研究会 第三回 超弦理論と宇宙 @ 城崎温泉 2010年2月18日

# Horava-Lifshitz重力と宇宙論的揺らぎの進化

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Based on: work with 浦川優子 (早稲田), 山口昌英 (青山学院)

arXiv:0908.1005, JCAP 0911:015 (2009)
arXiv:1002.3101

# *Motivation*

- A quantum gravity candidate
  - Recently Horava proposed a power-counting renormalizable theory of gravitation
    - ~~Lorentz symmetry~~
- Can perform concrete calculations:
  - ✓ Validity needs to be checked; still controversial
  - ✓ Interesting to explore cosmological consequences

# *Plan*

- Motivation
- Horava-Lifshitz gravity (10 slides)
- Cosmology in Horava-Lifshitz gravity (7 slides + 2)
- Summary

# Lorentz symmetry breaking as regulator

Visser 0902.0590

- Pedagogical example: scalar field theory

  *Anisotropic scaling*

  - Free Lagrangian invariant under $\vec{x} \to E^{-1}\vec{x},\ t \to E^{-z}t$

$$\frac{1}{2}\int dt d^3x \left[\dot{\phi}^2 - \phi(-\Delta)^z\phi\right] \quad \Rightarrow \quad \phi \to E^{(3-z)/2}\phi$$

  - Interactions $\int dt d^3x\, \phi^n \propto E^{-s}$ $\quad s = 3 + z + n(z-3)/2$

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| | |
|---|---|
| ✓ Lorentz invariant (z = 1) | ✓ ~~Lorentz invariant~~ (z = 3) |
| ✓ Renormalizable if $s = 4 - n \geq 0$ | ✓ Renormalizable for any *n* $s = 6$ |

# *Horava's idea for quantum gravity*

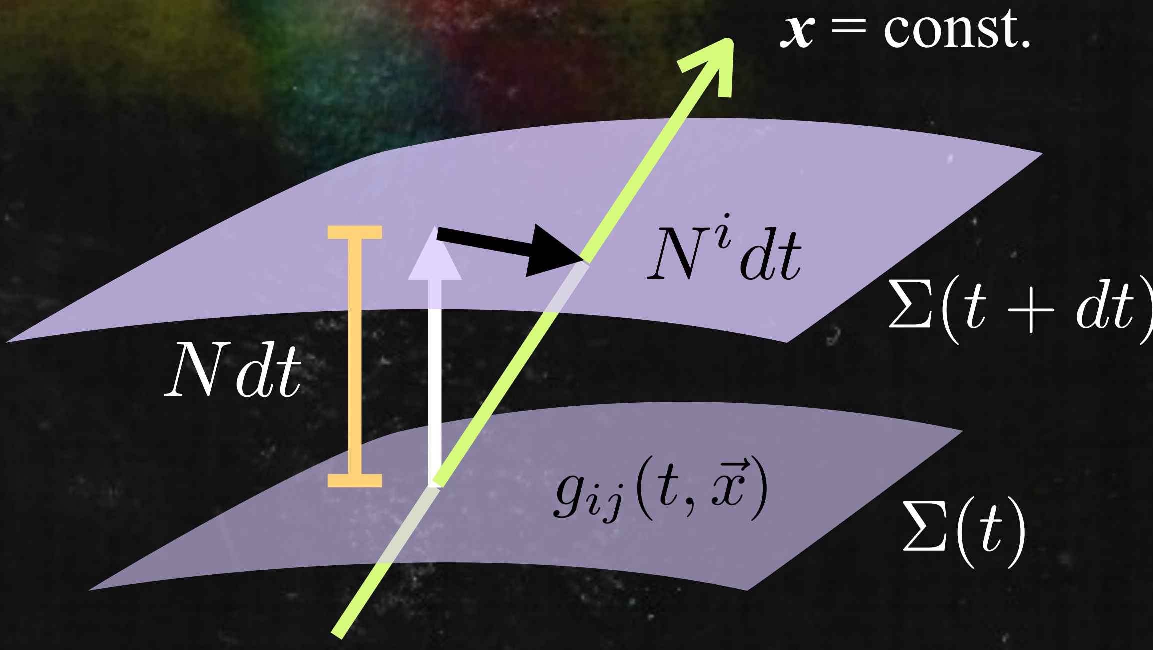
$$\text{Curvature}[g_{ij} = \delta_{ij} + h_{ij}] \sim \sum_{n=0}^{\infty} h^n \Delta h$$

- Abandon Lorentz invariance in exchange for (power-counting) renormalizability
- Apply this idea to gravity and consider z=3 theory

Horava 0812.4287; 0901.3775

# *Horava-Lifshitz gravity*

- Variables:
  - Lapse function $N$
  - Shift vector $N_i$
  - Spatial metric $g_{ij}$




- Symmetry: *foliation-preserving diffeomorphism invariance*

$$t \to \tilde{t}(t), \quad x^i \to \tilde{x}^i(t, \vec{x})$$

- cf. ADM decomposition of spacetime

$$ds^2 = -N^2 dt^2 + g_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

Horava 0812.4287, 0901.3775; Sotiriou, Visser, Weinfurtner 0904.4464, 0905.2798

# Action

- Volume elements $N dt, \ \sqrt{g} d^3 x$
- 3D Ricci tensor $R_{ij}$
- Extrinsic curvature $K_{ij} := \frac{1}{2N}\left(\partial_t g_{ij} - \nabla_i N_j - \nabla_j N_i\right)$

**Covariant under**

$t \to \tilde{t}(t), \quad x^i \to \tilde{x}^i(t, \vec{x})$

- Kinetic term

$$S_K = \int dt d^3 x \sqrt{g} N \mathcal{L}_K \qquad \mathcal{L}_K = g_K \left(K_{ij} K^{ij} - \lambda K^2\right)$$

GR $\longrightarrow$ $\lambda = 1$

- Potential term

$$S_{\mathcal{V}} = -\int dt d^3 x \sqrt{g} N \mathcal{V} \qquad \mathcal{V} \supset R, R_{ij} R^{ij}, \ldots$$

(nest slide)

# Potential

- Up to 6th derivative of metric $dtd^3x\sqrt{g}N\,\partial^6 g \propto E^{3-z}$
- 6th derivative $R^3, RR_{ij}R^{ij}, R_{ij}R^{jk}R_{ki},$ $R\Delta R, \nabla_i R_{jk}\nabla^i R^{jk}$
- 5th (violating parity) $\epsilon^{ijk}R_{il}\nabla_j R^l{}_k$
- 4th $R^2, R_{ij}R^{ij}$
- 2nd $R$
- 0th $\Lambda$

✳ Can also include terms such as $a_i a^i, R\nabla_i a^i, \ldots,\ \ a_i := \partial_i \ln N$

Blas, Pujolas, Sibiryakov 0909.3525

# *Projectability condition*

- Infinitesimal foliation-preserving diff. transformation

$$t \to t + f(t), \; x^i \to x^i + \chi^i(t, \vec{x})$$

$$\begin{array}{lcl} g_{ij} & \to & g_{ij} - f\dot{g}_{ij} - \nabla_i \chi^k g_{jk} - \nabla_j \chi^k g_{ik} \\ N_i & \to & N_i + \ldots \\ N & \to & N - \partial_t (fN) - \chi^i \partial_i N \end{array}$$

- Space-independent *N* remains to be space-independent

→ Natural to assume $N = N(t)$

*Projectable*

# *Field equations*

- Action:

$$S = \int dt d^3x \sqrt{g} N \left( \mathcal{L}_K - \mathcal{V} + \mathcal{L}_m \right)$$

Matter

- Evolution eq. $\longleftarrow \delta g^{ij}$

$$T_{ij} = \mathcal{L}_m g_{ij} - 2\frac{\delta \mathcal{L}_m}{\delta g^{ij}}$$

$$\frac{1}{N\sqrt{g}} g_{ik} g_{jl} \partial_t \left[ \sqrt{g} \left( K^{kl} - \lambda K g^{kl} \right) \right] + \ldots = \frac{T_{ij}}{M_{\mathrm{Pl}}^2}$$

- Momentum constraints $\longleftarrow \delta N_i$

$$J^i = -N \frac{\delta \mathcal{L}_m}{\delta N_i}$$

$$\nabla_j \left( K^{ij} - \lambda K g^{ij} \right) = \frac{J^i}{M_{\mathrm{Pl}}^2}$$

- Hamiltonian constraint $\longleftarrow \delta N$ (nest slide)

# *Hamiltonian constraint*

- Projectability condition $\delta N(t)$

→ Hamiltonian constraint is not a local equation

$$\int d^3x \sqrt{g}\,(\mathcal{L}_K + \mathcal{V} + 2M_{\rm Pl}^{-2}\rho) = 0$$

Integration over the whole space

$$\rho = -\mathcal{L}_m - N\frac{\delta \mathcal{L}_m}{\delta N}$$

# *Matter*

- Energy conservation: $\int d^3x \, [\partial_t(\sqrt{g}\rho) + ...] = 0$
- Momentum conservation: $\nabla^j T_{ij} + ... = 0$

- **Example of matter Lagrangian**

Calcagni 0904.0829;
Kiritsis, Kofinas 0904.1334;
Mukohyama 0904.2190;
Wang, Wands, Maartens 0909.5167

$$\mathcal{L}_m = \frac{1}{2N^2}\left(\dot{\varphi} - N^i \partial_i \varphi\right)^2 - V$$

$$V = V_0(\varphi) + \frac{1}{2}(\partial_i \varphi)^2 + ... + V_6(\varphi)\Delta\varphi\Delta^2\varphi$$

- EOM

$$\frac{1}{N\sqrt{g}}\partial_t\left[\frac{\sqrt{g}}{N}\left(\dot{\varphi} - N^i \partial_i \varphi\right)\right] = ...$$

# *Physical d.o.f.*

- Transverse-traceless graviton + longitudinal graviton

$$g_{ij} = \delta_{ij} \quad + \quad h_{ij} \quad - \quad 2\psi\delta_{ij}$$

- Quadratic action

$$S_2 = -\int dt d^3x \left[ \frac{1}{c_g^2}\dot{\psi}^2 - (\partial\psi)^2 \right], \; c_g^2 := \frac{1-\lambda}{3\lambda - 1}$$

✓ $c_g^2 > 0:$ ghost

Horava 0901.3775;
Blas, Pujolas, Sibiryakov 0909.3525;
Koyama, Arroja 0910.1998

✓ $c_g^2 < 0:$ unstable

See, however: Izumi, Mukohyama 0911.1814

✓ Strong self-coupling for $c_g^2 \to 0, \quad S_3 \supset \frac{1}{c_g^4}\psi\dot{\psi}^2$

# *Horava-Lifshitz cosmology*

- Flat cosmological background: $N = 1, N_i = 0, g_{ij} = a^2(t)\delta_{ij}$
- Evolution eq. $2\dot{H} + 3H^2 = -8\pi G p$ $\quad H := \dot{a}/a$

$$8\pi G := 2/M_{\rm Pl}^2(3\lambda - 1)$$

*If matter is conserved locally*

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*If matter is conserved locally*

$$\dot{\rho} + 3H(\rho + p) = 0$$

*Integrate*

$$3H^2 = 8\pi G\left[\rho + \mathcal{E}\right], \; \mathcal{E} = \frac{C}{a^3}$$

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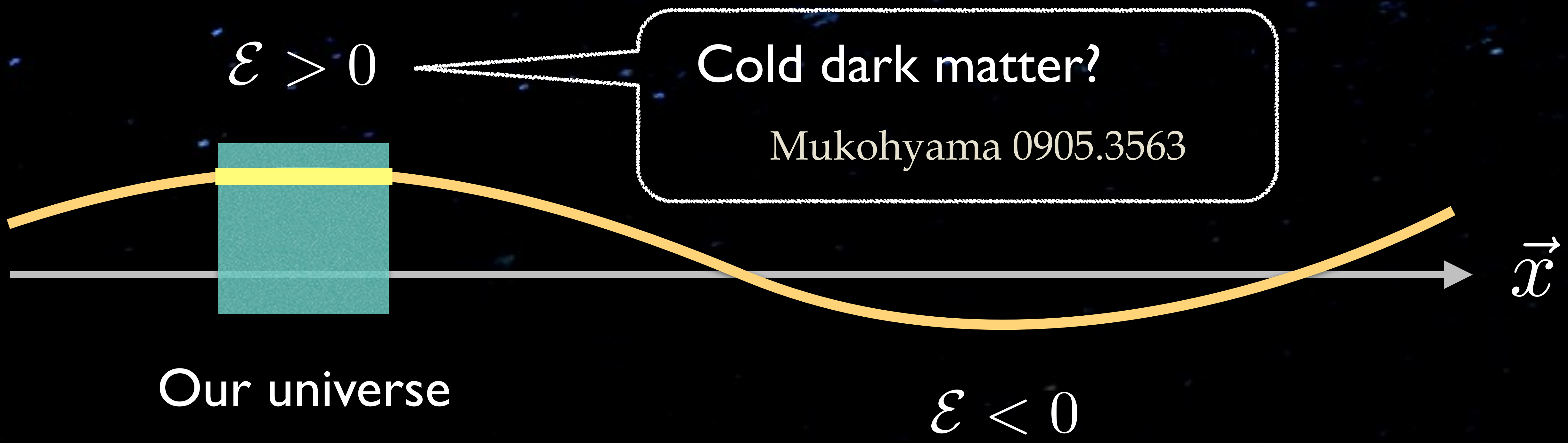
- In GR, local Hamiltonian constraint forces *C* to vanish
- But not in HL gravity
- Dust-like component (~ $a^{-3}$) ... cold dark matter?

Mukohyama 0905.3563

- *Global Hamiltonian constraint is not restrictive in cosmology*

$$\int d^3x\, a^3 \left(\frac{3H^2}{8\pi G} - \rho\right) = \int d^3x\, a^3 \mathcal{E} = 0$$

- Homogeneous in our observable patch of the universe
- Not necessarily homogeneous on lager scales



See also: Takahashi, Soda 0904.0554; Gao, Wang, Brandenberger, Riotto 0905.3821; Wang, Maartens 0907.1748; Gong, Koh, Sasaki 1002.1429; ......

# *Cosmological perturbations*

- Scalar-type variables:

$$N = 1 + A(t),\ N_i = a^2\beta_{,i},\ g_{ij} = a^2[(1-2\psi)\delta_{ij} + 2E_{,ij}]$$

See also: Takahashi, Soda 0904.0554; Gao, Wang, Brandenberger, Riotto 0905.3821; Wang, Maartens 0907.1748; Gong, Koh, Sasaki 1002.1429; ......

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Set $A$=0 by $t \to t + f(t)$

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Set A=0 by $t \to t + f(t)$ Set E=0 by $x^i \to x^i + \partial^i \chi(t, \vec{x})$

- Study large scale evolution of perturbations without using Hamiltonian constraint TK, Urakawa, Yamaguchi 0908.1005

$$\frac{\nabla^2}{a^2} \ll H^2$$

# *Perturbation equations*

- Evolution eq.

$$2\left(\ddot{\psi}+3H\dot{\psi}\right)+\mathcal{O}(\nabla^2)=8\pi G\delta p$$

- Define $\varepsilon(t,\vec{x})$ by

$$8\pi G\left[\varepsilon(t,\vec{x})+\delta\rho\right]=-6H\dot{\psi}$$

*Perturbation of "DM"*

$$8\pi G\varepsilon = \text{“}\ 8\pi G\delta T_0{}^0-\delta G_0{}^0\ \text{”}$$

*Hamiltonian constraint is not satisfied locally*

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*Hamiltonian constraint is not satisfied locally*

- Evolution eq. can be written in the form of coservation eq.

$$\dot{\varepsilon}+\dot{\delta\rho}+3H(\varepsilon+\delta\rho+\delta p)-3\dot{\psi}(\mathcal{E}+\rho+p)=\mathcal{O}(\nabla^2)$$

# *Translating into a suggestive form...*

- Define $\zeta := -\psi - H\frac{\varepsilon + \delta\rho}{\dot{\mathcal{E}} + \dot{\rho}} \left( = -\psi + \frac{H}{\dot{H}}\dot{\psi} \right)$
- Evolution eq. translates to:

$$\dot{\zeta} \simeq -\frac{H}{\mathcal{E} + \rho + p}\delta p_{\mathrm{nad}} + H c_s^2 f(1-f)\mathcal{S}_{\mathrm{HL}}$$

where $\delta p_{\mathrm{nad}} = \delta p - c_s^2 \delta\rho,\ c_s^2 = \dot{p}/\dot{\rho},\ f = \frac{\dot{\rho}}{\dot{\mathcal{E}} + \dot{\rho}}$

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- Relative entropy perturbation between "DM" and ordinary matter $\mathcal{S}_{\rm HL} = 3\left(\zeta_{\rm HL} - \zeta_{\rm m}\right)$

where $\zeta_{\rm HL} = -\psi - H\frac{\varepsilon}{\dot{\mathcal{E}}},\ \zeta_{\rm m} = -\psi - H\frac{\delta\rho}{\dot{\rho}}$

# *Conservation of $\zeta$ ?*

$$\dot{\zeta} \simeq -\frac{H}{\mathcal{E}+\rho+p}\delta p_{\rm nad} + H c_s^2 f(1-f)\mathcal{S}_{\rm HL}$$

- $\dot{\zeta} \simeq 0$ if $c_s^2 = 0$ / $\rho \gg \mathcal{E}$
- Matter energy "conservation"

$$\dot{\delta\rho} + 3H(\delta\rho + \delta p) - 3\dot{\psi}(\rho + p) - \frac{1}{a^2}\partial_i\left[J^i + (\rho+p)N^i\right] = \mathcal{O}(\nabla^4/M^2)$$

$$\longrightarrow \quad \dot{\zeta}_{\rm m} \simeq 0,\ \dot{\zeta}_{\rm HL} \simeq 0 \quad \longrightarrow \quad \dot{\mathcal{S}}_{\rm HL} \simeq 0$$

- Can solve for $\psi$ :

$$\psi(t,\vec{x}) = -\zeta_{\rm HL}(\vec{x}) + \mathcal{S}_{\rm HL}(\vec{x})\frac{H(t)}{2}\int\frac{(1+p/\rho)dt'}{1+\mathcal{E}/\rho}$$

- If "DM" is really DM, need natural mechanism to explain

$$\mathcal{S}_{\mathrm{HL}} \simeq 0$$

  - *If possible, would be very interesting*

- Observed perturbations are almost adiabatic

$$\frac{P_{\mathcal{S}}}{P_\zeta + P_{\mathcal{S}}} < 0.1$$

WMAP; Kawasaki, Sekiguchi 0705.2853

# *Healthy extension*

Blas, Pujolas, Sibiryakov 0909.3525

- Can construct "healthy" non-projectable model by adding

$$a_i a^i, R\nabla_i a^i, \ldots, \qquad a_i := \partial_i \ln N$$

- Gravitational Lagrangian:

$$\frac{M_{\rm Pl}^2}{2}\left[\mathcal{L}_K + R + \eta a_i a^i + \ldots\right]$$

- Longitudinal graviton is stable in IR if $0 < \eta < 2$

$$\mathcal{L}_2 \sim \frac{3\lambda - 1}{\lambda - 1}\dot{\psi}^2 - \frac{2-\eta}{\eta}(\partial\psi)^2$$

- IR limit is equivalent to special case of Einstein-Aether theory (*)

(*) *GR coupled to a dynamical, unit timelike vector* Jacobson 1001.4823

# *Healthy extension contd.*

- Low energy phenomenology
  - Gravitational field of static point source    Blas, Pujolas, Sibiryakov 0909.3525
    - PPN $\gamma = 1$
    - $8\pi G_N = \frac{1}{M_{\rm Pl}^2(1-\eta/2)} \neq \frac{2}{M_{\rm Pl}^2(3\lambda-1)} = 8\pi G_{\rm cosm}$
  - Density perturbations    TK, Urakawa, Yamaguchi 1002.3101

$$\ddot{\delta} + 2H\dot{\delta} = \frac{\nabla^2}{a^2}\phi,$$

$$\frac{\nabla^2}{a^2}\phi = 4\pi G_N \rho\delta$$

# *Summary*

- In Horava gravity Lorentz symmetry is broken in exchange for power-counting renormalizability
- *Natural to assume projectability condition*
  - Hamiltonian constraint is not satisfied locally
  - DM-like component: Need mechanism to make isocurvature contribution small

TK *et al*. 0908.1005

- *Non-projectable model can be extended to healthy one*
  - Cosmology and perturbation evolution are addressed

TK *et al*. 1002.3101