kkm(χ6789, ψ)
p(χ)



- $\hat{\hat{P}}$ contains free parameters χ (puff up direction) and θ (density)
- $\hat{\hat{\Pi}}Q = 0$ for **any** χ, θ

→ Can puff up into any surface!?

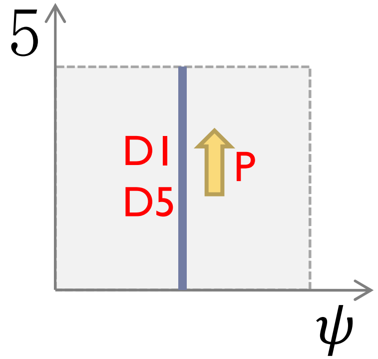


Projector analysis: 2-charge exercise (2)

- F1(1)+D0 → Straight D2(1ψ)+P(ψ)
- F1(1)-D0 preserves supercharge $Q = \begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix}$

$$\Pi_i Q = 0 \qquad \Pi_i = \frac{1}{2}(1 + P_i), \;\; i = 1,2$$

$$P_1 = \Gamma^{01}\sigma^3, \;\; P_2 = \Gamma^0 i\sigma^2$$



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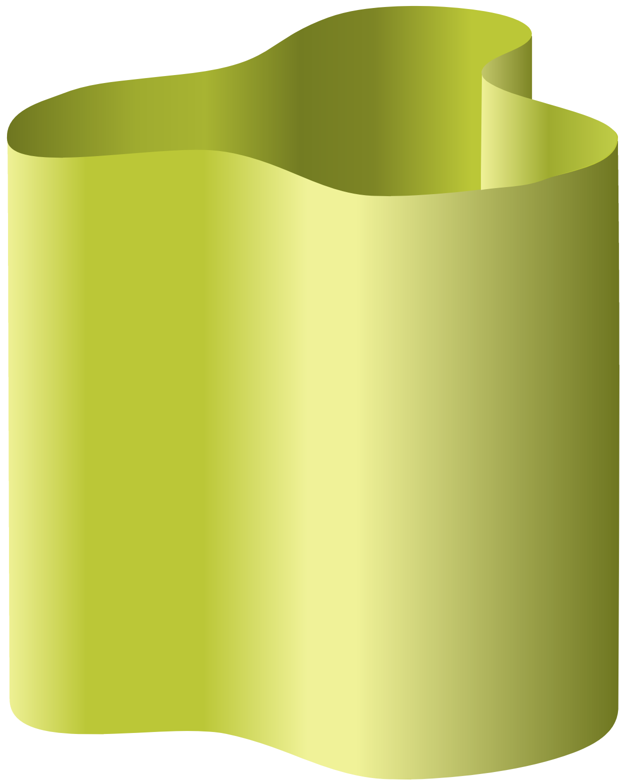

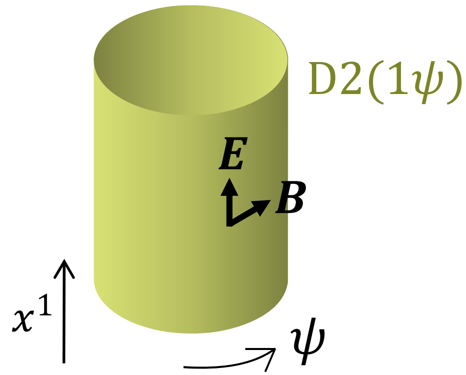

Projector analysis: 2-charge exercise (1)

- F1(1)+D0 → flat D2(1ψ)+P(ψ)
- F1(1)-D0 preserves supercharge $Q = \begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix}$

$$\Pi_i Q = 0 \qquad \Pi_i = \frac{1}{2}(1 + P_i), \quad i = 1,2$$

$$P_1 = \Gamma^{01}\sigma^3, \quad P_2 = \Gamma^0 i\sigma^2$$