超弦理論と宇宙 @城崎　(第3回2010年2月)

Lifshitz PointのAdS/CFTへの応用

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① Introduction

超弦理論は量子重力理論の最有力候補であるにも関わらず、その完全な(非摂動的な)定義は明確に未だ与えられていない。

しかしながら、ホログラフィーという考え方を用いると、重力を含まない理論(場の理論)で等価に記述できる。その代表例が、AdS/CFTである。

$$AdS_{d+1}: \qquad ds^2_{AdS} = -r^2 dt^2 + r^2 \sum_{i=1}^{d} dx_i^2 + \frac{dr^2}{r^2}.$$

Quantum Gravity

↕ Equivalent

$$d \text{ dim. CFT}$$

Quantum Mechanics
(We know non-perturbatively !)

今回はAdS/CFTの一つの単純な変形としてCFTのスケール不変性を非相対論的にしてみよう（Lifshitz固定点）。

CFTの相対論的スケール不変性 $(t, x_i, r) \rightarrow (\lambda t, \lambda x_i, \frac{r}{\lambda})$

⬇

非相対論的スケール不変性 $(t, x_i, y_j, r) \rightarrow (\lambda^z t, \lambda^z x_i, \lambda y_j, \frac{r}{\lambda})$

ホログラフィック双対として期待される重力解（Lifshitz解）:

$$ds_{Li}^2 = r^{2z}\left(-dt^2 + \sum_{i=1}^{p} dx_i^2\right) + r^2 \sum_{i=1}^{d-p} dy_i^2 + \frac{dr^2}{r^2}.$$

[Kachru-Liu-Mulligan 08']

[Cf. Non-relativistic conformal invariant theory (D. Son,...)→Refer to Yoshida's talk]

コメント

私の話では、重力側は、一般座標変換不変性を保つ通常の重力理論であり、Horava-Lifshitz理論とは全く関係がない。

ネーミングをするなら、Kachru-Lifshitz理論とでも呼ぶ？

でも、この「Lifshitz」とは何のことだろうか？

Lifshitz pointとは？

(classical) Lifshitz point とは、異方的 なスケール不変性を持つ点を意味する。

Multi-critical point

$$V(M) = aM^2 + bM^4 + cM^6 + \cdots \quad (c > 0).$$

$$a = 0 \quad \Rightarrow \quad \text{ordinary critical pt.}$$

$$a = b = 0 \quad \Rightarrow \quad \text{Tricritical pt.}$$

Lifshitz point

[Hornreich-Luban-Shtrikman 1975']

$$F(M) = aM^2 + bM^4 + c(\nabla_{//} M)^2 + d(\nabla_{//} M)^4 + f(\nabla_{\perp} M)^2$$

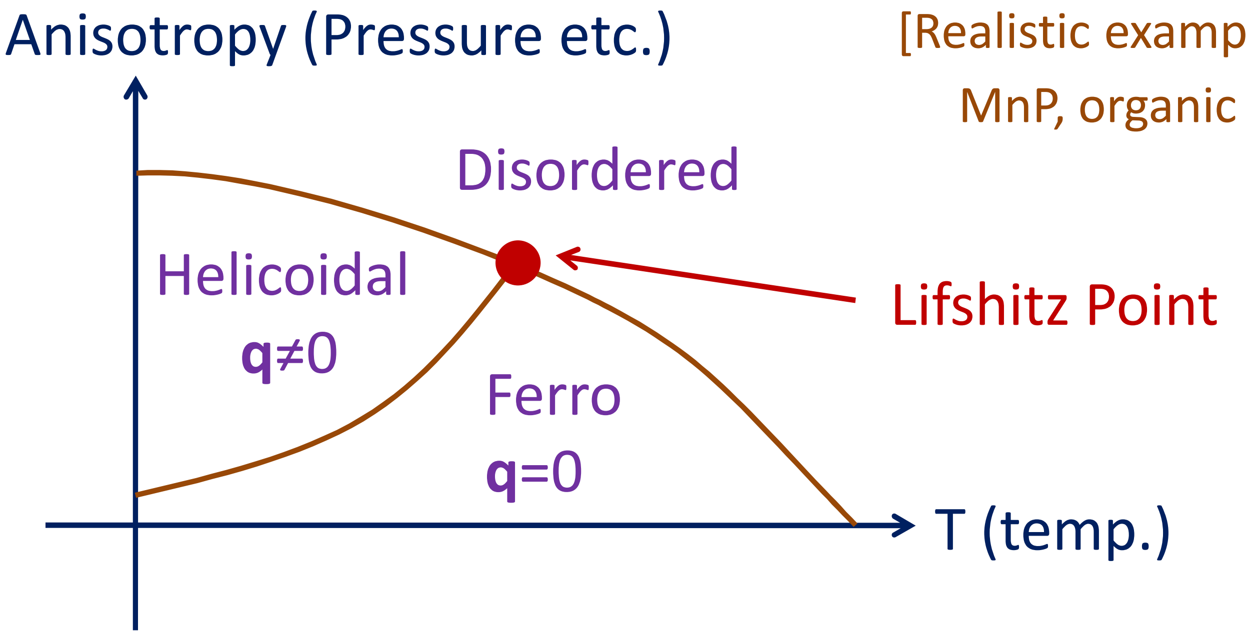
$$a = c = 0 \quad \Rightarrow \quad \text{(classical) Lifshitz point} \quad z = 2$$

Lifshitz points appear magnetic spin systems, typically when the following two interactions compete:

Nearest neighbor **ferro** interaction (**isotropic**)

\+ Next nearest neighbor **anti-ferro** interaction (**anisotropic**).

➡ The modulation wave vector **q** begins to be non-vanishing.



[Realistic examples:
MnP, organic crystals, alloy]

Classical Lifshitz model

$$S_E = \int dx^d \left[\frac{1}{2} (\nabla_{//}\phi)^2 + \frac{1}{2} (\nabla_{\perp}^2 \phi)^2 \right].$$

Quantum Lifshitz model

$$S_Q = \int dt dx^d \left[\frac{1}{2} (\partial_t \phi)^2 - \frac{1}{2} (\nabla^2 \phi)^2 \right].$$

Free field theory with z=2
The interaction changes
the value of z.

This theory is known to have the remarkable property:

$$\Psi_{Ground\ \ State} = e^{-\int dx^d \frac{1}{2} (\nabla \phi)^2}.$$

重力双対

Lifshitz解は、4次元Einstein-Maxwell-2 form理論の解であることがKachruたちによって示された(Einstein-Massive vectorと等価）。

$$S=\int dx^4\sqrt{-g}(R-2\Lambda)$$
$$-\int F_{(2)}\wedge{}^*F_{(2)}+H_{(3)}\wedge{}^*H_{(3)}-c\int B_{(2)}\wedge F_{(2)}.$$

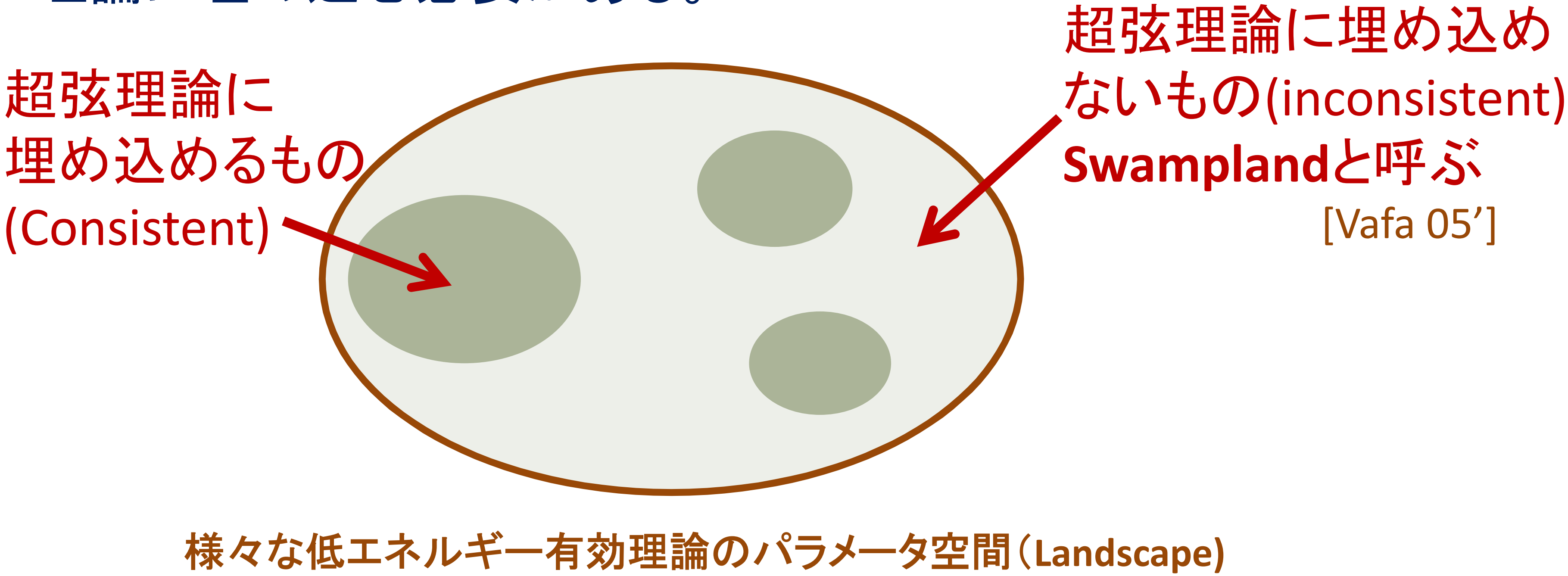
⇨

$$ds^2_{Li4}=L^2\left(-r^{2z}dt^2+r^2(dx^2+dy^2)+\frac{dr^2}{r^2}\right),$$

$$F_2=\alpha\theta^t\wedge\theta^r,\qquad H_3=\beta\theta^x\wedge\theta^y\wedge\theta^r.$$

$$\text{EOMs}\Rightarrow\quad \frac{\alpha}{\beta}=\sqrt{\frac{z}{2}}\ ,\qquad c=\frac{\sqrt{2z}}{L}\ .$$

しかし、双対な場の理論を微視的な立場で特定するには、超弦理論に埋め込む必要がある。



そこで、以下では、Lifshitz解が超弦理論に埋め込めるか調べたい。意外なことに、我々の解析の範囲内の結果としては、Swamplandになっている可能性を示唆しているといえる。

② No-go Results for Lifshitz Solutions

[Li-Nishioka-TT 09']

Below we would like study string/M-theory embeddings of the **4 dim. quantum Lifshitz metric** with a constant dilaton:

$$ds^2_{Li4} = -r^{2z}dt^2 + r^2(dx^2+dy^2) + \frac{dr^2}{r^2}$$

$$\equiv -(\theta_t)^2 + (\theta_x)^2 + (\theta_y)^2 + (\theta_r)^2.$$

(2-1) No-go Argument in Massive IIA Supergravity

The first setup is the spacetime $Li_4 \times M_6$ in (massive) type IIA supergravity.

This is motivated by the fact that the AdS4 × CP3 background dual to the ABJM theory realizes an Einstein gravity
+ 2-form and 3-form flux with CS term.

The dilaton is stabilized and there are no moduli.

Our ansatz with even forms J_1, J_2, J_3, V_4

$$M = m,$$

$$\widetilde{F}_2 = \alpha\theta_t\theta_r + \eta\theta_x\theta_y + J_1,$$

$$H_3 = \beta\theta_x\theta_y\theta_r,$$

$$\widetilde{F}_4 = f\theta_t\theta_x\theta_y\theta_r + \theta_x\theta_y J_2 + \theta_t\theta_r J_3 + V_4.$$

By employing all of the equations of motion, we can show

$$\beta^2 L^2 = 4(z-1), \quad \text{and} \quad z = -4 \quad ,$$

which leads to contradiction $\beta^2 < 0$.

(We can actually find *imaginary* solutions.)

[More general No-go results have been obtained recently by Blaback-Danielsson-Riet 10']

(2-2) No-go Argument in M-theory supergravity

Next we want to find solutions which look like $Li_4 \times M_7$ in eleven dim. supergravity.

Our flux ansatz

$$F_4 = f\theta_t\theta_x\theta_y\theta_r + \theta_x\theta_y\theta_r \wedge \alpha_1 + \theta_t\theta_r \wedge \beta_2 + \theta_r \wedge \Omega_3 + V_4 \ .$$

After some algebras with equations of motion, we can show

$$R_{ij}\alpha^i\alpha^j = \frac{4z(z-1)(z+4)}{L^4} > 0.$$

Weitzenbock formula :

$$\alpha^i \Delta \alpha_i = -\alpha^i \nabla_j \nabla^j \alpha_i + R_{ij} \alpha^i \alpha^j$$

This is vanishing Because α is harmonic one form due to EOMs.

This is positive after a partial integration.

This is positive !

Contradiction !

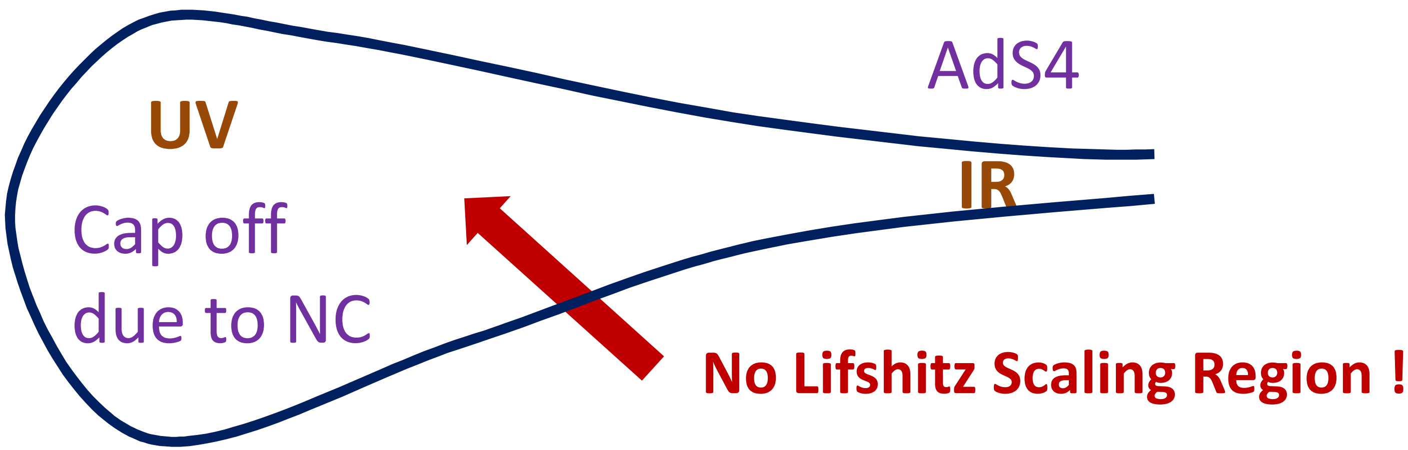
In this way, we find that there are no Lifshitz solutions under our flux ansatz for any compactifications.

(We can also extend our argument to warped compactifications.)

(2-3) But, why ? : an indirect argument

In previous examples, we have the H-flux in (x,y,r) direction.

This is holographically dual to the *non-commutativity.*



However, still the direct reason of no-go is not clear…

[Some possibilities of stringy constructions (using F-theory, fermi surfaces) have been given in Hartnoll-Silverstein-Polchinski-Tong 09']

③ Analogous Solutions in Type IIB Supergravity

[Azeyanagi-Li-TT 09']

しかしながら、ディラトン場のr 依存性を許すと実はTypeIIB
超重力理論で解を構成できることが分かる。D-braneの言葉で
言うとD3－D7系に相当する。

$$ds^2_{string} = e^{2h(r)}(-dt^2+dx^2+dy^2) + e^{2h(r)+2a(r)}dv^2 + e^{2c(r)-2a(r)}dr^2 + e^{2c(r)}r^2 ds^2_{X_5} \,,$$

Any Einstein Manifold

$$\phi = \phi(r), \quad B_2 = C_2 = 0,$$

$$F_5 = \alpha\left(\mathrm{Vol}(X_5) + {}^*\mathrm{Vol}(X_5)\right),$$

$$F_1 = d\chi = \beta dw$$

Equations of motion look like

$$[b'e^{2z}]' = \frac{\beta^2}{4}e^{-2a-h+3b+6c}r^5 + \frac{\alpha^2}{4}e^{-4c+3b+h}r^{-5},$$
$$[(a+h)'e^{2z}]' = -\frac{\beta^2}{4}e^{-2a-h+3b+6c}r^5 + \frac{\alpha^2}{4}e^{-4c+3b+h}r^{-5},$$
$$[(c+\log r)'e^{2z}]' = \frac{4}{r^2}e^{2z-2a} + \frac{\beta^2}{4}e^{-2a-h+3b+6c}r^5 - \frac{\alpha^2}{4}e^{-4c+3b+h}r^{-5},$$
$$[(2z+c-a)'e^{2z}]' = \frac{20}{r^2}e^{2z-2a} - \frac{\beta^2}{4}e^{-2a-h+3b+6c}r^5 - \frac{\alpha^2}{4}e^{-4c+3b+h}r^{-5},$$
$$2z'' + c'' - a'' + 2(z')^2 + \frac{1}{2}(h')^2 + a'h' + 2(c')^2 + (\frac{5}{r} + a')c' + \frac{3}{2}(b')^2 - \frac{10e^{-2a}}{r^2} + \frac{5}{2r^2} = 0,$$

where we have defined

$$z \equiv \frac{3}{2}b + \frac{5}{2}\log r + a + 2c + \frac{1}{2}h - \phi.$$

The derivative of a function f with respect to r is denoted by $f'(r)$. An observation, which will be useful in the next section, is that a linear combination of the first four equations gives

$$[(2b - 2a - \phi - 2h)'e^{2z}]' = 0.$$

To solve EOMs, we assume that a(r),b(r),h(r),c(r) and ϕ(r) are all proportional to log r. (``Scaling ansatz'')

Then we obtain the scaling solutions:

$$ds^2_{Einstein} = \widetilde{R}^2 \left[r^2 \left(-dt^2 + dx^2 + dy^2 \right) + r^{\frac{4}{3}} dw^2 + \frac{dr^2}{r^2} \right] + R^2 ds^2_{X5} \quad ,$$

$$e^{\phi(r)} = e^{\phi_0} r^{\frac{2}{3}} \quad , \qquad \left(R^2 = \frac{12}{11} \widetilde{R}^2 \right)$$

Notice: After the redefinition $\rho = r^{2/3}$, we obtain

$$ds^2_{Einstein} = \frac{9}{4} \widetilde{R}^2 \left[\rho^3 \left(-dt^2 + dx^2 + dy^2 \right) + \rho^2 dw^2 + \frac{d\rho^2}{\rho^2} \right] + R^2 ds^2_{X5} \quad .$$

In terms of N and k (= the number of D3 and D7 - branes),

$$\alpha = \frac{(2\pi)^4 N}{\mathrm{Vol}(X_5)} \quad , \qquad \beta = \frac{k}{L} \quad , \qquad R^2 = 2\sqrt{\frac{\pi^4 N}{\mathrm{Vol}(X_5)}} \quad ,$$

where we assumed the radius of w is L .

Moreover, our solutions allow the black hole generalization:

Lifshitz Black Hole Solution

$$ds^2_{Einstein} = \tilde{R}^2\left[r^2\left(-F(r)dt^2 + dx^2 + dy^2\right) + r^{\frac{4}{3}}dw^2 + \frac{dr^2}{r^2F(r)}\right] + R^2 ds^2_{X5} \quad ,$$

$$e^{\phi(r)} = e^{\phi_0} r^{\frac{2}{3}} \quad , \quad F(r) = 1 - \frac{\mu}{r^{11/3}} \quad , \quad \left(R^2 = \frac{12}{11}\tilde{R}^2\right) \quad .$$

The temperature and entropy are consistent with the scaling:

$$T = \frac{11}{12\pi}\mu^{3/11}, \qquad S_{BH} = \gamma \cdot N^2 T^{8/3} V_2 L \qquad (\gamma = 3.729...).$$

(1+1+2/3) dim. space → Agree with [w]=2/3 !

④ Conclusions

- 超重力理論におけるLifshitz解は、多くの場合で試してみたが、構成できなかった。

  ➡ de Sitterと似た状況？それとも超弦理論に存在しない？

- しかし、ディラトンの座標依存性を許すと解は存在し、エントロピーなどの物理量は、スケール普遍性を保つ。

[今後の課題：GRの方々へ]

超重力理論におけるLifshitz解のNO-GO定理の一般的な証明、もしくは反例を見つける。

（注：Lifshitz解は、AdSに次ぐ基本的な重力解といえる！）