# RECENT DEVELOPMENT OF THE KERR/CFT CORRESPONDENCE

Tatsuma Nishioka (Princeton)

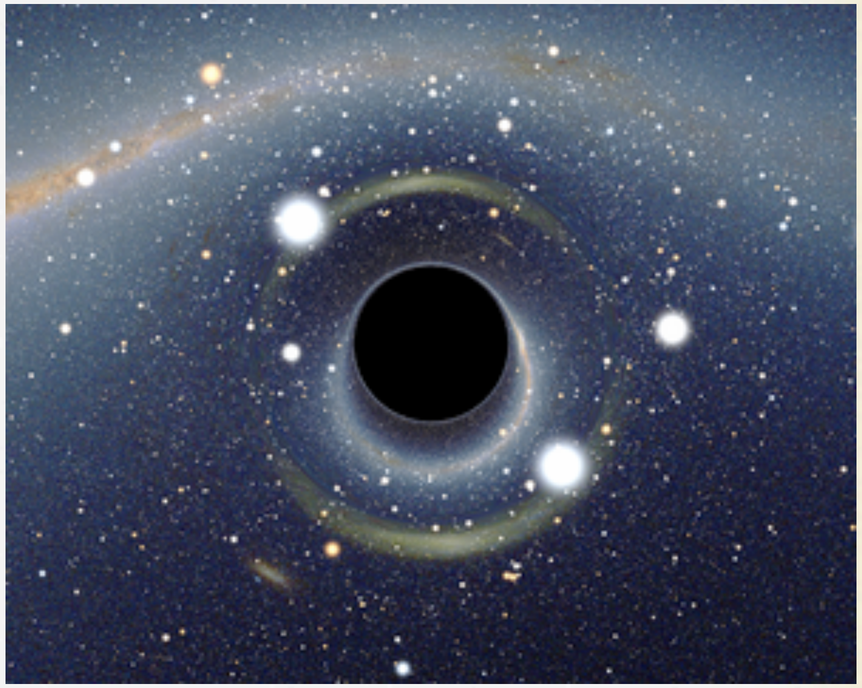
*The fourth workshop on Superstring theory and Universe @ Hakone, 2/17-19, 2011*

# INTRODUCTION

- Black hole:

  *Nothing can escape from its horizon*

  *It seems to exist in our universe*



- Thermodynamics:

  *Theoretically, it has a temperature and an entropy proportional to the horizon area*

# HAWKING TEMPERATURE

- Schwarzschild black hole

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \frac{dr^2}{1 - \frac{2M}{r}} + r^2 d\Omega_2^2$$

- Near horizon $r = 2M + \rho^2/8M$ and Wick rotation

$$ds^2 = \frac{\rho^2}{16M^2} d\tau^2 + d\rho^2 + (2M)^2 d\Omega_2^2$$

- The Hawking temperature is determined as

$$\tau \sim \tau + \frac{1}{T_H}, \qquad T_H = \frac{1}{8\pi M}$$

to avoid the conical singularity $\rho = 0$

# BLACK HOLE ENTROPY

- Small mass $dM$ increases an entropy

$$dS_{BH} = \frac{dM}{T_H} = d(4\pi M^2) \quad \Rightarrow \quad S_{BH} = \pi(2M)^2$$

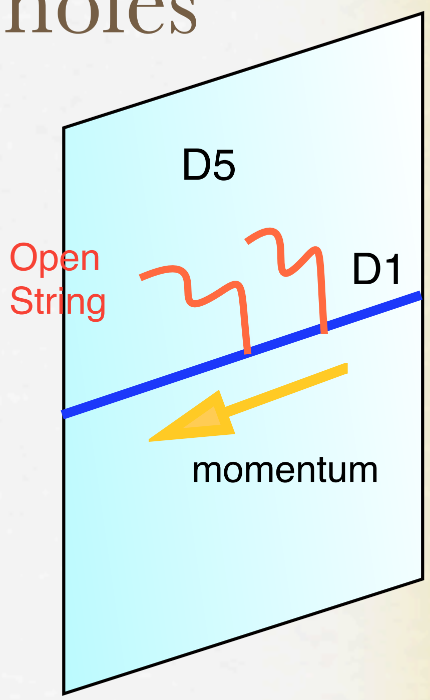
- The Bekenstein-Hawking area law

$$S_{BH} = \frac{\text{Area(Horizon)}}{4}$$

can be applied to the other black holes including charges and angular momentums

# ORIGIN OF BLACK HOLE ENTROPY

- In general relativity, we don’t know what is inside it
- In string theory, one can construct some black holes
  - D1-D5-P system (Strominger-Vafa ’96) : BTZ
  - Entropy comes from dof of the open strings
- The Kerr/CFT correspondence
  - New way of understanding the extremal black hole



# PLAN OF TALK

- The Kerr/CFT correspondence
- Hidden conformal symmetry
- Toward the correspondence at finite temperature
- Future directions

# THE KERR/CFT CORRESPONDENCE

*(Guica-Hartman-Song-Strominger '08)*

- The extremal Kerr black hole is conjectured to be dual to a two-dimensional conformal field theory
- Fundamental data to characterize CFT
  - Central charge $c$
  - Temperature $T$
- They are determined from the black hole and reproduce the black hole entropy!

$$S_{CFT} = \frac{\pi^2}{3} cT = S_{BH}$$

# CFT DUALS FOR EXTREMAL BH

*(Hartman-TN-Murata-Strominger '08  Compere-TN-Murata '09)*

- It is known that arbitrary extremal black holes in the Einstein-Maxwell-scalar theory (Kunduri-Lucietti-Reall '07)

$$S = \frac{1}{16\pi}\int d^4x\sqrt{-g}\Big(R - \frac{1}{2}f_{AB}(\chi)\partial_\mu\chi^A\partial^\mu\chi^B - V(\chi) - \frac{1}{4}g_{IJ}(\chi)F^I_{\mu\nu}F^{J\mu\nu}\Big) + \frac{1}{2}\int h_{IJ}(\chi)F^I\wedge F^J$$

- The near horizon geometry always takes the form of

$$ds^2 = \Gamma(\theta)\Big[\underbrace{-r^2dt^2 + \frac{dr^2}{r^2}}_{SL(2,R)} + \alpha(\theta)d\theta^2\Big] + \gamma(\theta)(\underbrace{d\phi}_{U(1)} + krdt)^2$$

- The correspondence can be generalized with the use of the universal form of the near horizon geometry

# DUAL CFT

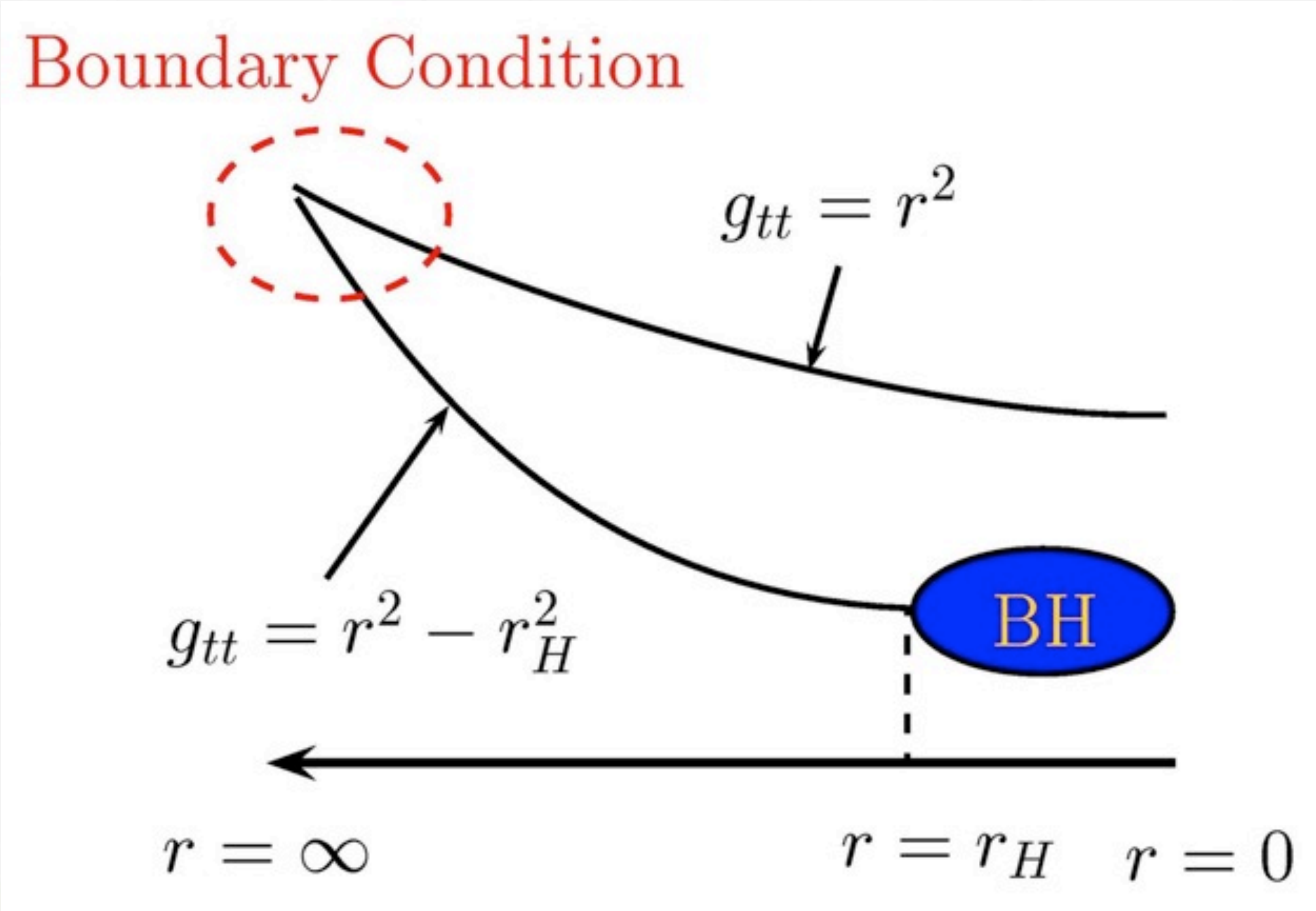
- Need to know
  - Central charges of Virasoro symmetries

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{c}{12}(m^3 - m)\delta_{m+n,0}$$

  - Temperature

- Virasoro symmetry is realized as an Asymptotic Symmetry of spacetime
  - A part of diffeomorphism which obeys the boundary condition

# BOUNDARY CONDITION



# ASYMPTOTIC SYMMETRY AND BOUNDARY CONDITION

- Let's take the following boundary condition

$$\mathcal{L}_\xi g_{\mu\nu} = \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu = \mathcal{O} \begin{array}{cccc} t & \phi & \theta & r \\ \end{array} \begin{pmatrix} r^2 & 1 & 1/r & 1/r^2 \\ & 1 & 1/r & 1/r \\ & & 1/r & 1/r^2 \\ & & & 1/r^3 \end{pmatrix}$$

- Then we find

$$\zeta_\epsilon = \epsilon(\phi)\partial_\phi - r\epsilon'(\phi)\partial_r$$

satisfying

$$i[\xi_m, \xi_n] = (m-n)\xi_{m+n} \ , \qquad \xi_n = \xi(\epsilon = -e^{-in\phi})$$

# CONSERVED CHARGES

- We found the Virasoro algebra as the asymptotic symmetry
- We need to construct the associated conserved charge $Q_\xi$ following the Noether procedure

$$\{Q_\zeta, Q_{\tilde{\zeta}}\}_{DB} = Q_{[\zeta,\tilde{\zeta}]} + \underbrace{\frac{1}{8\pi}\int k_\zeta[\mathcal{L}_{\tilde{\zeta}}\bar{g};\bar{g}]}_{\text{central extension}}$$

- The central charge can be read off from the central extension term

# CENTRAL CHARGE

- Substitute the near horizon metric

$$i\{Q_{\zeta_\epsilon}, Q_{\zeta_{\tilde{\epsilon}}}\}_{DB} = iQ_{[\zeta_\epsilon,\zeta_{\tilde{\epsilon}}]}$$

$$-\frac{ik}{16\pi}\int d\theta d\phi \sqrt{\frac{\alpha(\theta)\gamma(\theta)}{\Gamma(\theta)}}\left(\Gamma(\theta)\epsilon'\tilde{\epsilon}'' + \gamma(\theta)\epsilon\tilde{\epsilon}' - (\epsilon \leftrightarrow \tilde{\epsilon})\right)$$

- Then we find

$$c = 3k\int_0^\pi d\theta\sqrt{\Gamma(\theta)\alpha(\theta)\gamma(\theta)}$$

- Note that the other fields could  contribute to the central charge, but doesn't

# TEMPERATURE

- Given the metric before taking near horizon one can define the Frolov-Thorne vacuum
- Quantum fields with $(\omega, m)$ will be characterized by near horizon charges $(n_L, n_R)$

$$e^{-i\omega\hat{t}+im\hat{\phi}} = e^{-in_R t+in_L\phi}$$

- The Boltzmann factor will be

$$e^{-\frac{\omega-\Omega_H m}{T_H}} = e^{-\frac{n_L}{T_L}-\frac{n_R}{T_R}}$$

# ENTROPY

- Several examples show

$$T_L = \frac{1}{2\pi k} \,, \qquad T_R = 0$$

- Using the Cardy formula

$$S_{CFT} = \frac{\pi^2}{3} c T_L = \frac{\pi}{2} \int_0^{\pi} d\theta \sqrt{\Gamma(\theta)\alpha(\theta)\gamma(\theta)} = S_{BH}$$

- We have reproduced the black hole entropy!

# IN HIGHER DIMENSION

- Five dimensional Myers-Perry black hole has two rotations
- We can construct two corresponding CFTs $(c_1, T_1), (c_2, T_2)$
- They give the same entropies

$$S_{CFT} = \frac{\pi^2}{3} c_1 T_1 = \frac{\pi^2}{3} c_2 T_2 = S_{BH}$$

without summation

# RIGHT SECTOR

- We found only one Virasoro algebra for the boundary condition we have adopted
- Another boundary condition is proposed (Matsuo-Tsukioka-Yoo '09) which gives two Virasoro algebras
- They calculated the central charge for the right sector by introducing a cut-off at the boundary

$$c_R \sim \frac{1}{\Lambda}$$

# SUMMARY I

- We have seen that fairly general extremal black holes seem to be dual to two-dimensional CFTs
- The CFTs are characterized by the central charges $(c_L, c_R)$ and temperatures $(T_L, T_R)$
- They are chiral CFTs and the right sector seems to be responsible for the excitation from the extremality

# PLAN OF TALK

- The Kerr/CFT correspondence
- Hidden conformal symmetry
- Toward the correspondence at finite temperature
- Future directions

# HIDDEN CONFORMAL SYMMETRY

*(Castro-Maloney-Strominger '10)*

- Consider a scalar field on the Kerr black hole

$$\Phi(t,r,\theta,\phi) = e^{-i\omega t + im\phi} R(r) S(\theta)$$

- Then the equation of motion can be separable

$$\left[\frac{1}{\sin\theta}\partial_\theta\left(\sin\theta\partial_\theta\right) - \frac{m^2}{\sin^2\theta} + \omega^2 a^2 \cos^2\theta\right] S(\theta) = -K_\ell S(\theta) \ ,$$

$$\left[\partial_r \Delta \partial_r + \frac{(2Mr_+\omega - am)^2}{(r-r_+)(r_+-r_-)} - \frac{(2Mr_-\omega - am)^2}{(r-r_-)(r_+-r_-)} + (r^2 + 2M(r+2M))\omega^2\right] R(r) = K_\ell R(r) \ .$$

$$M \ : \text{mass} \ , \qquad J = aM \ : \text{angular momentum}$$

# NEAR REGION

- Define "near" and "far" regions

$$r \ll \frac{1}{\omega} \quad \text{(near)}\ , \qquad M \ll r \quad \text{(far)}$$

- Matching region exists when $M\omega \ll 1$

$$M \ll r \ll \frac{1}{r}$$

- Then the equations of motion are simplified and

$$K_l = l(l+1)$$

# CONFORMAL SYMMETRY

- The other equation can be solved by a hypergeometric function

$$\left[\partial_r \Delta \partial_r + \frac{(2Mr_+\omega - am)^2}{(r-r_+)(r_+-r_-)} - \frac{(2Mr_-\omega - am)^2}{(r-r_-)(r_+-r_-)}\right] R(r) = \ell(\ell+1)R(r)$$

- SL(2,R) symmetry appears in the equation
- This suggests the existence of a hidden conformal symmetry

# SL(2,R)$_L$ × SL(2,R)$_R$

- Coordinate transformation

$$w^+ = \sqrt{\frac{r-r_+}{r-r_-}}\, e^{2\pi T_R \phi}$$
$$w^- = \sqrt{\frac{r-r_+}{r-r_-}}\, e^{2\pi T_L \phi - \frac{t}{2M}}$$
$$y = \sqrt{\frac{r_+-r_-}{r-r_-}}\, e^{\pi (T_L+T_R)\phi - \frac{t}{4M}}$$

$$T_R \equiv \frac{r_+-r_-}{4\pi a}\ , \quad T_L \equiv \frac{r_++r_-}{4\pi a}$$

- The equation of motion reduces to that on the AdS3

$$ds^2 = \frac{dy^2 + dw^+ dw^-}{y^2}$$

# SL(2,R) GENERATORS

- Using the following vectors

$$H_1 = i\partial_+ \ , \qquad \bar{H}_1 = i\partial_- \ ,$$

$$H_0 = i(w^+\partial_+ + \frac{1}{2}y\partial_y) \ , \qquad \bar{H}_0 = i(w^-\partial_- + \frac{1}{2}y\partial_y) \ ,$$

$$H_{-1} = i(w^{+2}\partial_+ + w^+ y\partial_y - y^2\partial_-) \ , \qquad \bar{H}_{-1} = i(w^{-2}\partial_- + w^- y\partial_y - y^2\partial_+)$$

satisfying SL(2,R) algebras $[H_0, H_{\pm1}] = \mp i H_{\pm1} \ , \quad [H_{-1}, H_1] = -2iH_0$

- The equation of motion for the radial part becomes

$$\bar{\mathcal{H}}^2\Phi = \mathcal{H}^2\Phi = \ell(\ell+1)\Phi$$

$$\mathcal{H}^2 = \bar{\mathcal{H}}^2 = -H_0^2 + \frac{1}{2}(H_1H_{-1} + H_{-1}H_1)$$

# DUAL CFT

- The eom implies that $\Phi$ is an operator with conformal dimensions $(h_L, h_R) = (l, l)$
- At $r \longrightarrow \infty$

$$w^{\pm} = e^{2\pi T_{R,L}\hat{t}^{\pm}} \qquad \hat{t}_{+} = \phi\ , \qquad \hat{t}_{-} = \phi - \frac{t}{4\pi M T_L}$$

- Relation of stress-energy tensor between w- and t-planes

$$T_{++}(\hat{t}^{+}) = \left(\frac{\partial w^{+}}{\partial t^{+}}\right)^{2} T_{++}(w^{+}) + \frac{1}{12\pi}\sqrt{\frac{\partial w^{+}}{\partial \hat{t}^{+}}}\frac{\partial^{2}}{\partial \hat{t}_{+}^{2}}\sqrt{\frac{\partial \hat{t}^{+}}{\partial w^{+}}}$$

# DUAL CFT

- This gives $\langle T_{++}(\hat{t}^{+})\rangle = \frac{\pi^2}{12}T_R^2$
- Then the temperatures are $(T_L, T_R)$
- Assume $c_L = c_R = 12J$ *(cf. Castro-Larsen '09 obtained it by dimensional reduction to 2d theory)*
- Entropy

$$S = \frac{\pi^2}{3}(c_L T_L + c_R T_R)$$

$$= 2\pi M r_+ = \frac{\text{Area}}{4}$$

# SUMMARY II

- The hidden conformal symmetry appears in the "near" region of the scalar equation of motion
- Dual CFT lives on $(\hat{t}^+, \hat{t}^-)$ coordinates with the central charges $c_L = c_R = 12J$ and the temperatures $(T_L, T_R)$
- This symmetry is also found in other black holes with charges and cosmological constant
- Evidence for the Kerr/CFT at finite temperature?

# PLAN OF TALK

- The Kerr/CFT correspondence
- Hidden conformal symmetry
- Toward the correspondence at finite temperature
- Future directions

# TOWARD THE CORRESPONDENCE AT FINITE TEMPERATURE

- The previous analysis with the hidden conformal symmetry indicates that CFT dual for the Kerr black hole is non-chiral, i.e. $c_L = c_R = 12J$
- On the other hand, no one derives $c_R = 12J$ in the usual manner
- It is quite important to see how this value can be obtained for understanding the correspondence at finite temperature

# PUZZLE

- Although an indirect argument for the right central charge is presented, we'd like to derive it directly
- The difficulty comes from the fact that the Virasoro symmetry for the right sector is associated with a reparametrization of time direction
- Naive calculation leads to time-dependent central charge

# NEAR HORIZON LIMIT

- The Kerr geometry

$$ds^2 = -\frac{\Delta}{\rho^2}(dt - a\sin^2\theta\, d\phi)^2 + \frac{\sin^2\theta}{\rho^2}\left[(r^2+a^2)d\phi - adt\right]^2 + \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2$$

$$\Delta = r^2 - 2Mr + a^2 \ , \quad \rho^2 = r^2 + a^2\cos^2\theta$$

- Usual near horizon limit

$$\hat{t} = \frac{\epsilon}{2M}t \ , \qquad \hat{\phi} = \phi - \frac{t}{2M} \ , \qquad r = a(1+\epsilon\hat{r})$$

- Then the metric reduces to the previous form

# COMMENTS

- CFT lives on $(\hat{t}, \hat{\phi})$ coordinates
- Virasoro symmetries are $(\xi_L = \xi_L(\hat{\phi}),\ \xi_R = \xi_R(\hat{t}))$
- There is no natural period for time direction
- Expand $\xi_R(\hat{t}) = \hat{t}^n$ then associated central charge vanishes $c_R = 0$

# NEW NEAR HORIZON LIMIT

*(Matsuo-TN '10)*

- Define the following coordinates instead

$$x^+ = \epsilon\phi \ , \qquad x^- = \phi - \frac{a\,t}{2M^2} \ , \qquad r = a(1+\epsilon\hat{r})$$

- The near horizon metric takes the same form as before with replacing $(t, \phi)$ with $(x^+, x^-)$
- So we can find Virasoro symmetries for this near horizon geometry

$$(\xi_L = \xi_L(x^-), \xi_R = \xi_R(x^+))$$

# CENTRAL CHARGES

- The periods of the near horizon coordinates

$$x^+ \sim x^+ + 2\pi\epsilon \,, \qquad x^- \sim x^- + 2\pi$$

- Expand $\xi_L = -e^{-inx^-} \,, \quad \xi_R = -\epsilon e^{-inx^+/\epsilon}$

  to make the Lie bracket relation canonical

$$[\xi_m, \xi_n] = (m-n)\xi_{m+n}$$

- Using this basis, we obtain $c_L = c_R = 12J$

# SUMMARY III

- We obtain the expected values of the central charges by using the new near horizon limit
- The CFT with $c_L = c_R = 12J$ should be considered to live on $(x^+/\epsilon, x^-)$
- Our limit is useful to describe the right sector since the central charge can be calculated explicitly

# PLAN OF TALK

- The Kerr/CFT correspondence
- Hidden conformal symmetry
- Toward the correspondence at finite temperature
- Future directions

# FUTURE DIRECTIONS

- The extremal Kerr black hole is dual to the CFT with

  $$c_L = c_R = 12J$$

- Hidden conformal symmetry implies the correspondence for the non-extremal case
- Non-extremal Kerr cannot be studied in a similar manner
- Need a new way of "near" horizon?

# FUTURE DIRECTIONS

*(Guica-Strominger '10)*

**5D Myers-Perry**

$J_L$

**Dual CFT**

$c_L = c_R = 6J_L$

*?*

**5D Kerr-Newmann**

$(J_L, Q) \quad J^2 > Q^3$

*deformation*

**Maximal KN**
**= Maximal BMPV**

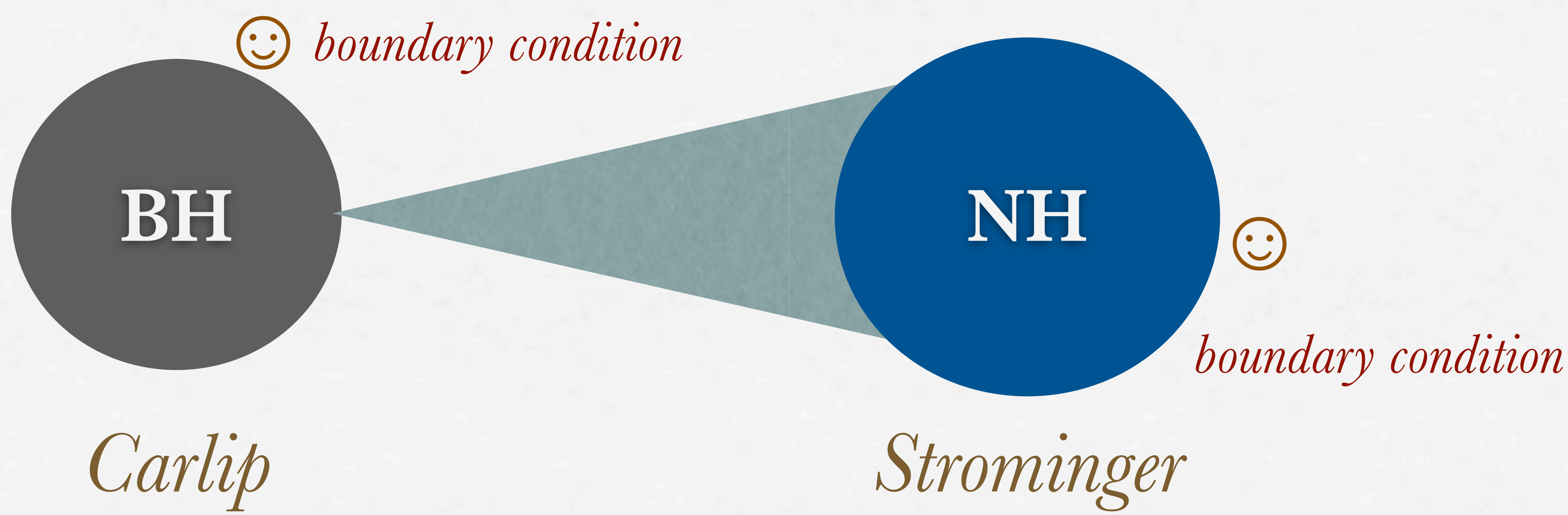
$(J_L, Q) \quad J^2 = Q^3$

**Dual CFT**

$c_L = c_R = 6J_L$

# FUTURE DIRECTIONS

- Another way to derive the Virasoro algebra *(Carlip '98)*




- Can be applied to the non-extremal black hole
- Can account for BH entropy with tuning one parameter
- Relation to the Kerr/CFT?