Primordial Non-Gaussianity in Inflation

初期密度ゆらぎの非ガウス性とインフレーションモデル

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研究会 第二回　超弦理論と宇宙 in 城崎温泉 2/19

Outlook

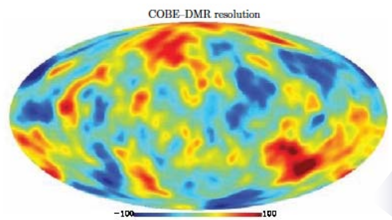
- Introduction
- Non-Gaussianity
- Formulation (delta N formula)
- Model
- Discussion

Introduction *– Observation-*

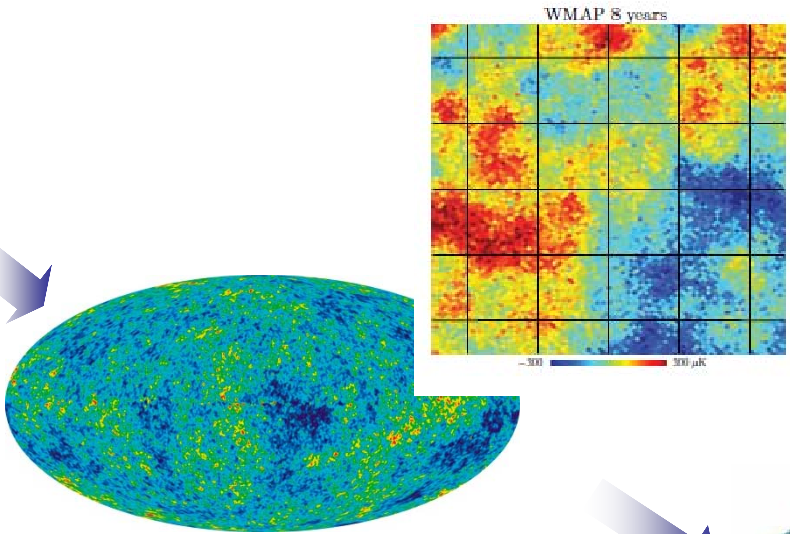
- Progress of observational technology

➔ Precision cosmology

We can determine/constrain possible models/theories of the early universe from observations.

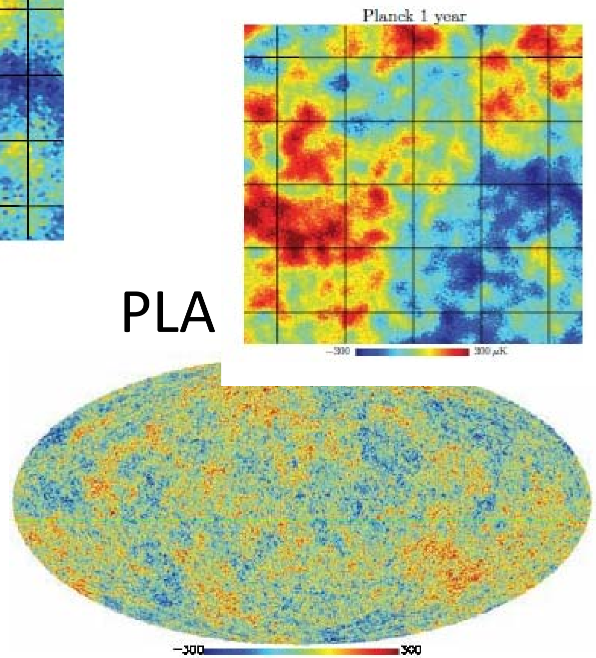


COBE(1993)



WMAP(2003,2006,2008)

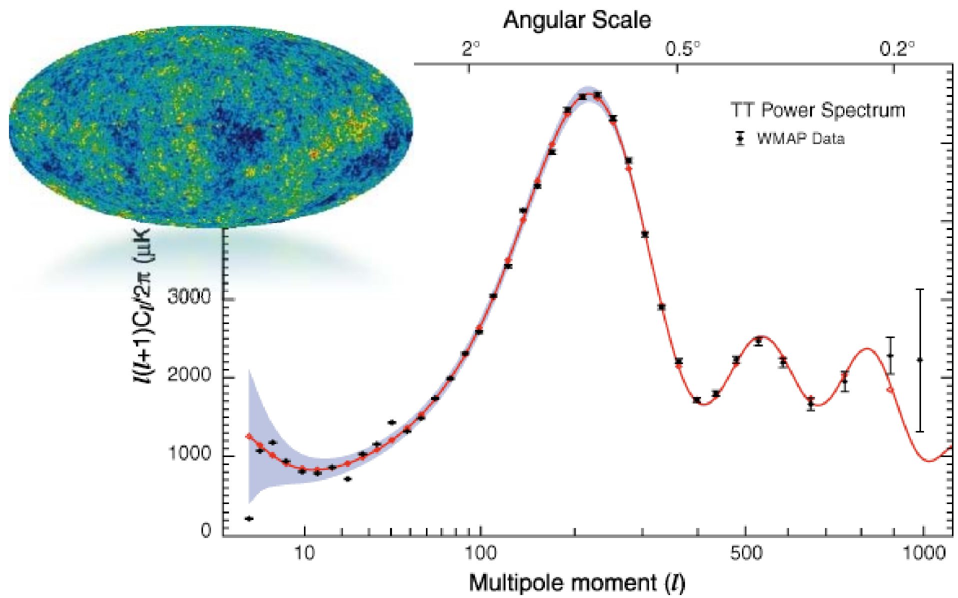
PLA



(PLANCK team)

Introduction *– CMB observation-*

◆ Cosmic Microwave Background (CMB) Anisotropy



(WMAP Five year data (WMAP team))

•Information about
  the evolution of perturbations

+

•Information about
  the primordial perturbations

• amplitude
$$\sim 10^{-5}$$

• spectral index

Almost scale-inv.

•statistics

Almost Gaussian

•statistical isotropy

SY and Soda (2008)

Test of the mechanism of generating primordial perturbations
(Inflation model)

@ Paul Shellard

S-dimensional assisted inflation
assisted brane inflation
anomoly-induced inflation
assisted inflation
assisted chaotic inflation
boundary inflation
brane inflation
brane-assisted inflation
brane gas inflation
brane-antibrane inflation
braneworld inflation
Brans-Dicke chaotic inflation
Brans-Dicke inflation
bulky brane inflation
chaotic inflation
chaotic hybrid inflation
chaotic new inflation
D-brane inflation
D-term inflation
dilaton-driven inflation
dilaton-driven brane inflation
double inflation
double D-term inflation
dual inflation
dynamical inflation
dynamical SUSY inflation
eternal inflation
extended inflation
extended open inflation
extended warm inflation
extra dimensional inflation
F-term inflation
F-term hybrid inflation
false-vacuum inflation
false-vacuum chaotic inflation
fast-roll inflation
first-order inflation
gauged inflation
Hagedorn inflation
higher-curvature inflation
hybrid inflation
hyperextended inflation
induced gravity inflation
intermediate inflation
inverted hybrid inflation
isocurvature inflation......................

Introduction – *WMAP constraint* -

Table 1
Summary of the Cosmological Parameters of ΛCDM Model and the Corresponding 68% Intervals

Class	Parameter	*WMAP* 5 Year ML[a]	*WMAP*+BAO+SN ML	*WMAP* 5 Year Mean[b]	*WMAP*+BAO+SN Mean
Primary	$100\Omega_b h^2$	2.268	2.262	2.273 ± 0.062	$2.267^{+0.058}_{-0.059}$
	$\Omega_c h^2$	0.1081	0.1138	0.1099 ± 0.0062	0.1131 ± 0.0034
	Ω_Λ	0.751	0.723	0.742 ± 0.030	0.726 ± 0.015
	n_s	0.961	0.962	$0.963^{+0.014}_{-0.015}$	0.960 ± 0.013
	τ	0.089	0.088	0.087 ± 0.017	0.084 ± 0.016
	$\Delta^2_{\mathcal{R}}(k_0^c)$	2.41×10^{-9}	2.46×10^{-9}	$(2.41 \pm 0.11) \times 10^{-9}$	$(2.445 \pm 0.096) \times 10^{-9}$
Derived	σ_8	0.787	0.817	0.796 ± 0.036	0.812 ± 0.026
	H_0	$72.4\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$	$70.2\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$	$71.9^{+2.6}_{-2.7}\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$	$70.5 \pm 1.3\,\mathrm{km\,s^{-}\,Mpc^{-}}$
	Ω_b	0.0432	0.0459	0.0441 ± 0.0030	0.0456 ± 0.0015
	Ω_c	0.206	0.231	0.214 ± 0.027	0.228 ± 0.013
	$\Omega_m h^2$	0.1308	0.1364	0.1326 ± 0.0063	$0.1358^{+0.0037}_{-0.0036}$
	z^d_{reion}	11.2	11.3	11.0 ± 1.4	10.9 ± 1.4
	t^e_0	13.69 Gyr	13.72 Gyr	13.69 ± 0.13 Gyr	13.72 ± 0.12 Gyr

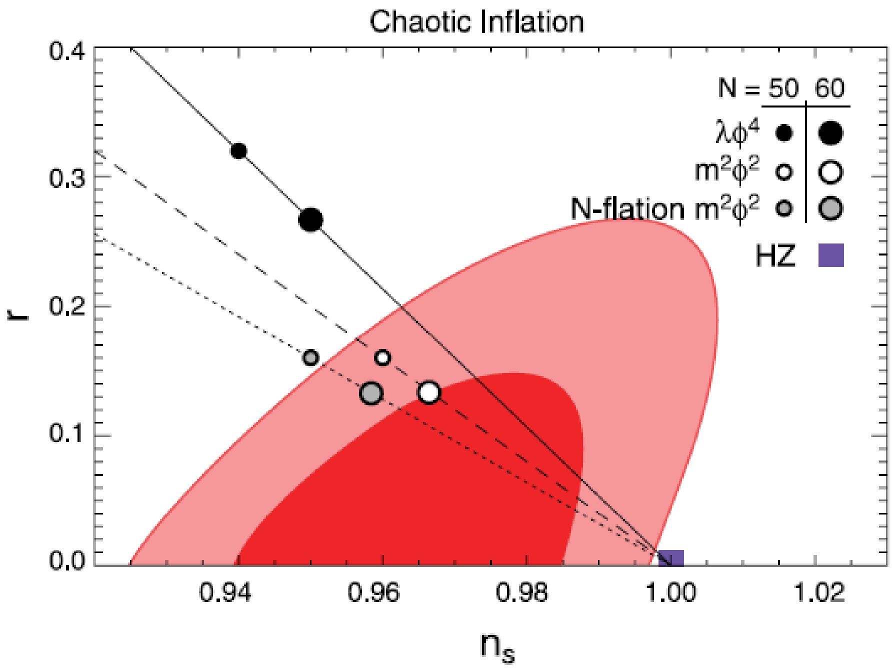
(WMAP 5 year data)

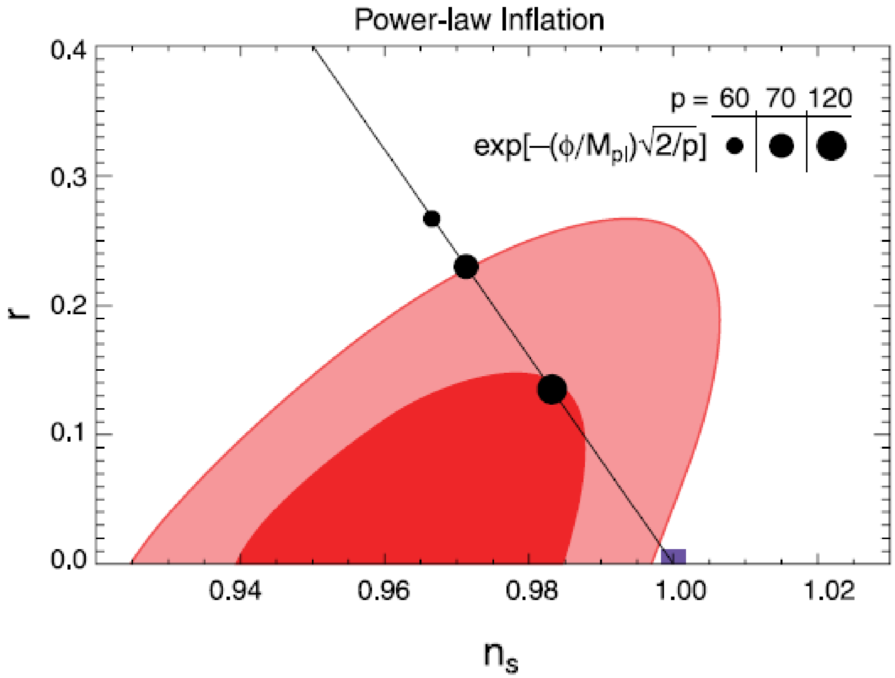
Table 2
Summary of the 95% Confidence Limits on Deviations from the Simple (Flat, Gaussian, Adiabatic, Power-Law) ΛCDM Model

Section	Name	Type	*WMAP* 5 Year	*WMAP*+BAO+SN
Section 3.2	Gravitational wave[a]	No running index	$r < 0.43$[b]	$r < 0.22$
Section 3.1.3	Running index	No grav. wave	$-0.090 < dn_s/d\ln k < 0.019$[c]	$-0.068 < dn_s/d\ln k < 0.012$
Section 3.4	Curvature[d]		$-0.063 < \Omega_k < 0.017$[e]	$-0.0179 < \Omega_k < 0.0081$[f]
	Curvature radius[g]	Positive curv.	$R_{\mathrm{curv}} > 12\,h^{-1}$ Gpc	$R_{\mathrm{curv}} > 22\,h^{-1}$ Gpc
		Negative curv.	$R_{\mathrm{curv}} > 23\,h^{-1}$ Gpc	$R_{\mathrm{curv}} > 33\,h^{-1}$ Gpc
Section 3.5	Gaussianity	Local	$-9 < f_{\mathrm{NL}}^{\mathrm{local}} < 111$[h]	N/A
		Equilateral	$-151 < f_{\mathrm{NL}}^{\mathrm{equil}} < 253$[i]	N/A
Section 3.6	Adiabaticity	Axion	$\alpha_0 < 0.16$[j]	$\alpha_0 < 0.072$[k]
		Curvaton	$\alpha_{-1} < 0.011$[l]	$\alpha_{-1} < 0.0041$[m]
Section 4	Parity violation	Chern–Simons[n]	$-5.^\circ9 < \Delta\alpha < 2.^\circ4$	N/A
Section 5	Dark energy	Constant w[o]	$-1.37 < 1 + w < 0.32$[p]	$-0.14 < 1 + w < 0.12$
		Evolving $w(z)$[q]	N/A	$-0.33 < 1 + w_0 < 0.21$[r]
Section 6.1	Neutrino mass[s]		$\sum m_\nu < 1.3$ eV[t]	$\sum m_\nu < 0.67$ eV[u]
Section 6.2	Neutrino species		$N_{\mathrm{eff}} > 2.3$[v]	$N_{\mathrm{eff}} = 4.4 \pm 1.5$[w] (68%)

Introduction – *WMAP constraint* -

(WMAP+BAO+SN)





(WMAP 5 year data)

$$n_s - 1 \equiv \frac{d\ln \mathcal{P}_\zeta}{d\ln k}$$; spectral index

$$r \equiv \frac{\mathcal{P}_T}{\mathcal{P}_\zeta}$$; tensor to scalar ratio

Introduction *- construction of realistic inflation models -*

Constructing realistic models based on SUGRA or string theory

✓the energy scale of inflation is much lower

✓ the scalar field may have multi-components during inflation

(Multi-scalar inflation)

Tensor perturbation

Tensor perturbation

The discrimination of the simplest single-field model from the other low energy models will be mostly clearly done by the future observation of CMB B-mode polarization.

(Primordial tensor perturbation)

Second order perturbation

Second order perturbation

Recently, the non-linearity (non-Gaussianity) of the primordial perturbations also has been a focus of constant attention by many authors.

(Komatsu & Spergel (2001), ……….)

Introduction *– Planck era -*

- PLANCK (2009 April)

(PLANCK team)

power spectrum

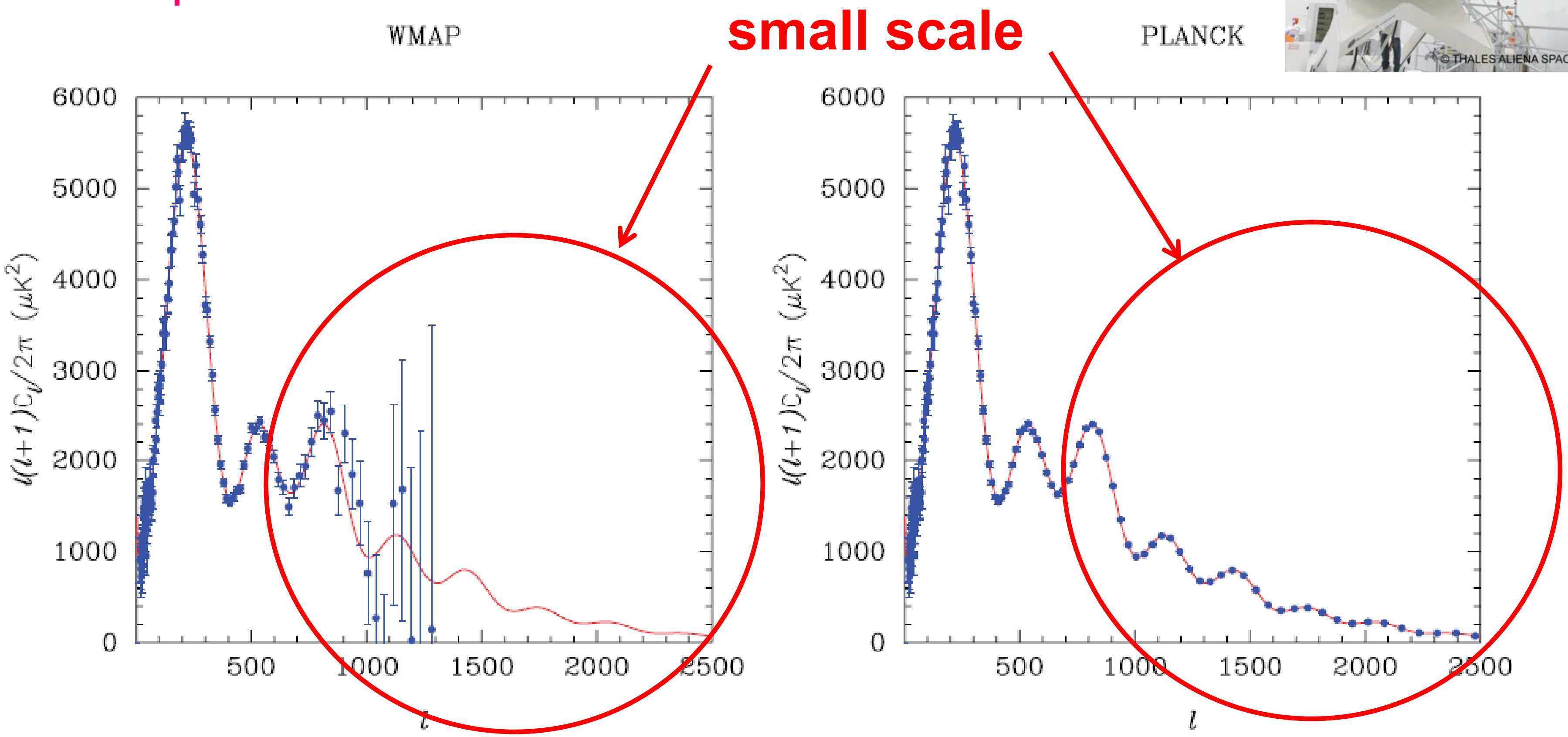


FIG 2.8.—The left panel shows a realisation of the CMB power spectrum of the concordance ΛCDM model (red line) after 4 years of *WMAP* observations. The right panel shows the same realisation observed with the sensitivity and angular resolution of *Planck*.

Introduction – *Planck era* -

- PLANCK (2009 April)

probability of detecting B-mode polarization

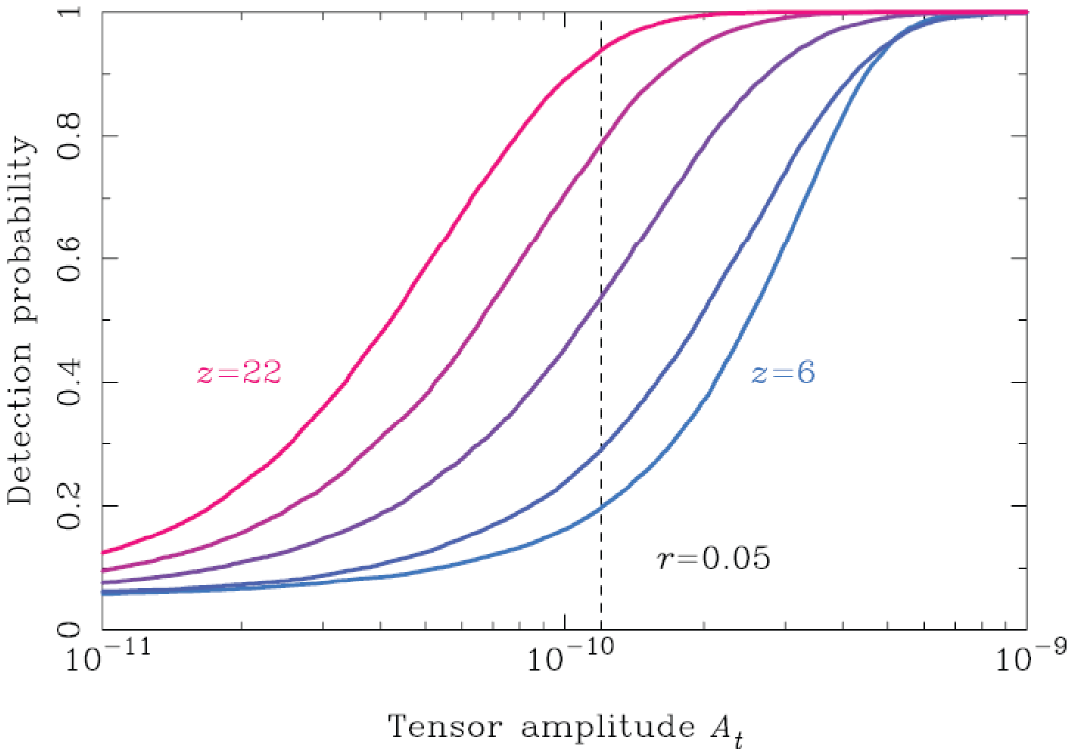


FIG 2.16.—The probability of detecting B-mode polarization at 95% confidence as a function of A_T, the amplitude of the primordial tensor power spectrum (assumed scale-invariant), for *Planck* observations using 65% of the sky. The curves correspond to different assumed epochs of (instantaneous) reionization: $z = 6$, 10, 14, 18 and 22. The dashed line corresponds to a tensor-to-scalar ratio $r = 0.05$ for the best-fit scalar normalisation, $A_S = 2.7 \times 10^{-9}$, from the one-year *WMAP* observations.

Introduction – *B-mode polarization -*

CMBPol mission (0811.3919[astro-ph])

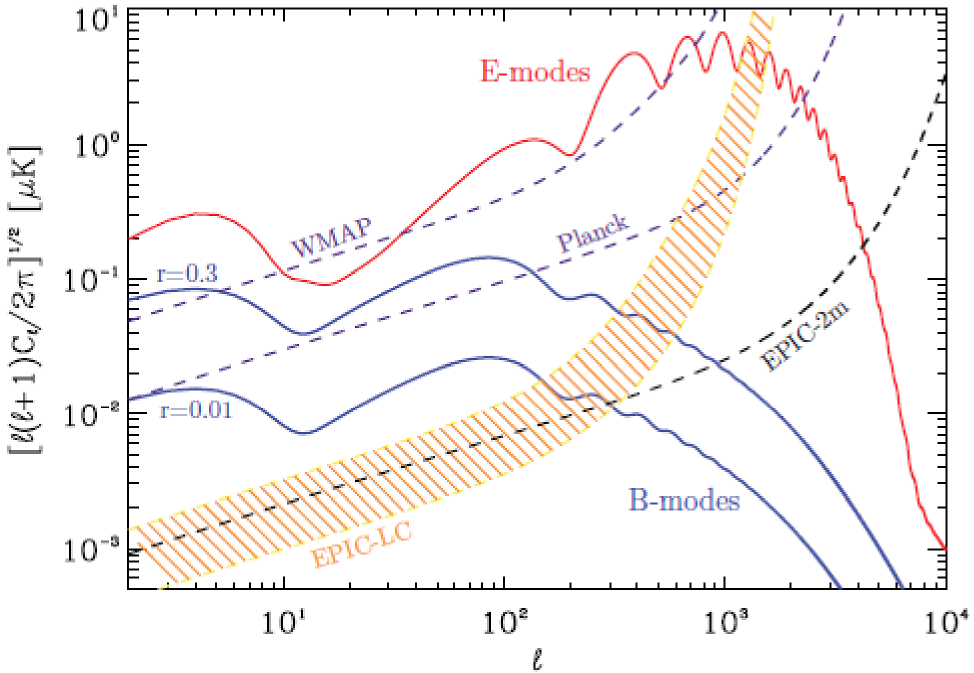
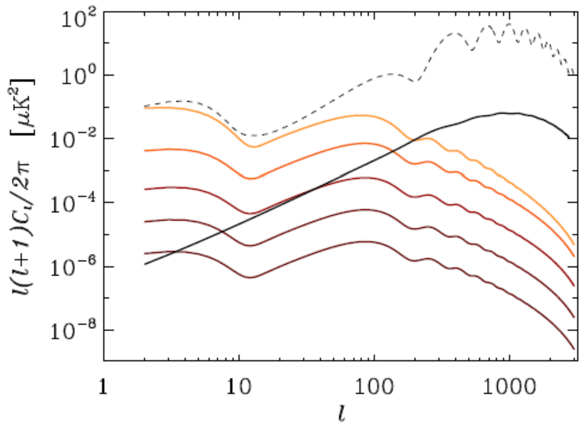


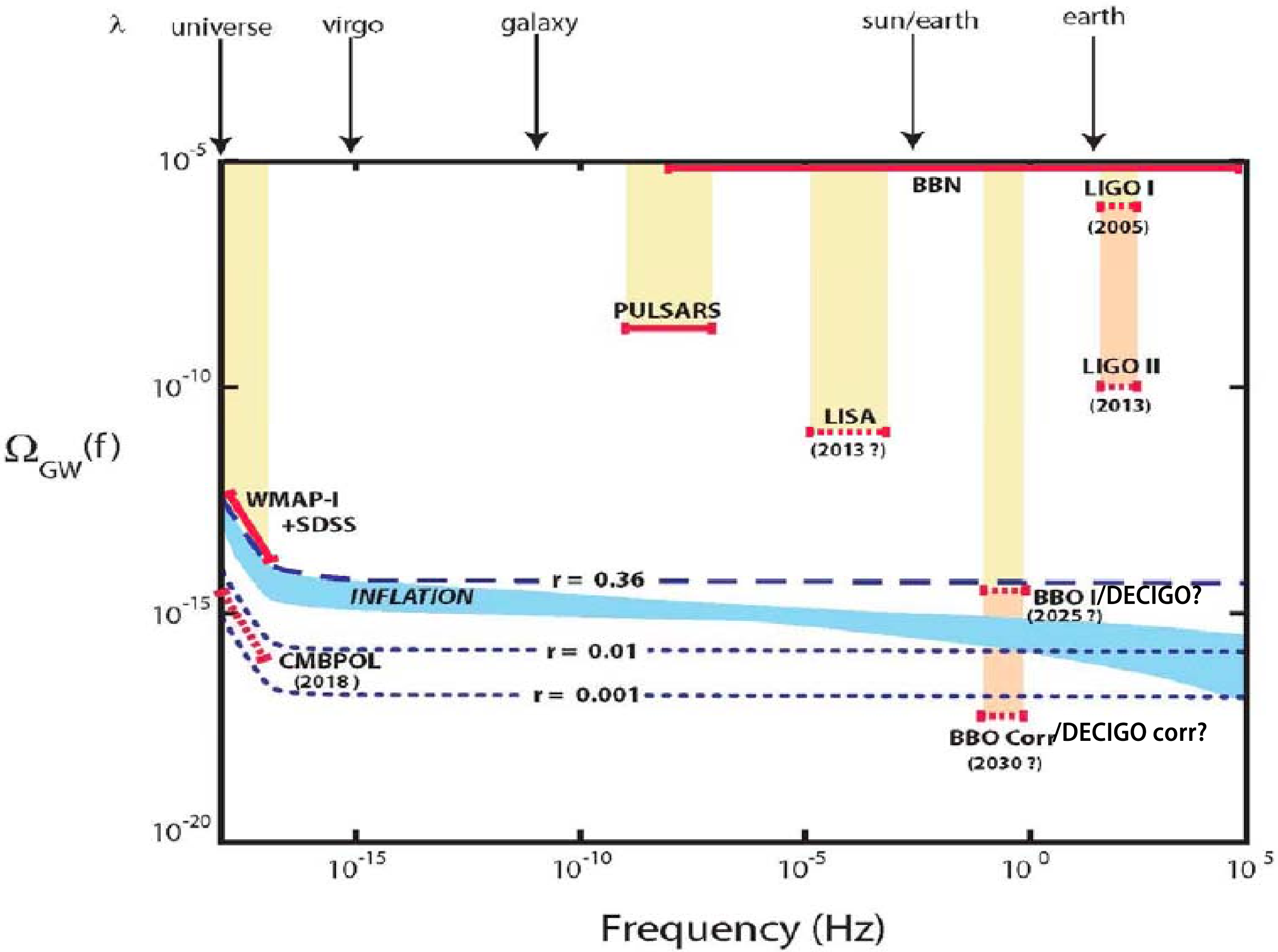
Figure 6: *E*- and *B*-mode power spectra for a tensor-to-scalar ratio saturating current bounds, $r = 0.3$, and for $r = 0.01$. Shown are also the experimental sensitivities for *WMAP*, *Planck* and two different realizations of *CMBPol* (EPIC-LC and EPIC-2m). (Figure adapted from Bock *et al.* [56].)

tensor mode vs lensing



Space mission (submitted to Astro 2010)

Introduction *– tensor mode –direct detection- -*



Gravitational wave background

CMBPol webpage

Non-Gaussianity

Non-linearity parameter

◆ Non-linear parameter

Curvature perturbation on uniform density slicing (a gauge invariant variable);

Komatsu and Spergel (2001)

$$\zeta(\mathbf{x}) = \underline{\zeta_G(\mathbf{x})} + \frac{3}{5}\underline{f_{NL}}\zeta_G^2(\mathbf{x})$$

Gaussian statistics Non-linear parameter

The power spectrum of curvature perturbation is leadingly identical to that of Gaussian part, while the three point correlation function is affected by the non-linear part.

$$\langle \zeta_{\mathbf{k}_1}\zeta_{\mathbf{k}_2}\zeta_{\mathbf{k}_3}\rangle \equiv \delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)B_\zeta(k_1,k_2,k_3)$$:bispectrum

$$B_\zeta(k_1,k_2,k_3) = -\frac{6}{5}\frac{f_{NL}}{(2\pi)^{3/2}}\left[P_\zeta(k_1)P_\zeta(k_2)+P_\zeta(k_2)P_\zeta(k_3)+P_\zeta(k_3)\underline{P_\zeta(k_1)}\right]$$

:power spectrum

WMAP (current obs.) $-9 < f_{NL}^{\text{local}} < 111$ (95% CL) → PLANCK $|f_{NL}| \gtrsim 5$

Non-linearity in cosmological perturbation

- Einstein equation …….

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$: nonlinear differential equation

- Primordial perturbation

Quantum fluctuation of Inflaton（scalar field）

If free field, the fluctuation is Gaussian

Through the interactions, the deviation from Gaussian (non-Gaussianity) can be generated.

Theoretical predictions

◆ non-linear parameter f_{NL}

- Standard single-scalar slow-roll inflation

$$f_{\rm NL} = \mathcal{O}(10^{-2}) \ll 1$$

(Maldacena (2003), Lyth & Rodoriguez (2005), ...)

- Curvaton scenario, Modulated reheating scenario

(Lyth et al (2003), Sasaki et al (2006), Dvali et al (2004)...)

- (DBI inflation (non-slow-roll model)) (Alishahisa et al (2004), ...)

$$f_{\rm NL} \gg 1$$

Detectable by the future experiments (PLANCK,....)!!

We cannot distinguish these scenario at the linear (power spectrum) level. (tensor mode?)

f_{NL} is expected as a new cosmological parameter, which brings us valuable information.

Theoretical predictions

How to generate the non-linearity？

1）. "scalar field –scalar field" coupling during inflation

$\delta\phi - \delta\phi$ （self-interaction term in potential)

✓Standard single slow-roll model ; coupling ⟺ po $f_{\rm NL} = \mathcal{O}(10^{-2}) \ll 1$ ameter

✓DBI inflation – (fast roll inflation) - $f_{\rm NL} \gg 1$ (equilateral type) $(k_1 = k_2 = k_3)$

2）. "curvature-scalar field" coupling

$\zeta - \delta\phi$ (c.f. "curvature - iso-curvature mode" coupling)

✓Curvaton model ; couplir $f_{\rm NL} \gg 1$ (local type)
an extra light scalar ... sity / radiation density
$(k_1 = k_2 \gg k_3)$

✓ Multi-scalar inflation

3）. Non-linearity in the transfer functions of temperature anisotropies

higher order correlation function

- The deviation from the pure Gaussian affects also higher order correlation functions (higher order spectrum).

ζ : primordial curvature perturbation

Bispectrum (connected part)

$$\langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle = \frac{6}{5}f_{NL}\left(P(k_1)P(k_2)+P(k_2)P(k_3)+P(k_3)P(k_1)\right)\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)$$

Trispectrum (connected part)

$$\langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\zeta(\mathbf{k}_4)\rangle = \Big[\tau_{NL}\left(P(k_{13})P(k_3)P(k_4)+(11\text{perms})\right)$$
$$+\frac{54}{25}g_{NL}\left(P(k_2)P(k_3)P(k_4)+(3\text{perms})\right)\Big]\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3+\mathbf{k}_4)$$

Non-linear parameters

$P(k_1)$; power spectrum

Can we detect？ (Komatsu & Spergel(2001), Kogo & Komatsu 2006)

If $f_{NL} > 5$ or $\tau_{NL} > 560$ we can detect the NG in future experiments (Planck(2009))

higher order correlation function

Roughly speaking,

- ✓ the leading order of n-point function is $O(P^{n-1})$ $\left(P \sim O(10^{-10})\right)$

- ✓ If the non-Gaussianity large, we can estimate $O(f_{NL}^{n-2} P^{n-1})$

  $(f_{NL} < O(100))$

- ✓ The number of argument wavenumbers of the n-point function is n-1. Naively, the number increases as $\ell_{max}^{2(n-1)}$

  where ℓ_{max} is the maximum angular momentum of CMB observations

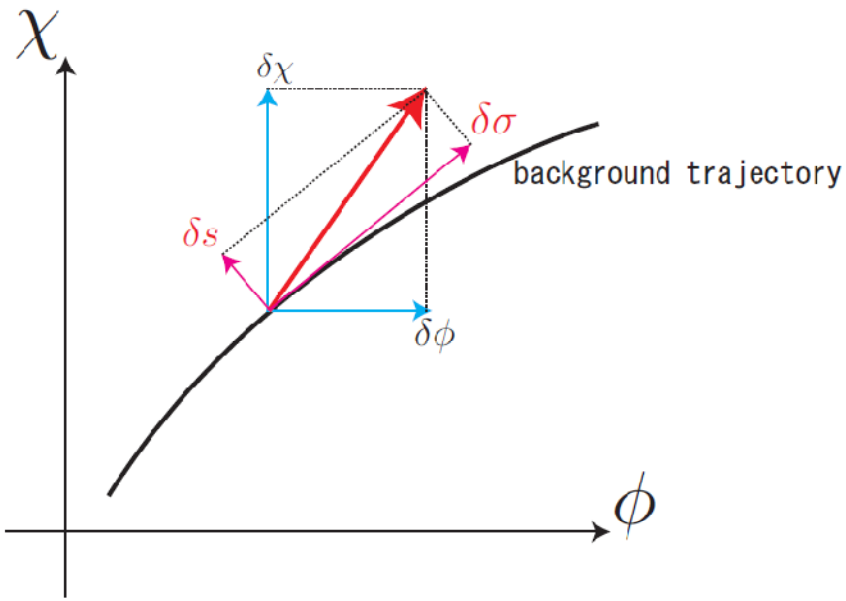
- ✓ This large number enhances the detectability of n-point functions to $O(f_{NL}^{-1} P^{-1/2} (f_{NL} \ell_{max} \sqrt{P})^{n-1})$

- ✓ Hence, if $f_{NL} \ell_{max} \sqrt{P}$ exceeds unity, in principle the higher order correlation functions are measurable.
- ✓ For Planck, $\ell_{max} \sim O(2000)$, and hence if $f_{NL} \sim O(50)$ then we can.

(Kogo and Komatsu (2006))

Motivation

◆ How is the primordial non-Gaussianity in Multi-scalar inflation ?



Multi-scalar models ;

- a lot of inflatons (c.f. curvaton scenario)
- a lot of dynamical degree of freedom during inflation

→

- **the effects of the iso-curvature mode**
- **the violation of the slow-roll conditions**

multi-field case

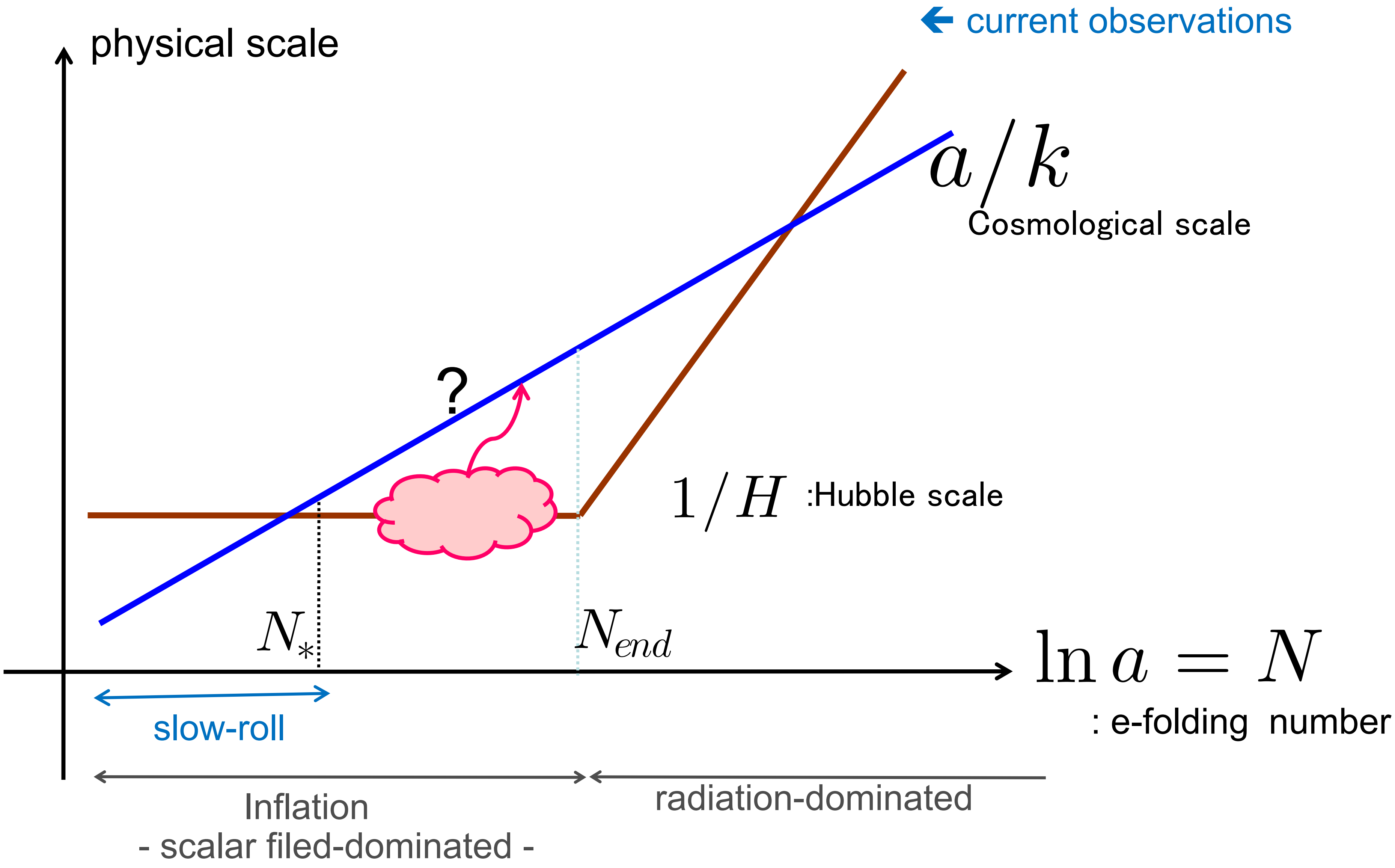
$$\zeta \neq \mathrm{const.}$$

on super-horizon scales

Local type non-Gaussianity

• without slow-roll condition on super-horizon scales

•The slow-roll conditions are satisfied for all directions on sub-horizon scales



δN formalism for the primordial non-Gaussianity

Second order CPT ;

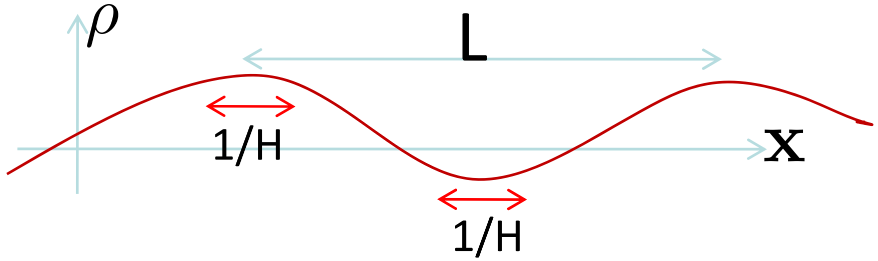
For example, {00}-Einstein tensor;

$$\delta^{(2)}G^{0}{}_{0} = \frac{1}{a^2}\Big(3\Big(\frac{a'}{a}\Big)^2\phi^{(2)} + 3\,\frac{a'}{a}\psi^{(2)'} - \nabla^2\psi^{(2)} + \frac{a'}{a}\nabla^2\omega^{(2)} - \frac{1}{4}\partial_k\partial_i\, D^{ki}\chi^{(2)} \tag{A.39}$$
$$- 12\left(\frac{a'}{a}\right)^2\left(\phi^{(1)}\right)^2 - 12\,\frac{a'}{a}\,\phi^{(1)}\,\psi^{(1)'} - 3\,\partial_i\psi^{(1)}\,\partial^i\psi^{(1)} - 8\,\psi^{(1)}\,\nabla^2\psi^{(1)} + 12\,\frac{a'}{a}\psi^{(1)}\,\psi^{(1)'}$$
$$- 3\left(\psi^{(1)'}\right)^2 + 4\,\frac{a'}{a}\,\phi^{(1)}\,\nabla^2\omega^{(1)} - 2\,\frac{a'}{a}\partial_k\omega^{(1)}\,\partial^k\phi^{(1)} - \frac{1}{2}\frac{a''}{a}\,\partial_k\omega^{(1)}\,\partial^k\omega^{(1)}$$
$$+ \frac{1}{2}\,\partial_i\partial_k\omega^{(1)}\,\partial^i\partial^k\omega^{(1)} - \frac{1}{2}\,\partial_k\partial^k\omega^{(1)}\,\partial_k\partial^k\omega^{(1)} - 2\,\frac{a'}{a}\partial_k\psi^{(1)}\,\partial^k\omega^{(1)} + 4\,\frac{a'}{a}\psi^{(1)}\,\nabla^2\omega^{(1)}$$
$$- 2\,\partial_k\omega^{(1)}\,\partial^k\psi^{(1)'} - 2\psi^{(1)'}\nabla^2\omega^{(1)} - \phi^{(1)}\,\partial_i\partial^k\, D^i{}_k\chi^{(1)} - 2\,\psi^{(1)}\partial_k\partial^i\, D^k{}_i\chi^{(1)}$$
$$+ \partial_k\partial_i\psi^{(1)}\, D^{ki}\chi^{(1)} - 2\,\frac{a'}{a}\partial_i\partial_k\omega^{(1)}\, D^{ik}\chi^{(1)} - 2\,\frac{a'}{a}\partial_k\omega^{(1)}\,\partial_i\, D^{ik}\chi^{(1)} - \partial_k\omega^{(1)}\,\partial^i D^k{}_i\chi^{(1)'}$$
$$- \frac{1}{2}\,\nabla^2 D_{mk}\chi^{(1)}\, D^{km}\chi^{(1)} + \partial_m\partial^k\, D_{ik}\chi^{(1)}\, D^{im}\chi^{(1)} + \frac{1}{2}\,\partial_k D^{km}\chi^{(1)}\,\partial^i D_{mi}\chi^{(1)}$$
$$- \frac{1}{8}\,\partial^i D^{km}\chi^{(1)}\,\partial_i D_{km}\chi^{(1)} + \frac{1}{8}\, D^{ik}\chi^{(1)'}\, D_{ki}\chi^{(1)'} + \frac{a'}{a} D^{ki}\chi^{(1)}\, D_{ik}\chi^{(1)'}\Big)\,;$$

It seemes complicated to solve fully… ➔ delta N

Formulation – *δN formalism-* *(Sasaki & Tanaka(1998),Lyth et al.(2005))*

- Focus on the Super-horizon fluctuations
- Smoothing on sub-horizon scales
- Locally, the universe seems FRW.



Naively,

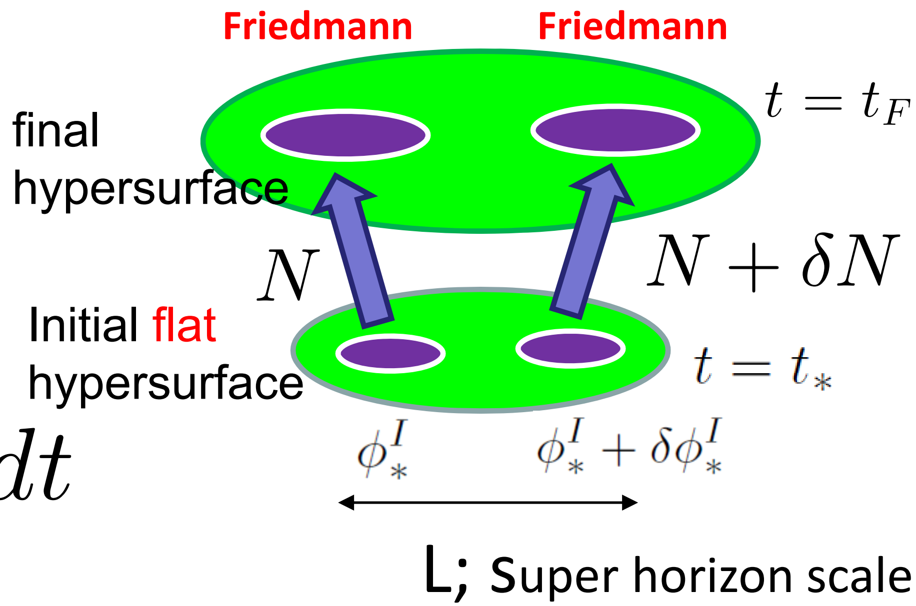
$$ds^2 = -dt^2 + \underline{a^2 e^{2\zeta}}\delta_{ij}dx^i dx^j$$

$$\gamma_{ij} = O((\tfrac{1}{LH})^2)$$

$$e^{2(N+\delta N)}$$

e-folding number ;

$$N(t_F, t_*) \equiv \int_{t_*}^{t_F} H dt$$



$$\zeta(t_F) \simeq \delta N(t_F, \phi^I(t_*, \vec{x}))$$

Taking Final hypersurface to be uniform energy density one and Initial hypersurface to be flat one

Formulation *– δN formalism-*

δN formalism (Sasaki & Tanaka (1998)) (separte universe)

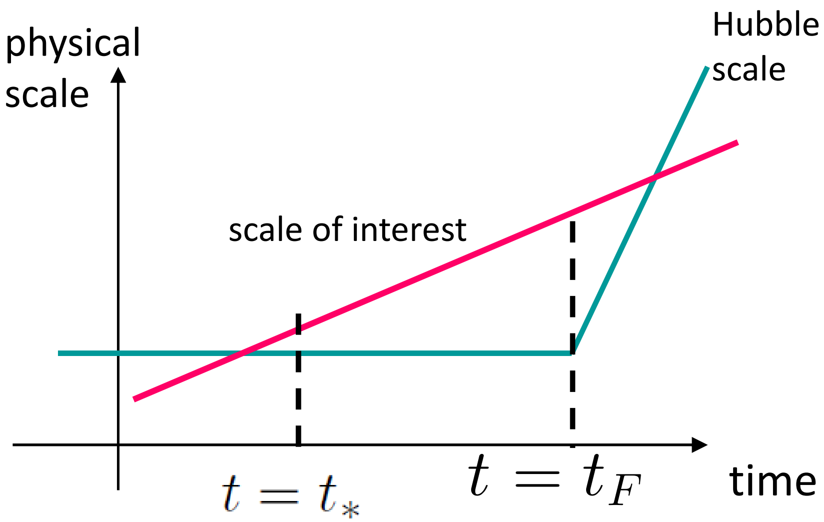
$$\zeta(t_F) \simeq \delta N = N_I^* \delta\phi_*^I + \frac{1}{2} N_{IJ}^* \delta\phi_*^I \delta\phi_*^J + \cdots$$

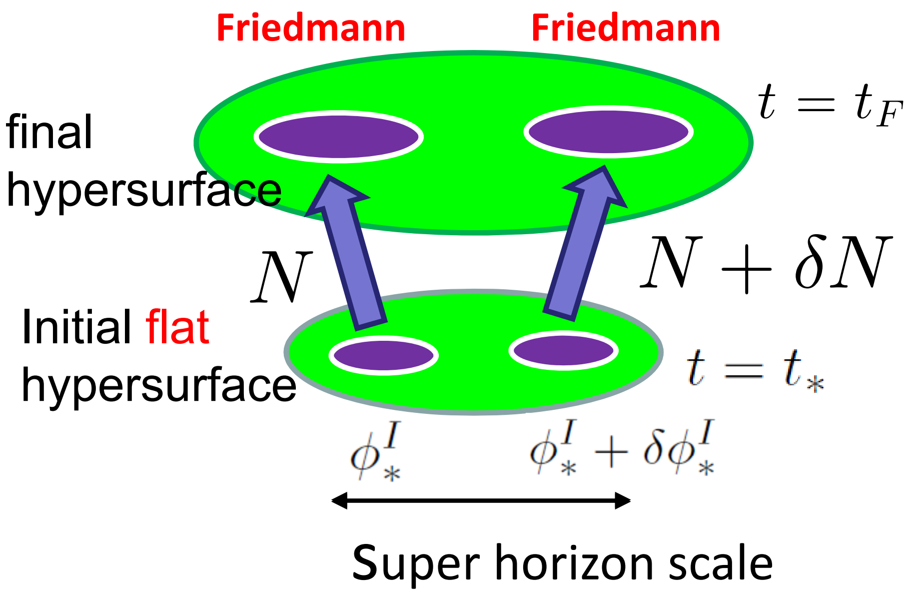
uniform energy density slicing

$$N_I(t) \equiv \left.\frac{\partial N(t_F, \phi^I)}{\partial \phi^I}\right|_{\phi^I = \phi^I(t)},$$

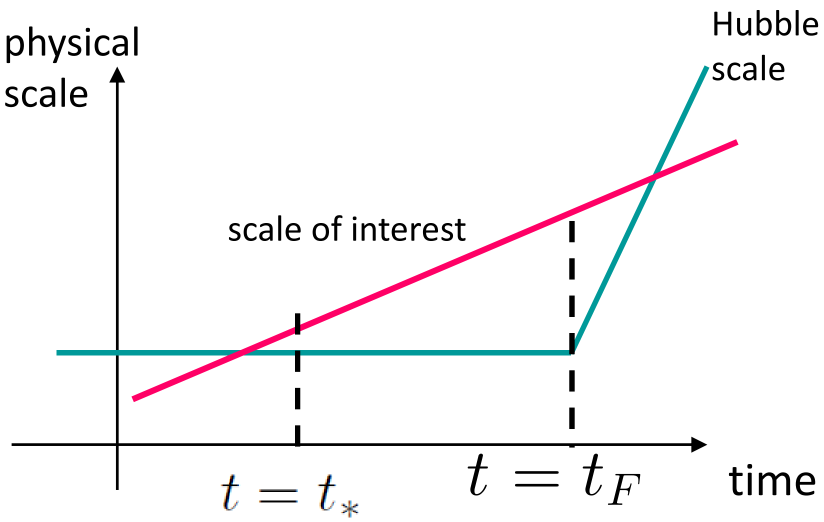
$$N_{IJ}(t) \equiv \left.\frac{\partial^2 N(t_F, \phi^I)}{\partial \phi^I \partial \phi^J}\right|_{\phi^I = \phi^I(t)}$$

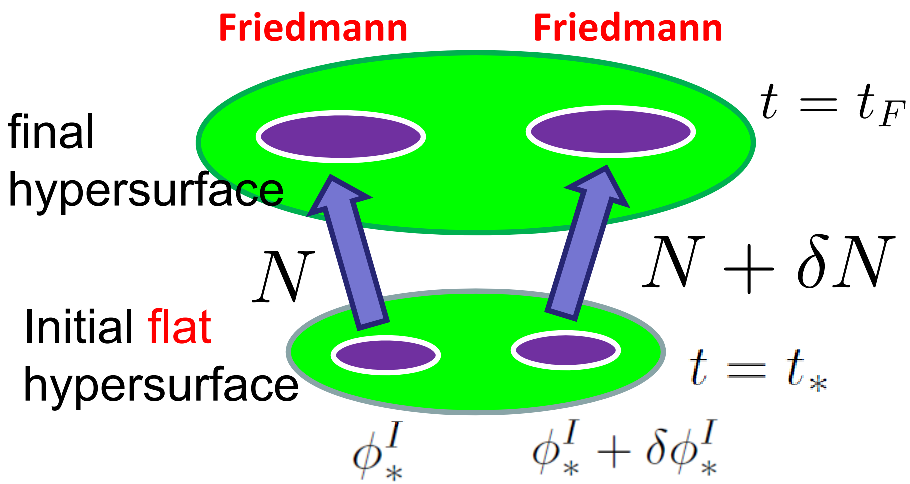
✓Super-horizon scale curvature perturbation is given by the perturbation of the background e-folding number





Formulation *– δN formalism-*





If,

$t_F > t_c$: a time when the background trajectory has converged.

⇔ pure adiabatic perturbation (no-iso-curvature)

then,

$$\zeta(t_F) = \text{const.}$$ (Lyth, Malik, and Sasaki (2005))

Formulation – *Non-Gaussianity* -

δN can be expanded up to the second order as (Lyth & Rodriguez, 2005)

$$\zeta(t_c) \simeq \delta N = N_I^* \delta\phi_*^I + \frac{1}{2} N_{IJ}^* \delta\phi_*^I \delta\phi_*^J + \cdots$$

to the second order

$$N_I(t) = \left.\frac{\partial N(t_c, \phi^I)}{\partial \phi^I}\right|_{\phi^I = \phi^I(t)},$$

$$N_{IJ}(t) \equiv \left.\frac{\partial^2 N(t_c, \phi^I)}{\partial \phi^I \partial \phi^J}\right|_{\phi^I = \phi^I(t)}$$

Bispectrum is leadingly given by

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle = \underbrace{N_I^* N_J^* N_K^* \langle \delta\phi_{\mathbf{k}_1 *}^I \delta\phi_{\mathbf{k}_2 *}^J \delta\phi_{\mathbf{k}_3 *}^K \rangle}_{①}$$

$$+ \underbrace{\frac{1}{2} N_I^* N_J^* N_{K_1 K_2}^* \left[\langle \delta\phi_{\mathbf{k}_1 *}^I \delta\phi_{\mathbf{k}_2 *}^J \left(\delta\phi^{K_1} \star \delta\phi^{K_2} \right)_{\mathbf{k}_3 *} \rangle \right.}_{②} \left. + \text{perms} \right]$$

① ←→ 1) ; "field-field" coupling → suppressed by slow-roll para. under the slow-roll approx.
(Seery & Lidsey (2005))

② ←→ 2) ; the non-linearity of the "curvature-field" coupling

Non-linearity parameter

Neglecting the contribution of ①, $\delta\phi^I - \delta\phi^I$
we have,

$$\frac{6}{5} f_{NL} \simeq \frac{N_*^I N_*^J N_{IJ}^*}{(N_K^* N_*^K)^2}$$

This represents the non-linearity which is generated on the super-horizon scales evolution.
（② term）"curvature - field" coupling ("curvature – iso-curvature")

What we need to calculate is only the evolution of background e-folding number

- Higher order correlation func.-

$\delta\phi_*^I$ is almost Gaussian (slow-roll inflation) (Seery and Lidsey(2006,2007))

$$\tau_{\rm NL} = \frac{N_{IJ}N^{IK}N^J N_K}{(N_L N^L)^3}, \quad g_{NL} = \frac{25}{54}\frac{N_{IJK}N^I N^J N^K}{(N_L N^L)^3}$$

(Byrnes, Sasaki & Wands(2006))

called as “local type ” spectrum

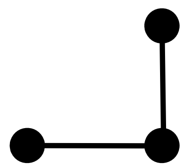
We have presented an efficient method for computing the non-linear parameters of the higher order correlation functions of local type curvature perturbations (SY, T. Suyama and T. Tanaka (2009))

delta N + Diagrammatic Approach (Byrnes, Koyama, Sasaki and Wands(2007))

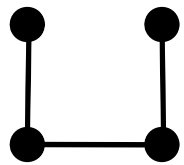
- Diagrammatic approach -

Byrnes, Koyama, Sasaki and Wands (2007)
SY, Suyama and Tanaka (2009)

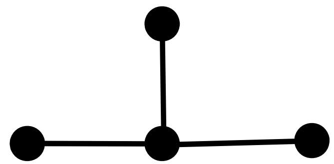
$$\frac{6}{5} f_{NL} = \frac{N_*^I N_*^J N_{IJ}^*}{(N_K^* N_*^K)^2}$$



$$\tau_{\rm NL} = \frac{N_{IJ} N^{IK} N^J N_K}{(N_L N^L)^3},$$



$$\frac{54}{25} g_{NL} = \frac{N_{IJK} N^I N^J N^K}{(N_L N^L)^3}$$



The number of the diagrams which have mutually different topology are corresponding to the number of parameters for n-point functions.

– the number of parameters for n-point functions --

The number of parameters for each correlation func. and diagram

TABLE 1

COUNTING TREE GRAPHS

N	(trees) t_N	(labeled trees) $T_N = N^{N-2}$
1	1	1
2	1	1
3	1	3
4	2	16
5	3	125
6	6	1 296
7	11	16 807
8	23	262 144
9	47	4 782 969
10	106	100 000 000
11	235	2 357 947 691
12	551	61 917 364 224
13	1301	1 792 160 394 037
14	3159	56 693 912 375 296
15	7741	1 946 195 068 359 375

(From Fry "Galaxy correlation" (1984))

Several examples

Example 1

Non-Gaussianity in slow-roll phase

(Analytic estimation)

Multi-scalar slow-roll

without specifying the form of potential

(SY, T. Suyama and T. Tanaka (2007))

$$-\frac{6}{5}f_{NL} = 2\left[\epsilon + \frac{\eta_{IJ}}{2V^K V_K}\left(2V^I V^J - 4V^I\tilde{\Theta}^J + \tilde{\Theta}^I\tilde{\Theta}^J\right)\right]_{\phi=\overset{(0)}{\phi}_{\rm f}}$$
$$+(N_*^I N_I^*)^{-2}\int_{N_*}^{N_{\rm f}} dN\, N_I(N) {Q^I}_{JK}(N)\,\Theta^J(N)\Theta^K(N)$$

assumption

$$\epsilon \equiv \frac{1}{2}\frac{V^I V_I}{V^2}$$

$$\epsilon \ll 1$$

- $V_{IJ}/V = O(\epsilon)$
- $V_{IJK}/V = O(\epsilon^{3/2})$:derivatives of potential
- $N \sim V/\frac{dV}{dN} \sim \frac{V}{V'\dot{\phi}} = \mathcal{O}(\epsilon^{-1})$:duration of inflation
- $N_I = \mathcal{O}(\epsilon^{-1/2})$
- $\Theta_I = \mathcal{O}(\epsilon^{-1/2})$
- $\tilde{\Theta}_I = \mathcal{O}(V\epsilon^{1/2})$

$$\Lambda^I{}_J(N,N') = \left[T\exp\left(\int_{N'}^{N} P(N'')dN''\right)\right]^I{}_J = O(1)$$

➔ loop hole ??

Naively, $f_{\rm NL} = \mathcal{O}(\epsilon) \ll 1$

➔ Next slide

Large non-Gaussianity in slow-roll inflation

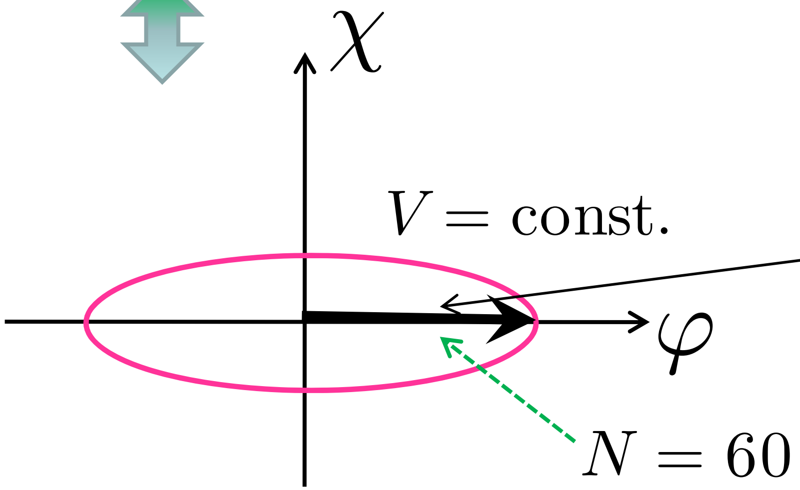
(Alabidi (2006), Byrnes, Choi and Hall(2008))

Potential;

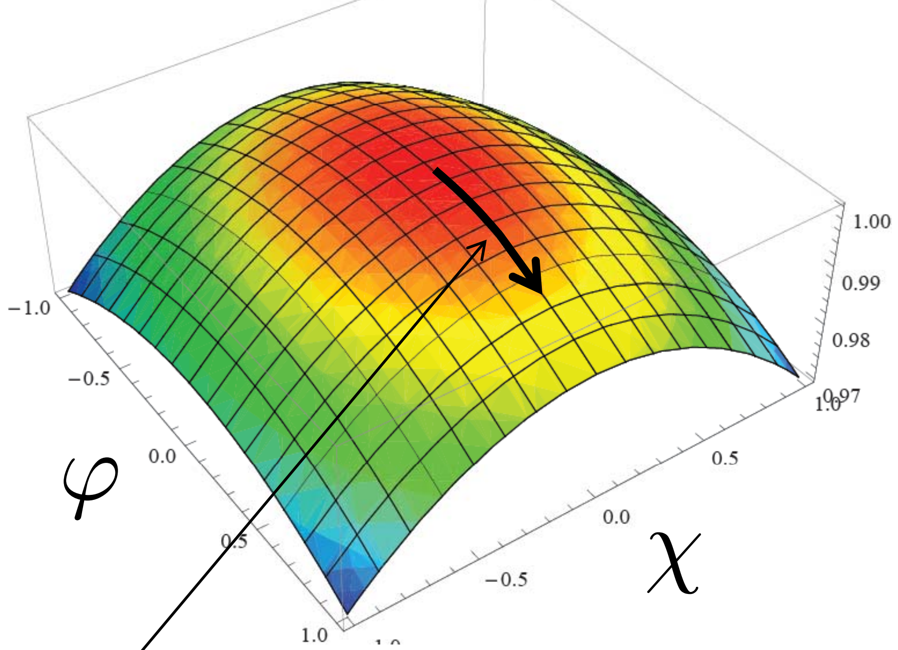
$$W(\varphi,\chi) = W_0 \left(1 + \frac{1}{2}\eta_{\varphi\varphi}\frac{\varphi^2}{M_P^2} + \frac{1}{2}\eta_{\chi\chi}\frac{\chi^2}{M_P^2}\right)$$

$|\eta_{\varphi\varphi}|, |\eta_{\chi\chi}| \ll 1$; slow-roll condition

The non-linearity of curvature perturbation has been evaluated at $V = \mathrm{const.}$ hypersurface.



potential form for $\eta_{\varphi\varphi}, \eta_{\chi\chi} < 0$



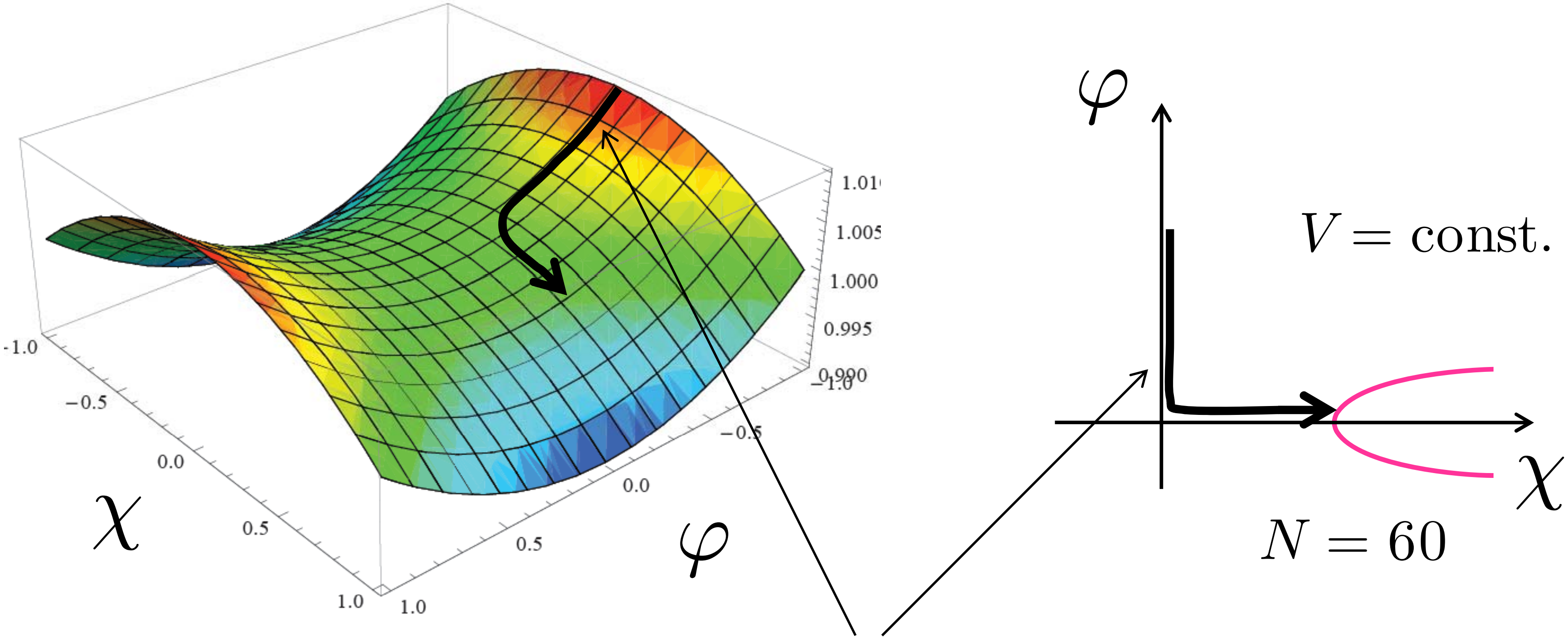
Black line; $\chi_* << M_{\mathrm{P}}$ trajectory

Small field model

Large non-Gaussianity in slow-roll inflation

(Alabidi (2006), Byrnes, Choi and Hall(2008))

potential form for $\eta_{\varphi\varphi} > 0, \eta_{\chi\chi} < 0$

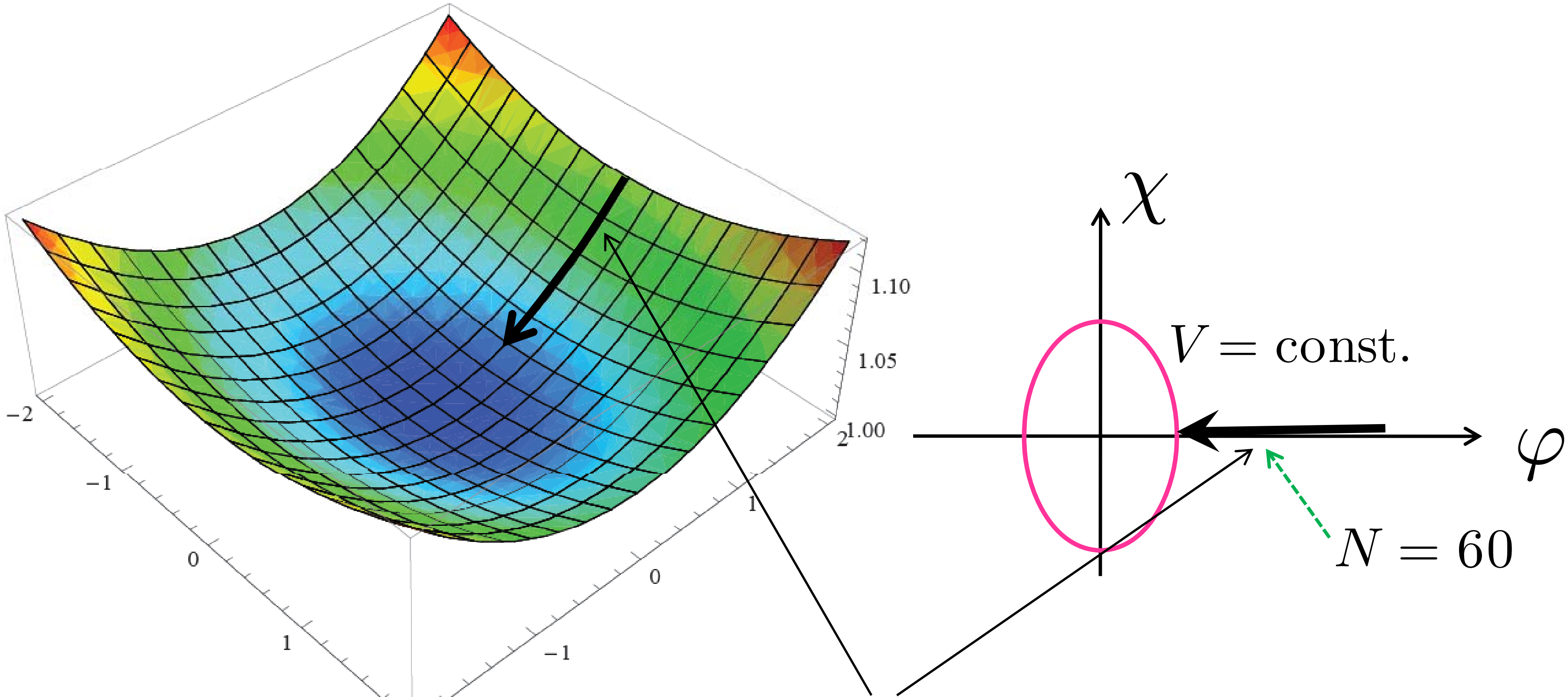


black line; $\chi_* << M_{\mathrm{P}}$ trajectory

saddle point model

Large non-Gaussianity in slow-roll inflation

(Alabidi (2006), Byrnes, Choi and Hall(2008))



black line; $\chi_* << M_{\rm P}$ trajectory

chaotic type

Large non-Gaussianity in slow-roll inflation

(Alabidi (2006), Byrnes, Choi and Hall(2008))

Results; various potential parameters and initial conditions

$\eta_{\varphi\varphi}$	$\eta_{\chi\chi}$	φ_*/M_p	χ_*/M_p	f_{NL}	
-0.01	-0.09	1.0	3.0×10^{-6}	-132	small field model
0.04	-0.04	1.0	6.8×10^{-5}	-123	saddle model
0.04	-0.04	1.0	1.5×10^{-4}	-68	
0.08	0.01	1.0	1.8×10^{-3}	9.3	chaotic type

(from Byrnes, Choi and Hall (2008))

Note that

the important point for generating large non-Gaussianity is $\chi_*/M_p \ll 1$

the non-linearity of $\delta\chi_*/\chi_*$ ➡ the large non-Gaussianity of curvature perturbation

Large non-Gaussianity in multi-brid inflation

(Lyth(2006), Naruko and Sasaki (2008), …)

Next, let me review a hybrid inflation type model, which have the possibility of generating large non-Gaussianity.

ϕ : inflaton

χ : waterfall field

Ordinary hybrid inflation model,

$$V(\phi,\chi) = \frac{\lambda}{4}(\chi^2 - v^2)^2 + \frac{m^2}{2}\phi^2 + \frac{g^2}{2}\phi^2\chi^2.$$

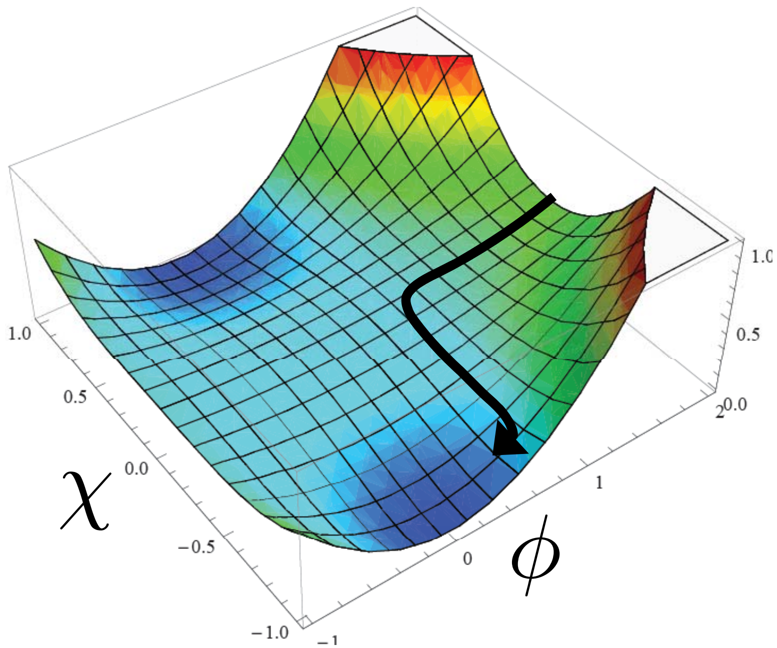
The effective mass of the waterfall field is

$$m_\chi^2 = -\lambda v^2 + g^2\phi^2$$

Inflation ends at $m_\chi^2 = 0$

$$\phi = \phi_e = \sqrt{\lambda}\, v/g$$

← tachyonic instability of the waterfall field



the end of inflation hypersurface ↔ $\boxed{m_\chi^2 = 0}$ hypersurface

In the previous models, the authors have considered the non-Gaussianity of the curvature perturbation on the V=constant hypersurface.

Large non-Gaussianity in multi-brid inflation

Hybrid inflation - generating curvature perturbation -

At the initial time (horizon crossing time), χ field is massive.

➔ The perturbation of χ field is suppressed.

➔ The perturbation of χ field does not contribute to generating the curvature perturbations. (almost single slow-roll inflation)

● Modification of the hybrid inflation models so that large non-Gaussianity can be generated

Hybrid inflation + light scalar field (curvaton like)

- Lyth (2005), Lyth and Alabidi (2006)
- Matsuda (2008)
- Sasaki, Naruko and Sasaki (2008)

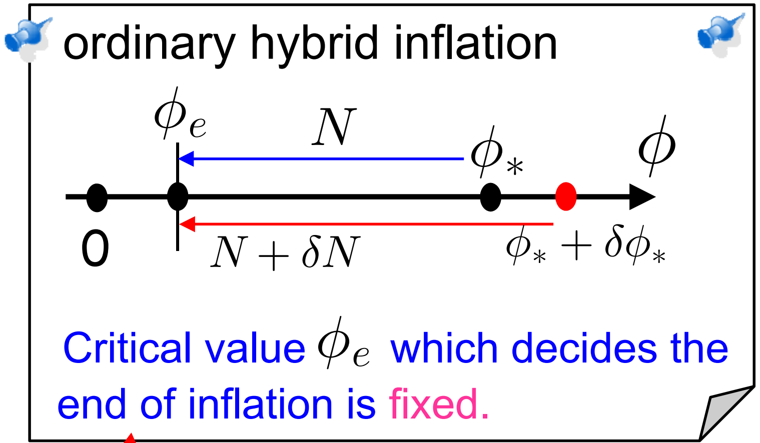
⋮

Large non-Gaussianity in multi-brid inflation

- curvature perturbation generated at the end of inflation

$$V = \frac{\lambda}{4}\left(v^2 - \chi^2\right)^2 + \frac{1}{2}m_\phi^2\phi^2 + \frac{g}{2}\phi^2\chi^2 \boxed{+ \frac{f}{2}\sigma^2\chi^2 + V(\sigma)}$$

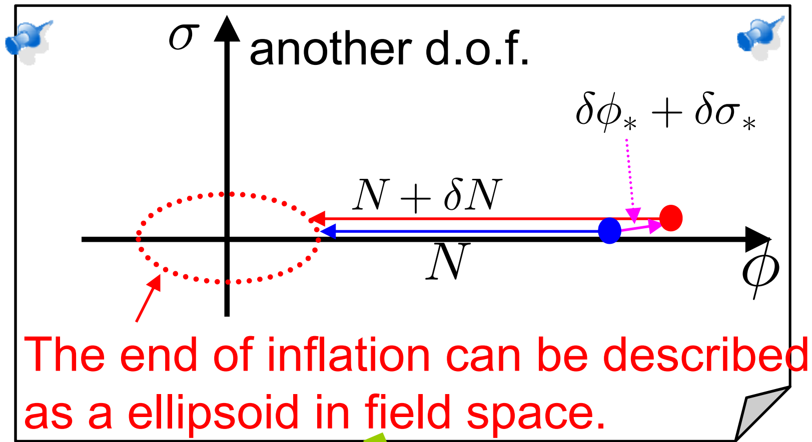
Ordinary hybrid inflation + light field



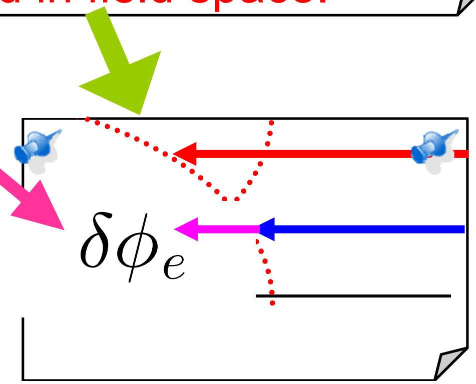
single field

+ a light field coupling with waterfall field

σ



Effectively, the critical value fluctuates due to the perturbation of a light field !!



Large non-Gaussianity in multi-brid inflation

Using δN formalism,

$$\zeta(t_e) = \delta N = \delta N_*(\phi_*) + \delta N_e(\sigma_*)$$

$$= N_{\phi_*}\delta\phi_* + \frac{1}{2}N_{\phi\phi *}\delta\phi_*^2 + N_{\phi e}\delta\phi_e + \frac{1}{2}N_{\phi\phi e}\delta\phi_e^2 \ ,$$

$$= N_{\phi *}\delta\phi_* + \frac{1}{2}N_{\phi\phi *}\delta\phi_*^2 + N_{\phi e}\phi'_e\delta\sigma_* + \frac{1}{2}\left(N_{\phi\phi e}\phi'^2_e + N_{\phi e}\phi''_e\right)\delta\sigma_*^2$$

$$\phi' = \partial\phi/\partial\sigma$$

Leadingly, the powerspectrum is given by

$$\mathcal{P}_\zeta = N_{\phi_*}^2\langle\delta\phi_*^2\rangle + N_{\phi e}^2\phi'^2_e\langle\delta\sigma_*^2\rangle = \left(N_{\phi_*}^2 + N_{\phi e}^2\phi'^2_e\right)\left(\frac{H_*}{2\pi}\right)^2$$

$$= (1+\alpha)\left(\frac{H_*}{2\pi\sqrt{\epsilon_*}}\right)^2$$

assuming $\langle\delta\phi_*^2\rangle = \langle\delta\sigma_*^2\rangle = \left(\frac{H_*}{2\pi}\right)^2$

$$\alpha = \frac{\text{the contribution from σ (light field)}}{\text{the contribution from Φ (inflaton)}} \equiv \frac{\epsilon_*}{\epsilon_e}\phi'^2_e$$

(cf. Mixed inflaton and curvaton model, mixed inflaton and modulated reheating scenario, Ichikawa , et al.(2008), Ichikawa et al. (2008))

Large non-Gaussianity in multi-brid inflation

Non-linearity parameters are given by

$$\frac{6}{5}f_{NL} = (2\epsilon_* - \eta_*)(1+\alpha)^{-2} + \alpha^2\left(2\epsilon_e - \eta_e + \sqrt{2\epsilon_e}\frac{\phi''_e}{\phi'^2_e}\right)(1+\alpha)^{-2}$$

$$\tau_{NL} = (2\epsilon_* - \eta_*)^2(1+\alpha)^{-3} + \alpha^3\left(2\epsilon_e - \eta_e + \sqrt{2\epsilon_e}\frac{\phi''_e}{\phi'^2_e}\right)^2(1+\alpha)^{-3}$$

$$\eta = \frac{V_{\phi\phi}}{V}$$

If one neglect the slow-roll parameters, we have

$$\frac{6}{5}f_{NL} = \left(\frac{\alpha}{1+\alpha}\right)^2\sqrt{2\epsilon_e}\frac{\phi''_e}{\phi'^2_e}\ ,$$

$$\tau_{NL} = \left(\frac{\alpha}{1+\alpha}\right)^3\left(\sqrt{2\epsilon_e}\frac{\phi''_e}{\phi'^2_e}\right)^2$$

The non-linearity is generated from the light field σ

Large non-Gaussianity in multi-brid inflation

Relation between trispectrum and bispectrum ;

$$\tau_{NL} = \left(\frac{1+\alpha}{\alpha}\right)\left(\frac{6}{5}f_{NL}\right)^2$$

$\alpha \gg 1$; The contribution from the fluctuation of σ dominates the curvature perturbation.

Consistency relation;

$$\tau_{NL} = \left(\frac{6}{5}f_{NL}\right)^2 \text{ + O(slow-roll parameter)}$$

Large non-Gaussianity in multi-brid inflation

$\alpha \ll 1$; The contribution from the fluctuation of Φ dominates the curvature perturbation.

Modified consistency relation ;

$$\tau_{NL} = \underline{\frac{1}{\alpha}} \left(\frac{6}{5} f_{NL} \right)^2$$

➔ There is a possibility of generating small bispectrum but large trispectrum.

➔ one of the importance of investigating the higher order correlation function

Large non-Gaussianity in multi-brid inflation

Lyth (2006)

Simple model

$$V = \frac{\lambda}{4}\left(v^2 - \chi^2\right)^2 + \frac{1}{2}m_\phi^2\phi^2 + \frac{g}{2}\phi^2\chi^2$$

$$+ \frac{f}{2}\sigma^2\chi^2 + \frac{1}{2}m_\sigma^2\sigma^2$$

parameter;

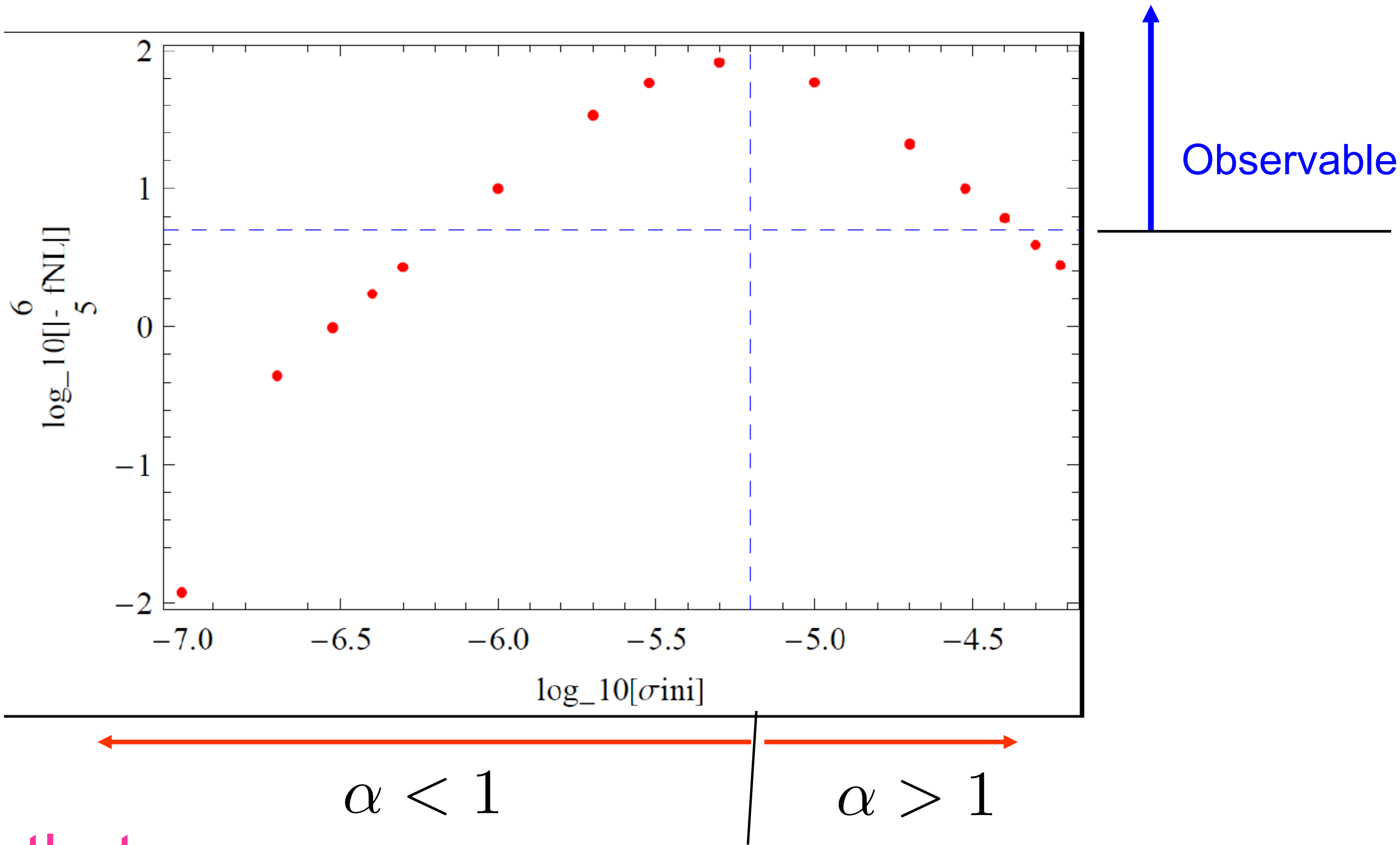
$$\lambda = 1.0 \times 10^{-10},\ v = 0.1,\ m_\phi = 5.0 \times 10^{-9},\ m_\sigma = 5.0 \times 10^{-10},$$
$$g = 1.0 \times 10^{-5},\ f = 1.0 \times 10^{-3},\ \sigma_* = 4.0 \times 10^{-7},\ \phi_* = 0.18.$$

$$\frac{6}{5}f_{NL} \simeq 1.734$$

$$\alpha \simeq 5.25 \times 10^{-3} \quad \tau_{NL} \simeq 569$$

Large non-Gaussianity in multi-brid inflation

f_{NL} vs σ_*

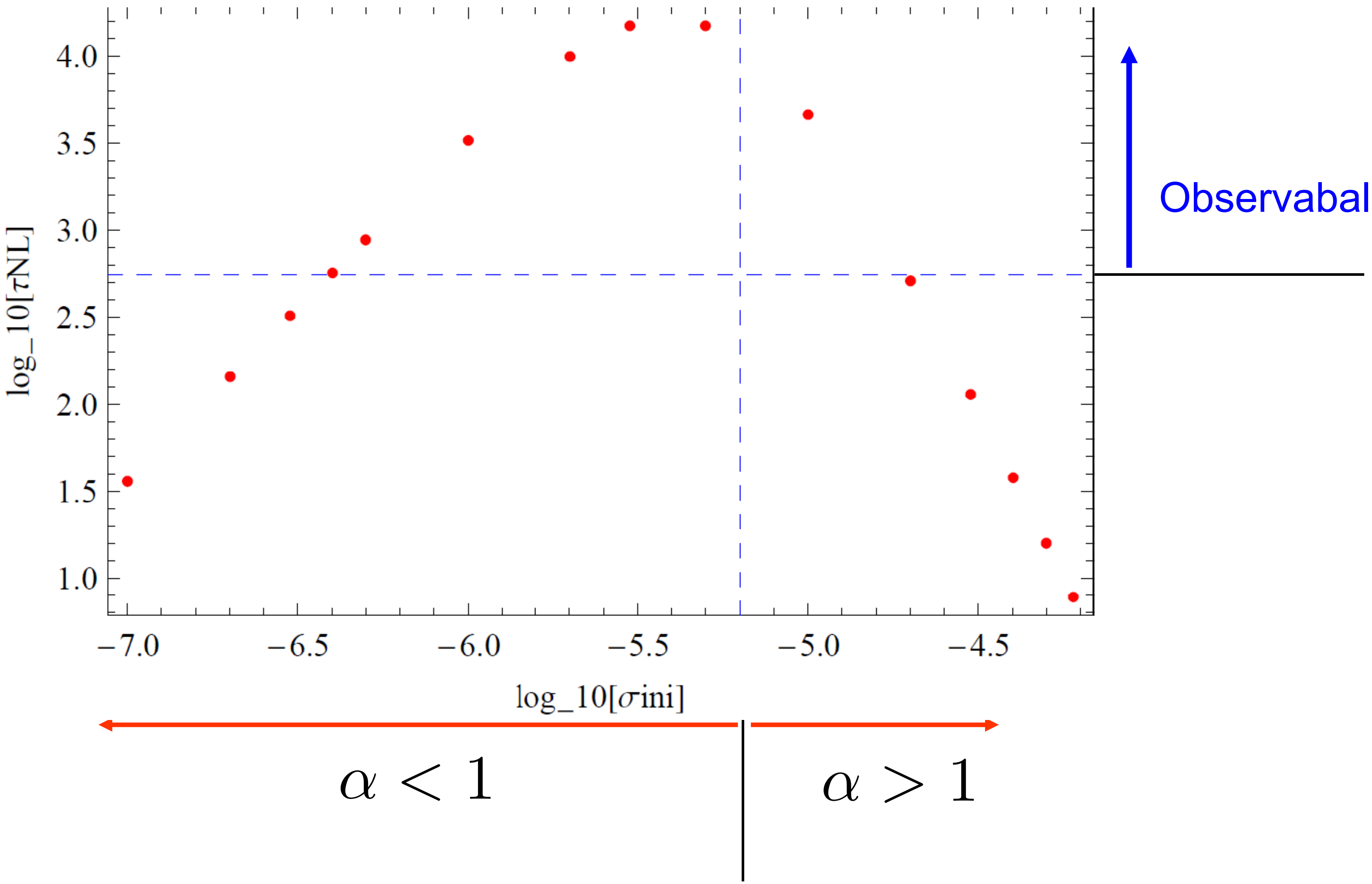


Note that

the important point for generating large non-Gaussianity is $\sigma_*/M_p \ll 1$

Large non-Gaussianity in multi-brid inflation

τ_{NL} VS σ_*



Large non-Gaussianity in multi-brid inflation

One loop effect ; "Ungaussiton";Suyama and Takahashi(2008), Cogollo et al.(2008)

$$\zeta = N_{\phi_*}\delta\phi_* + \frac{1}{2}N_{\phi\phi*}\delta\phi_*^2 + N_{\phi e}\delta\phi_e + \frac{1}{2}N_{\phi\phi e}\delta\phi_e^2 \,,$$

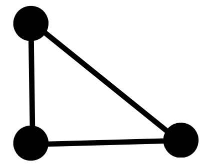
$$= N_{\phi*}\delta\phi_* + \frac{1}{2}N_{\phi\phi*}\delta\phi_*^2 + \underbrace{N_{\phi e}{\phi'}_e}_{N_\sigma}\delta\sigma_* + \frac{1}{2}\underbrace{\left(N_{\phi\phi e}{\phi'}_e^2 + N_{\phi e}{\phi''}_e\right)}_{N_{\sigma\sigma}}\delta\sigma_*^2$$

For $\sigma_* = 0$,

$N_\sigma = 0$ ➔ For the bispectrum (3-point function), the leading term comes from 2 x 2 x 2.

$$\frac{6}{5}f_{NL} = \frac{{N_{\sigma\sigma}}^3}{{N_\phi}^4}\mathcal{P}_\zeta \ln(kL)$$

box size



One loop

There seem to be various types of non-Gaussianity.

Example 2

The model in which the slow-roll conditions are temporarily violated after the horizon crossing time

(Numerical calculation)

Double inflation, N-flation type (large field model)

(SY, T. Suyama and T.Tanaka (2008))

Two fields chaotic inflation (double inflation) (Silk(1986))

$$V(\phi,\chi) = \frac{1}{2}m_\phi^2\phi^2 + \frac{1}{2}m_\chi^2\chi^2 \ .$$

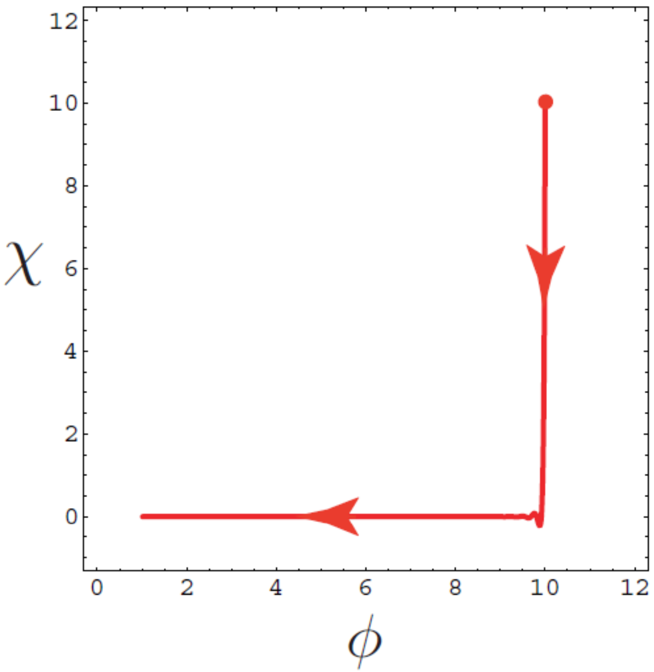
$M_{\rm Pl} = 1$

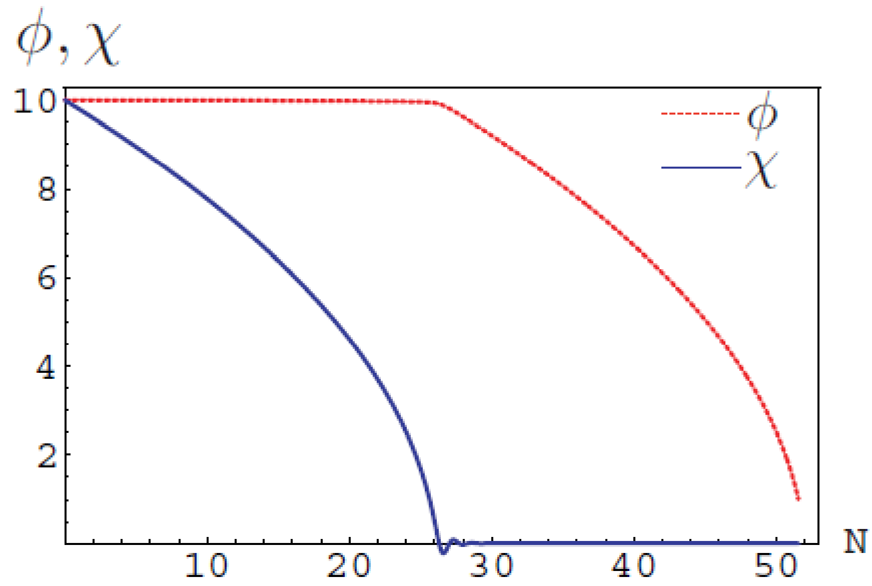
$m_\chi / m_\phi = 20$

$\phi_* = \chi_* = 10.$

Simple situation

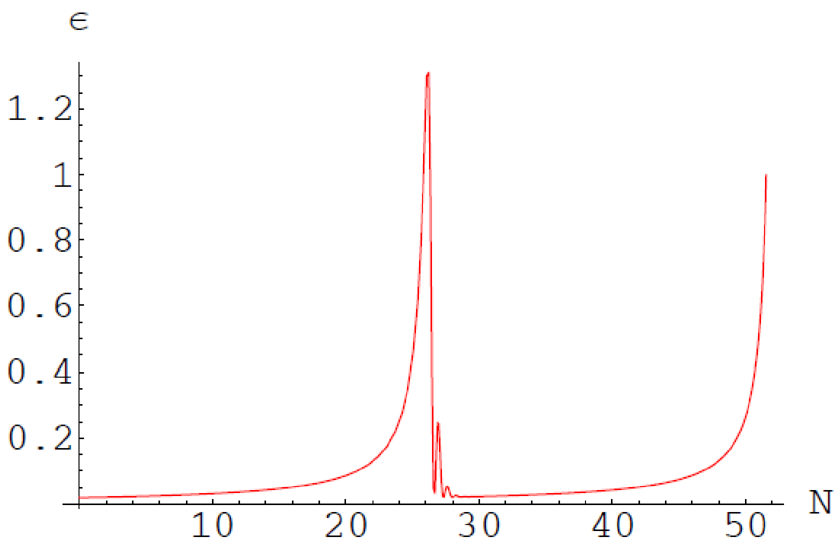
Background trajectory





Field evolution

Two fields chaotic inflation – slow-roll parameters -

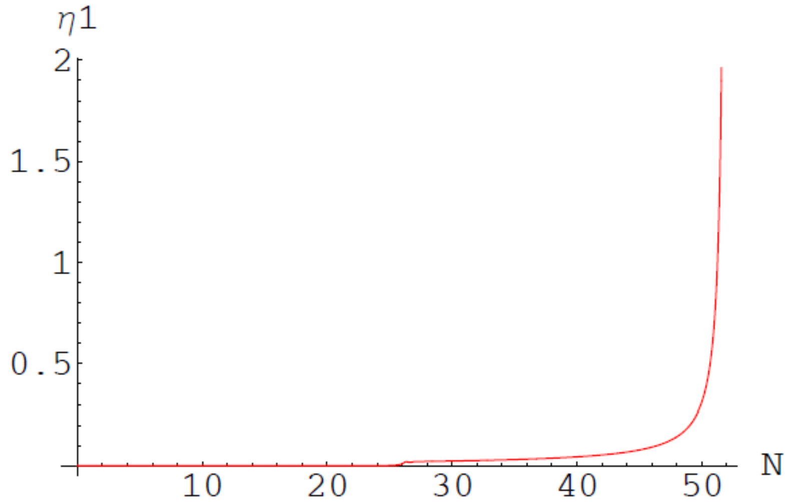


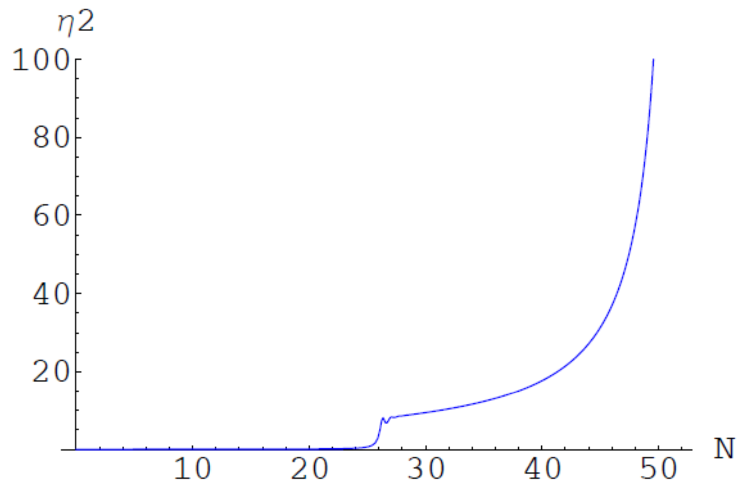
$$\epsilon \equiv -\frac{\dot{H}}{H^2}$$

$$\eta_1 \equiv \frac{V_{\phi\phi}}{V} \quad \eta_2 \equiv \frac{V_{\chi\chi}}{V}$$

The slow-roll conditions are violated (temporarily) !!

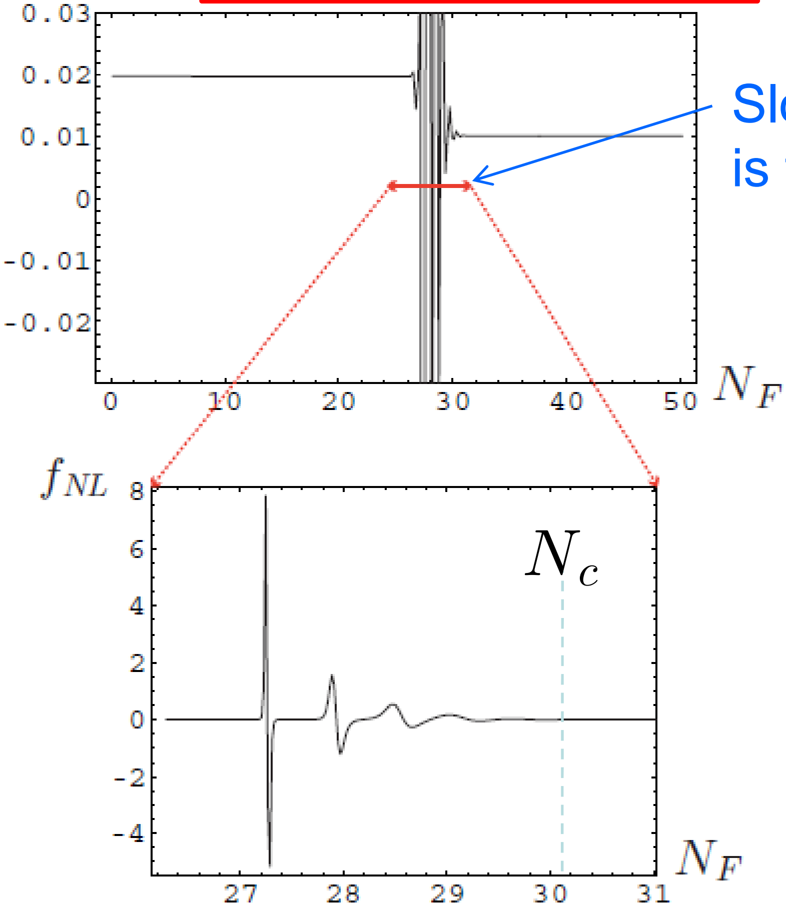
→ large non-Gaussianity?



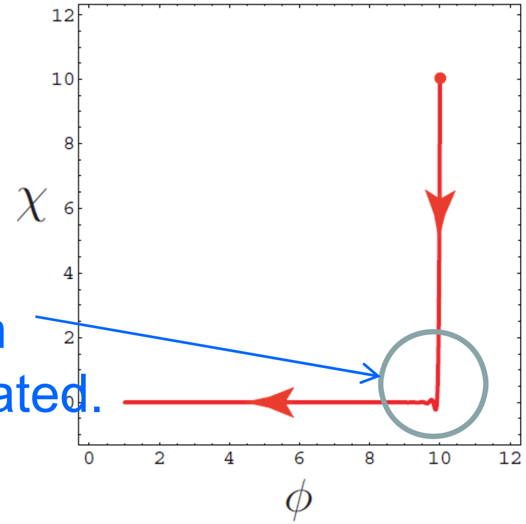


- non-linear parameter -

$$f_{NL} = f_{NL}(N_F)$$: function of N_F



Slow-roll condition is temporarily violated.



$$\zeta(N_F) = const.$$
$$f_{NL}(N_F) = const. \quad N_F > N_c$$

N_c ; the time when the complete convergence of the trajectories occurs

At the end of inflation,

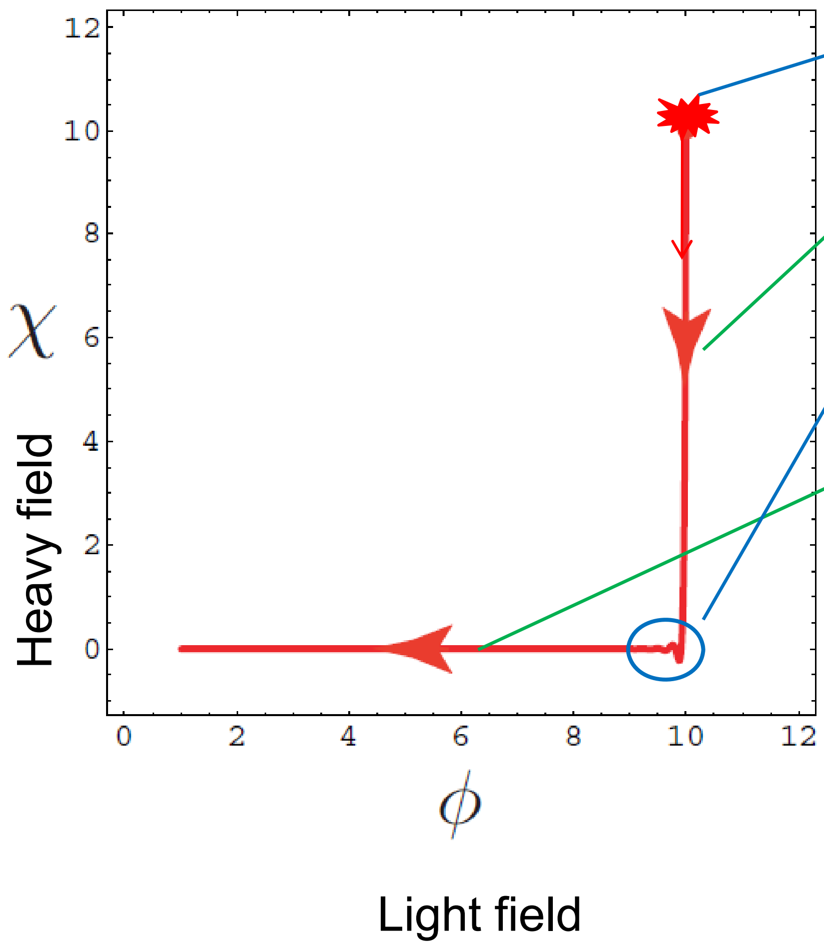
$$f_{NL} = 0.0108 \ll 1$$

small non-Gaussianity

means that horizon crossing approximation is valid.

Two fields chaotic inflation – non-linear parameter -

Background trajectory



Horizon crossing

First slow-roll phase

Slow-roll violation phase

(no effect for δN)

Second slow-roll phase

slow-roll + slow-roll

~~large f_{NL}~~

Example 3

The non-Gaussianity after the slow-roll phase / around the end of inflation

(numerical calculation)

In the estimation of the non-Gaussianity generated in the slow-roll phase, the authors have evaluated the curvature perturbation on the V=constant hypersurface or the mass of the waterfall field=constant hypersurface.

However, the curvature perturbation still evolve even on the super-horizon scales until the complete convergence of the trajectory in the field space has occurred.

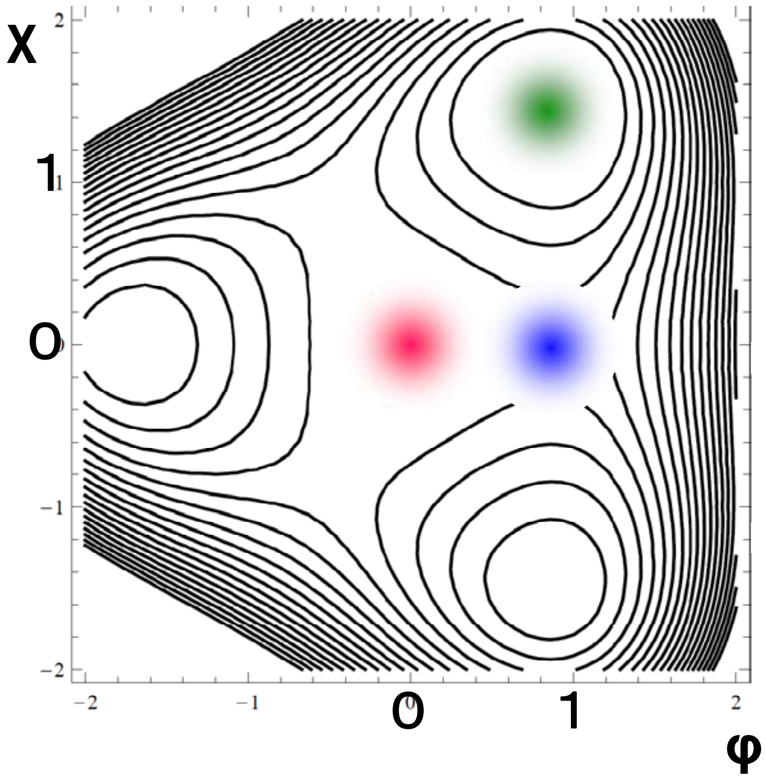
Here, we calculate the curvature perturbation on uniform energy density hypersurface using the δN formalism and check the convergence of the trajectory ($\zeta \rightarrow$ constant)

Then, we evaluate the amplitude of the non-Gaussianity.

- **Two models;**
  - Small field-hybrid model (modular inflation)
  - Multi-brid model

Small field – hybrid type model

(work in progress)



Ref.) Kadota and Stewart (2003) $M_{Pl}^2 = 1$

$$V = V_0 - m_0^2\Phi^2 + \frac{1}{3}Am_0^2|\Phi|^3 + \frac{1}{2}\nu(\nu+1)Am_0^2\Phi^4$$

$$\Phi = \frac{1}{\sqrt{2}}(\phi + i\chi)$$; complex scalar field

$$= \frac{1}{\sqrt{2}}\rho e^{i\theta}$$ (radial + angular component)

: maximum ; eternal inflation → generating perturbation (horizon crossing)

: saddle point ; waterfall phase → inflation ends

: minimum ; inflation ends → oscillation phase

↔ saddle point model ?

Small field – hybrid type model

Ref.) Kadota and Stewart (2003)

Originally, this model is motivated by supergravity/string theory.

➔ η - problem $m_0^2/H^2 \sim m_0^2/V_0 = O(1)$

➔ Introducing Qantum correction term (renormalized mass)

$$V_{loop1} = -\beta_1 m_0^2 \Phi^2 \left[\log|\Phi| - \frac{1}{2}\right]$$: effective around maximum point

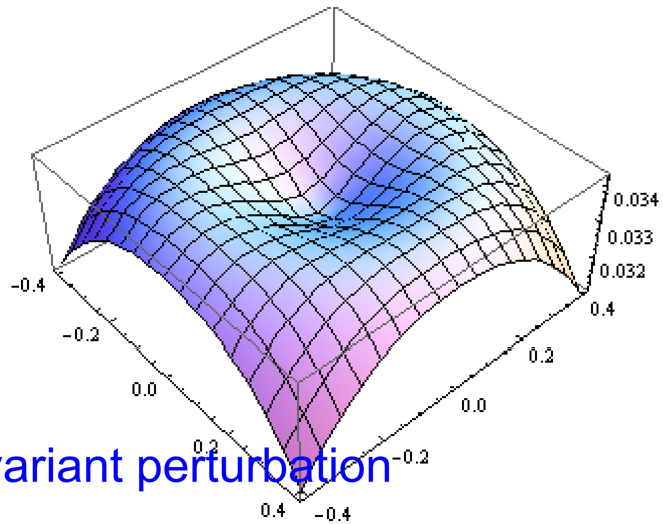
$$V_{loop2} = \beta_2 m_0^2 |\Phi - \Phi_s|^2 \left[\log|\Phi - \Phi_s| - \frac{1}{2}\right]$$: effective around saddle point

$\Phi = \Phi_s$; saddle point

Around top $V \sim V_0 - \frac{1}{2}\beta_1 m_0^2 \rho^2 \ln(\rho^2/\rho_t^2) +$

flattening

Radius; $\rho = \rho_t \sim e^{-\frac{1}{\beta_1}}$ ring maximum



Due to the coupling parameter β,
we can obtain enough e-folding number and scale-invariant perturbation

Small field – hybrid type model

Background dynamics

● around the top (Due to U(1) symmetry, consider only radius component.)

$$\frac{d^2}{dN^2}\rho + 3\frac{d}{dN}\rho - \frac{\beta_1 m^2}{H^2}\ln\frac{\rho}{\rho_t}\rho = 0\ .$$

slow-roll

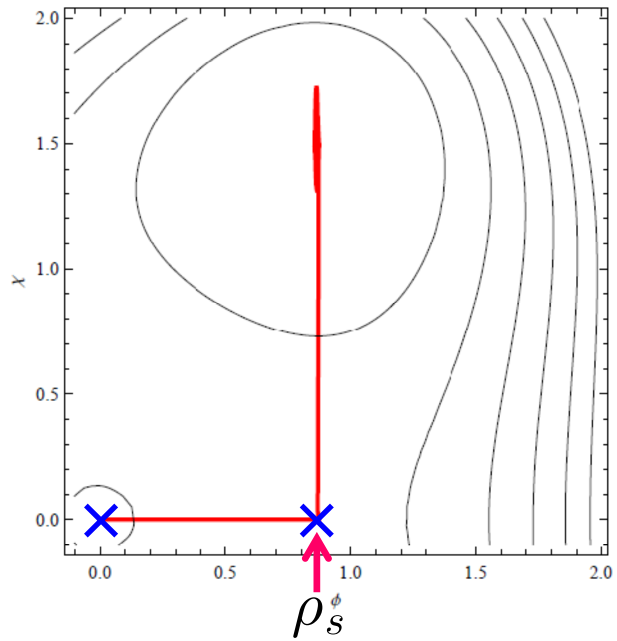
e-folding number as a time coordinate

(dN = H dt)

ρ_*: initial value (at the horizon crossing)

$$N_c \sim \frac{V_0}{\beta_1 m^2}\ln\left[\frac{1}{\beta_1}\frac{\rho_t}{\rho_* - \rho_t}\right]$$

$$\rho_c = \rho(N_c) \sim \rho_s, \quad \rho_* \sim \rho_t$$



Nc is a time when the U(1) symmetry is broken.

←→ close to the saddle point

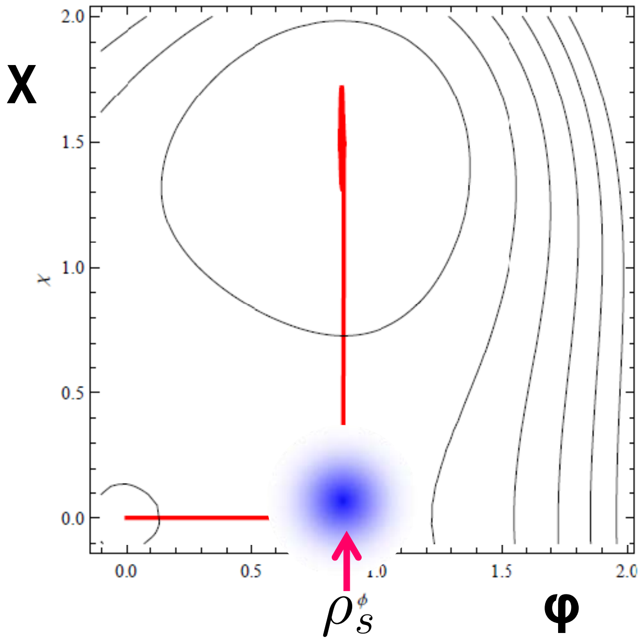
total e-folding number can be obtained in this phase and

The perturbation of φ,χ field are not suppressed and hold the scale invariance

Small field – hybrid type model

- around the saddle point $V = V_s - \mu_s^2 \chi^2$

$$+\frac{1}{2}\beta_2 m^2 \left(\ln \frac{\sqrt{\psi^2+\chi^2}}{\psi_s} - \frac{1}{2}\right)(\psi^2+\chi^2)\cdots$$



$\psi = \phi - \rho_s$: the value of Φ measured from the saddle point

$$\frac{\mu_s^2}{m^2} = \frac{\nu+2}{\nu+1}, \quad V_s/V_0 = (3\nu+1)^{-1}\left(\frac{2\nu+1}{\nu+1}\right)^3$$

$$\psi_s = \rho_s\sqrt{3}\exp\left(-\frac{2\nu+1}{\beta(\nu+1)}\right).$$

The dynamics of angular component = χ direction $\chi_c \simeq \rho_s\theta_* \sim \frac{\rho_s}{\rho_t}\chi_*$

$$\chi_e \sim \rho_s$$

$$N_e - N_c \approx \frac{3H^2}{2\mu_s^2}\ln\frac{|\psi_c|}{\chi_c}$$

$$+\alpha^{-1}\left[\ln\left(\left|\frac{2\mu_s^2}{\beta_2 m^2} - \left(\frac{\chi_c}{|\psi_c|}\right)^{\frac{\beta_2 m^2}{2\mu_s^2}}\ln\frac{|\psi_c|}{\psi_s}\right|\right) - \ln\left(\left|\ln\frac{\chi_e}{\chi_s}\right|\right)\right]$$

Small field – hybrid type model

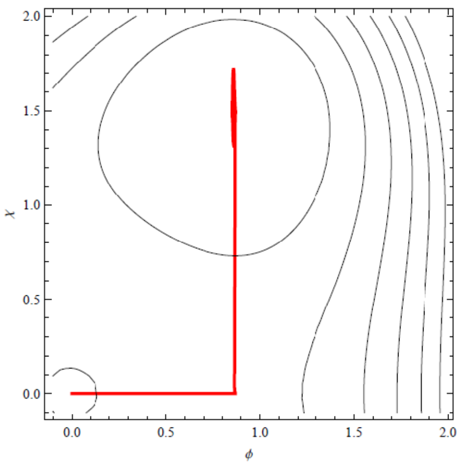
From the δN formalism,

$$\zeta = N_\phi \delta\phi_* + N_\chi \delta\chi_* + \frac{1}{2}N_{\phi\phi}\delta\phi_*^2 + \frac{1}{2}N_{\chi\chi}\delta\chi_*^2$$

If the contribution from the fluctuation of the waterfall direction (angular component) dominates, then we have

$$\frac{6}{5}f_{NL} = \frac{N_{\chi\chi}}{N_\chi{}^2} \simeq O\left(\frac{m^2}{V_0}\right) = O(1)$$

; detectable level !?



- Numerical result

$$\frac{m^2}{V_0} = 1.0,\ \nu = 1.0,\ \beta_1 = \beta_2 = 0.1$$

For $\theta_* = 1.0 \times 10^{-5}$

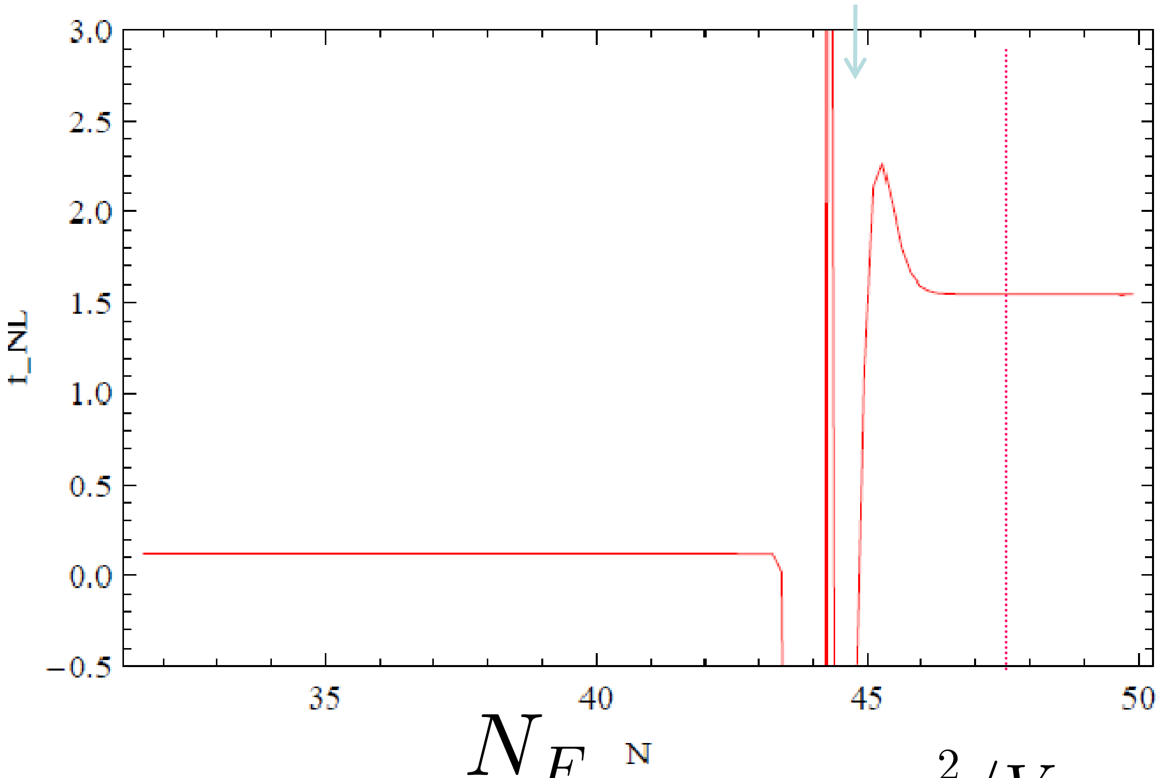
$$\frac{6}{5}f_{NL} = 1.552$$

Small field – hybrid type model

The evolution of non-linear parameter

Curved Trajectory = waterfall

f_{NL}



N_F

$m_\phi^2/V_0 = 1, \beta = 0.1$

At the end of inflation, we have

$$f_{NL} \simeq 1.5$$

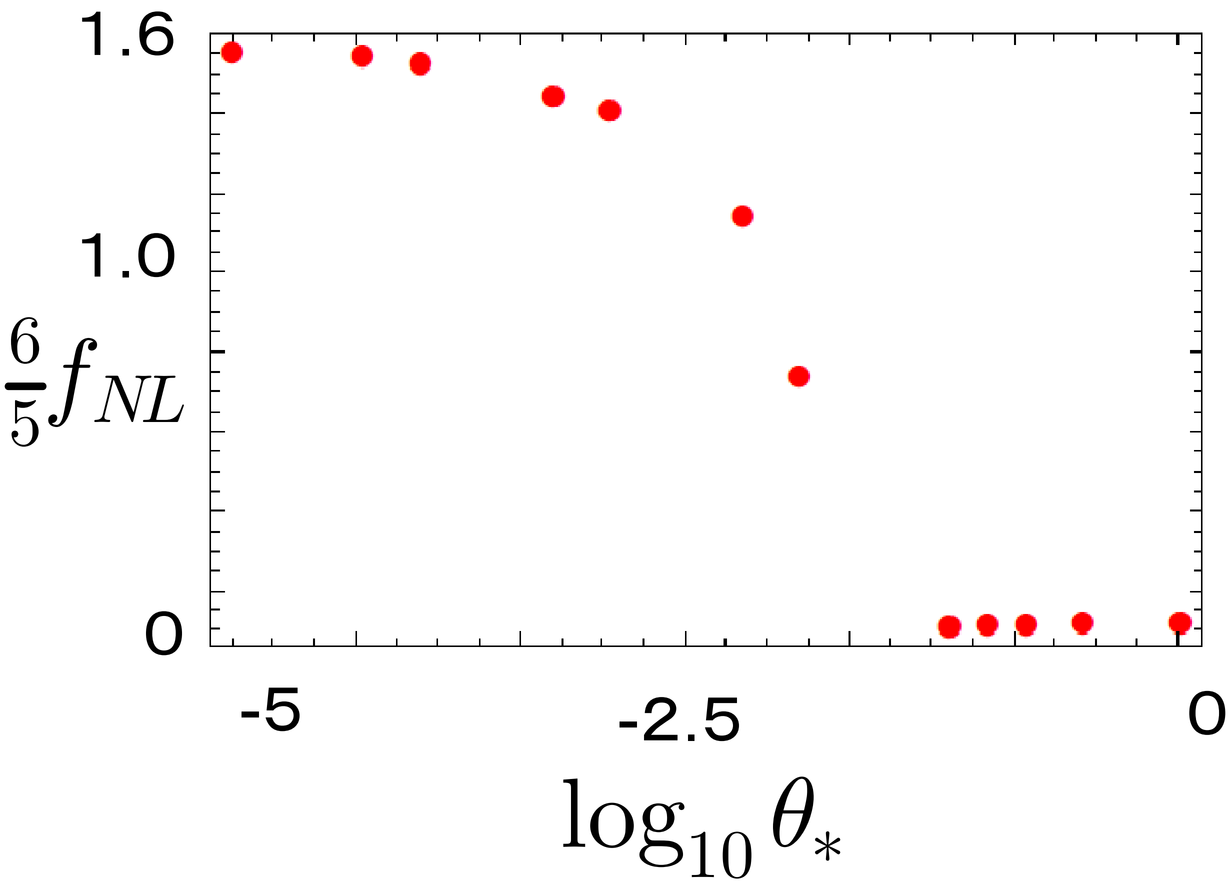
$\leftrightarrow$ Possible models of generating large (means O(1)) non-Gaussianity ?

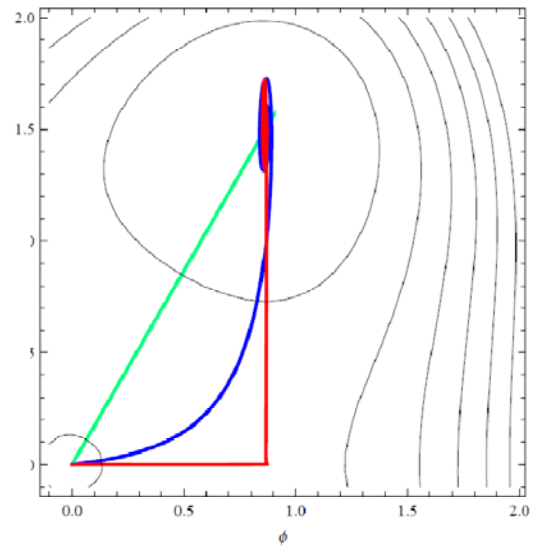
$$\neq O(\epsilon_*, \eta_*)$$

Small field – hybrid type model

Dependence on Parameters

Initial angle





large fNL

small angle is necessary.

through the saddle point

Small field – hybrid type model

Dependence on Parameters

mass parameter

$\frac{m^2}{V_0}$	1.0	2.0	3.0	4.0	5.0	6.0
$\frac{6}{5}f_{NL}$	1.55	2.57	3.28	3.94	4.60	5.09
τ_{NL}	2.41	6.58	10.71	15.46	20.55	25.77

; $\frac{\beta_1 m^2}{V_0} = 0.1$ is fixed (total e-folding number is also fixed.)

Future experiments ; $\frac{6}{5}f_{NL} \geq 3 - 5$

Trispectrum ; $\tau_{NL} \simeq \left(\frac{6}{5}f_{NL}\right)^2$

Small field – hybrid type model

- Probability Distribution Function of Initial Angle -

Preferred initial angle？？

$$P(\ln\theta_*) \propto \theta_* \, e^{3N(\ln\theta_*)}$$: Probability distribution function

Large volume is preferable

Avoid $\theta_* = 0$ (topological defect)

$$\frac{d\ln P(\ln\theta_*)}{d\ln\theta_*} = 3\frac{dN}{d\ln\theta_*} + 1 \Big|_{\theta_*=\theta_m} = 0 \ ,$$

$$\frac{d^2\ln P(\ln\theta_*)}{d\ln\theta_*^2} = 3\frac{d^2N}{d\ln\theta_*^2}\Big|_{\theta_*=\theta_m} < 0$$

: maximal at $\theta_* = \theta_m$

From the analysis of e-folding number

$$\ln\theta_m \sim -1/\beta_2$$ small angle is preferable!!

Small field – hybrid type model

Discussion

: maximal at $\theta_* = \theta_m$

$$\frac{d\ln P(\ln\theta_*)}{d\ln\theta_*} = 3\frac{dN}{d\ln\theta_*} + 1\Bigg|_{\theta_*=\theta_m} = 0 \ ,$$

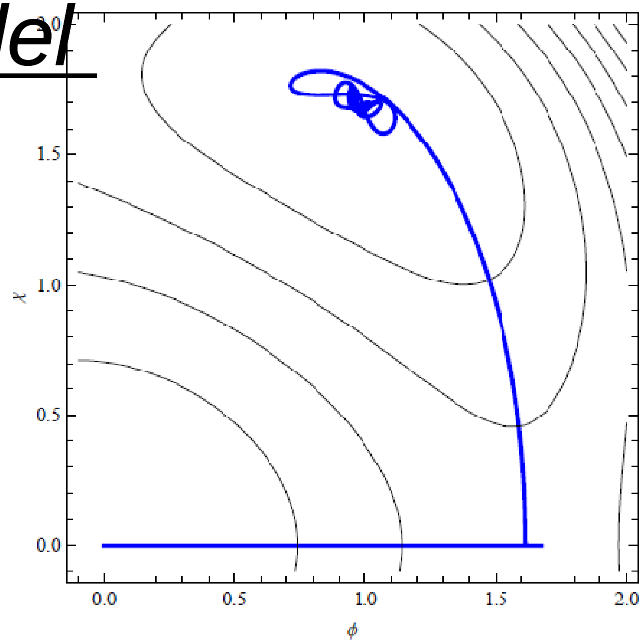
$$\frac{d^2\ln P(\ln\theta_*)}{d\ln\theta_*^2} = 3\frac{d^2N}{d\ln\theta_*^2}\Bigg|_{\theta_*=\theta_m} < 0$$

In order for the maximal value to exist, the necessary condition is

$$\frac{m_0^2}{V_0} \sim O(1)$$

: If we consider the large mass parameter,
→The angular dependence of the total e-foldings becomes small
→Hence, the PDF is $\propto \theta_*$
→and the trajectories which close the saddle point are not preferable
➔ Thus, f_NL seems also to be ~ 1 at the most ?? Can we detect ??

Small field – hybrid type model



Discussion

Another model parameter ν

➔ displace the saddle point

$$\nu = 5$$

ν	1.0	2.0	3.0	4.0	5.0
$\frac{6}{5}f_{NL}$	1.55	1.51	1.51	1.54	1.62

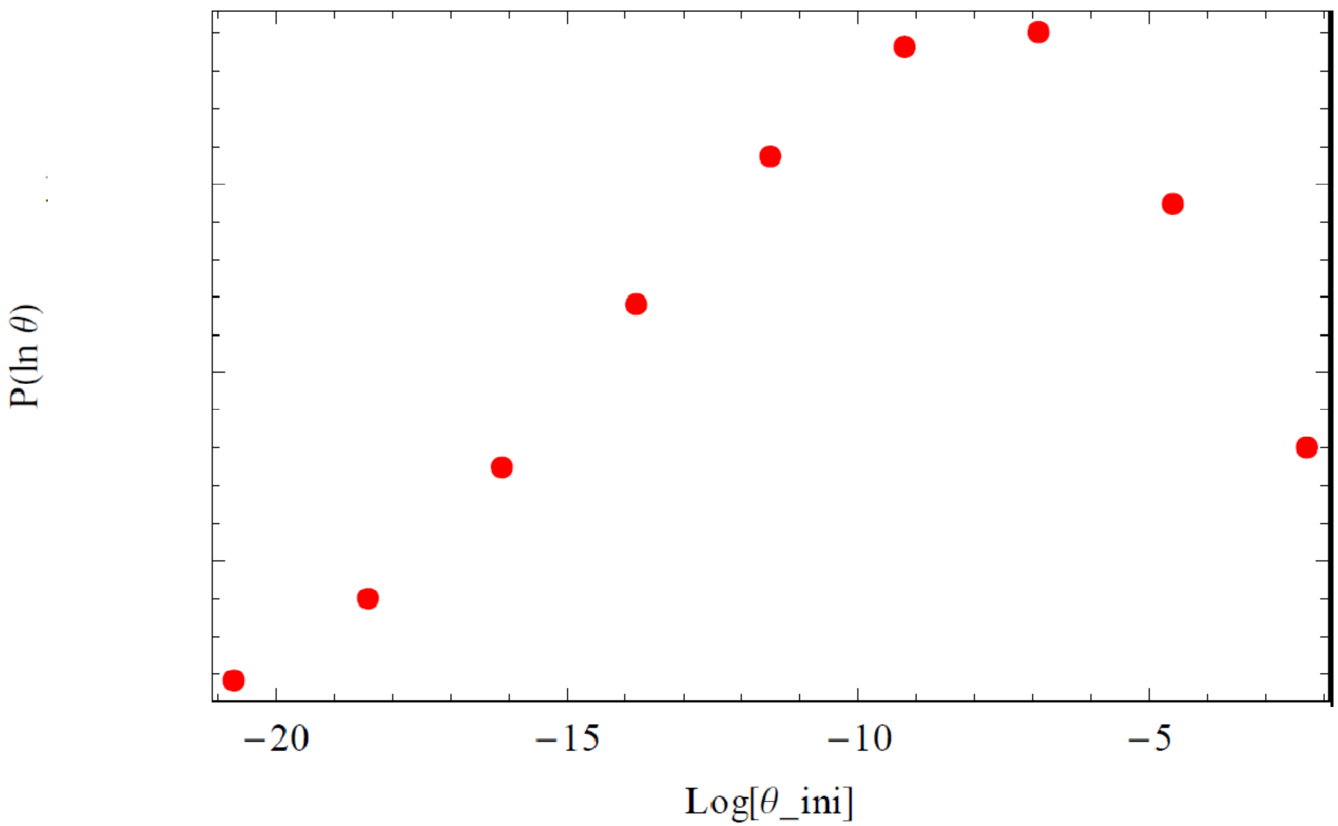
fixed $\beta_1 - 0.1, \frac{m^2}{V_0} - 1.0$

The dependence on the non-linear parameter seems to be weak.

Small field – hybrid type model

Discussion

Probability Distribution Function of Initial Angle



Multi-brid type

work in progress

In order to realize the tachyonic instability in the classical background level,
we introduce the artificial potential tem as;

$$V = \frac{\lambda}{4}\left(v^2 - \chi^2\right)^2 + \frac{1}{2}m_\phi^2\phi^2 + \frac{g}{2}\phi^2\chi^2$$

; conventional hybrid model

$$+ \frac{f}{2}\sigma^2\chi^2 + \frac{1}{2}m_\sigma^2\sigma^2 \quad + V_{add}$$

; “iso-curvature” field ; artificial term

$$V_{add} = \frac{1}{2}g_h^2\chi(\chi - 2v)\frac{\tanh\left[\frac{\phi_e - \phi}{\phi_n}\right] + 1}{2}$$

Multi-brid type

evolution of scalar fields

$\lambda = 1.0 \times 10^{-10},\ v = 0.1,\ m_\phi = 5.0 \times 10^{-9},\ m_\sigma = 5.0 \times 10^{-10},$

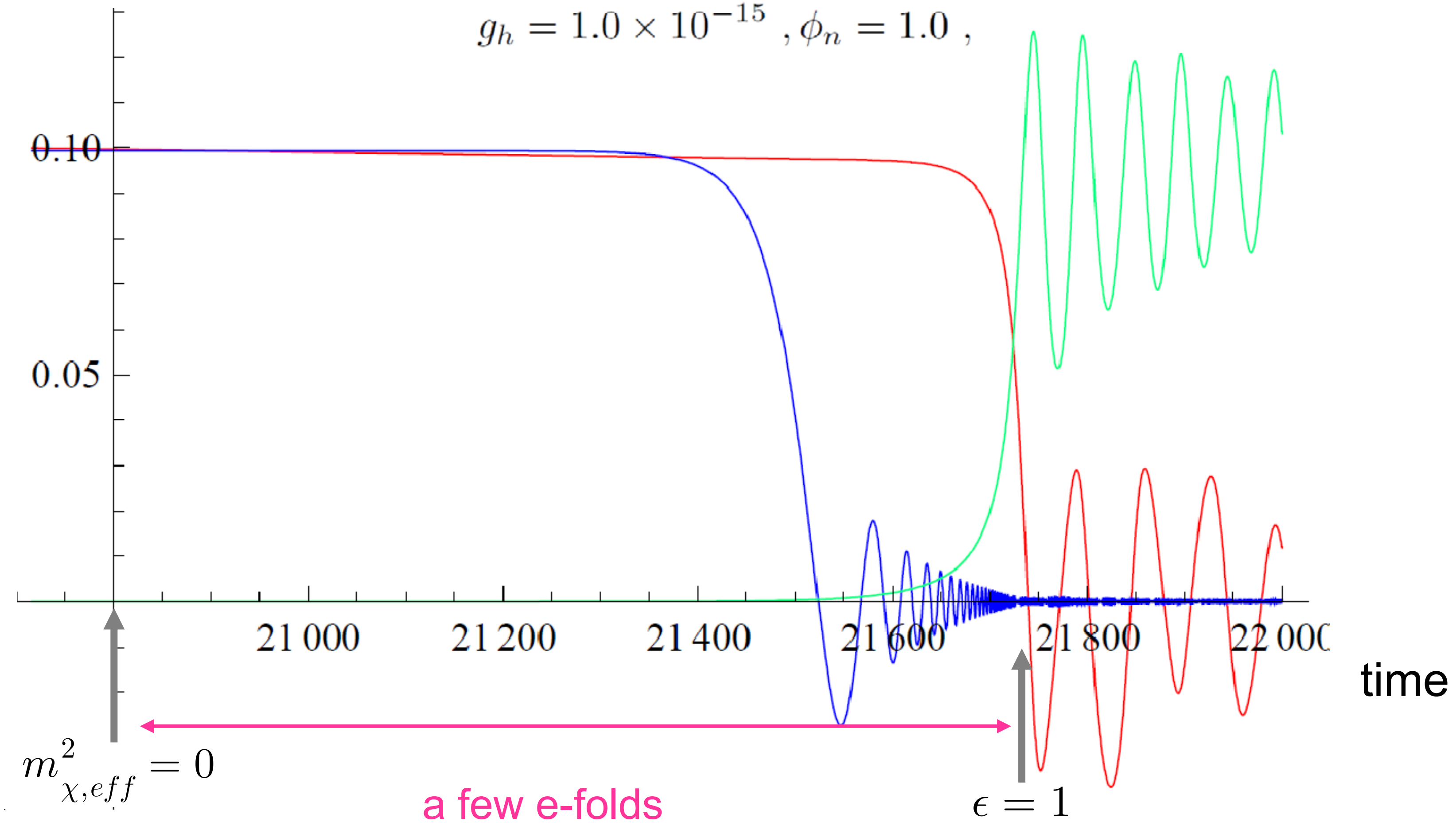
$g = 1.0 \times 10^{-5},\ f = 1.0 \times 10^{-3},\ \sigma_* = 4.0 \times 10^{-7},\ \phi_* = 0.18.$

$g_h = 1.0 \times 10^{-15}\ , \phi_n = 1.0\ ,$

blue; $\sigma/10\sigma_*$

red; ϕ

green; χ



Multi-brid type

Numerical results

$$\alpha = N_\sigma^2 / N_\phi^2$$

For $\sigma_* = 4.0 \times 10^{-7}$

$$\frac{6}{5} f_{NL} \simeq 1.26 \qquad \tau_{NL} \simeq 377$$

$$\alpha \simeq 4.26 \times 10^{-3}$$

We can obtain the relation as $\tau_{NL} \simeq \left(\frac{1+\alpha}{\alpha} \right) \left(\frac{6}{5} f_{NL} \right)^2 .$

For $\sigma_* = 4.0 \times 10^{-5}$

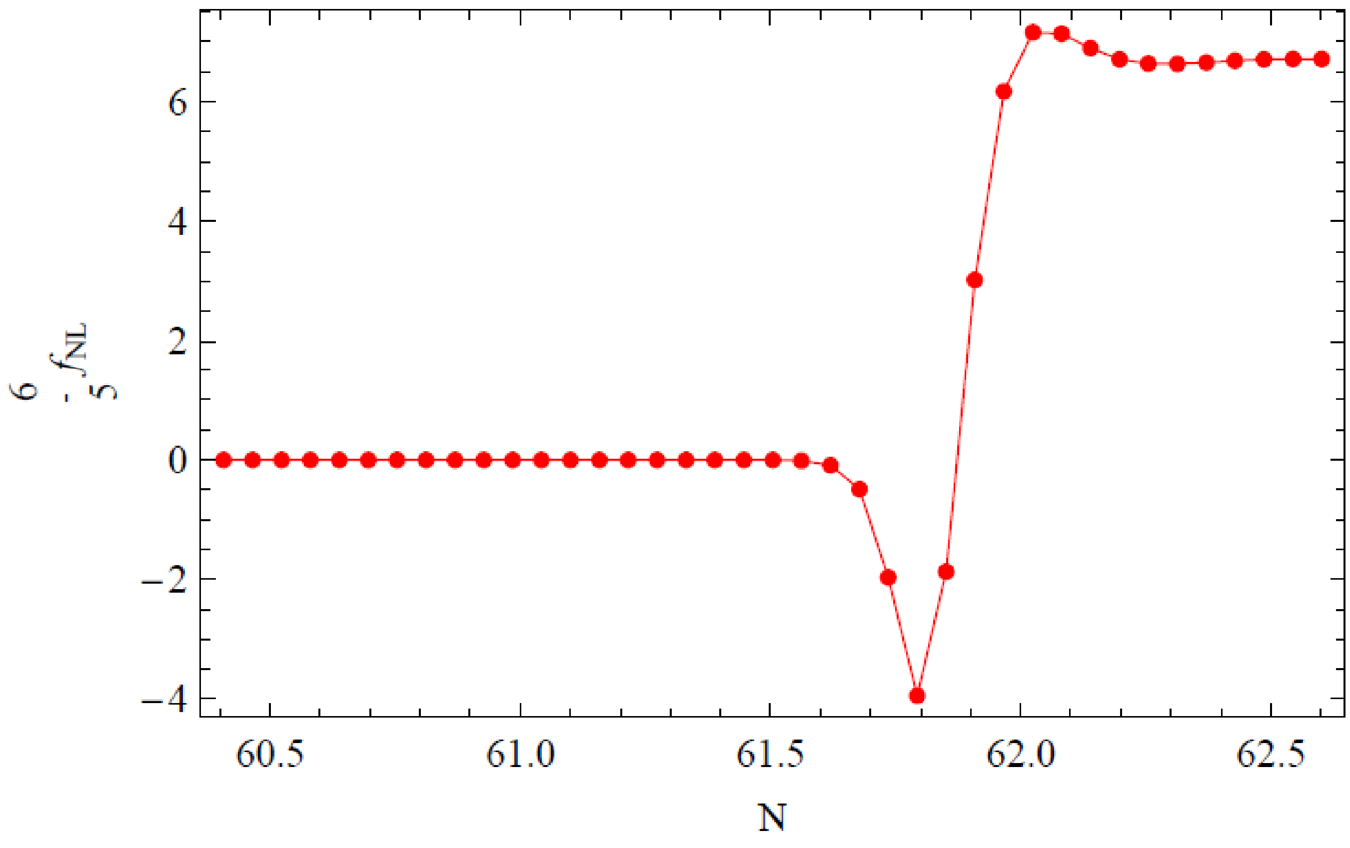
$$\frac{6}{5} f_{NL} \simeq 6.71 \qquad \tau_{NL} \simeq 46.0$$

$$\alpha \simeq 43.0$$

$$\tau_{NL} \simeq \left(\frac{6}{5} f_{NL} \right)^2$$

Multi-brid type

Time evolution of the non-linearity parameter fNL



Summary and Future issues

Precision cosmology era

We can determine/constrain possible models/theories of the early universe (inflation model, …) from observations (CMB anisotropies, …).

The primordial non-Gaussianity has been considered as one of the new cosmological parameters, which bring us valuable information about the mechanism of generating primordial perturbations (inflation model).

Many authors have proposed and analyzed the models in which there are possibilities of generating large non-Gaussianity.

Theoretical side

How can large non-Gaussianity (local type) be realized ?

- **Multi-component field**
  (inflaton Φ + a light field σ (generating NG field))
  dominant subdominant

  (←→ Single field case ; $\dot{\zeta} = 0$ on super-horizon scales)

- **Large** $\delta\sigma / \sigma$

$$\begin{cases} \delta\sigma \sim H_{\rm inf} \ll M_{\rm Pl} \\ \sigma \ll M_{\rm Pl} \end{cases}$$

(cf. GUT scale inflation; $H_{\rm inf} \sim 10^{-6} M_{\rm Pl}$)

← Is it natural ?

Observational side

- **Higher order correlation function of CMB anisotropies**
  - bispectrum, trispectrum, … higher … ?
  - local type, equilateral type

- **relation between NG and**
  residual iso-curvature perturbation
  (CDM iso-curv., Baryon iso-curv., …)

- **effects on the large scale structure**
  - **power spectrum**
  - **bi-spectrum**
  - **halo mass function**
  - **void abundance**

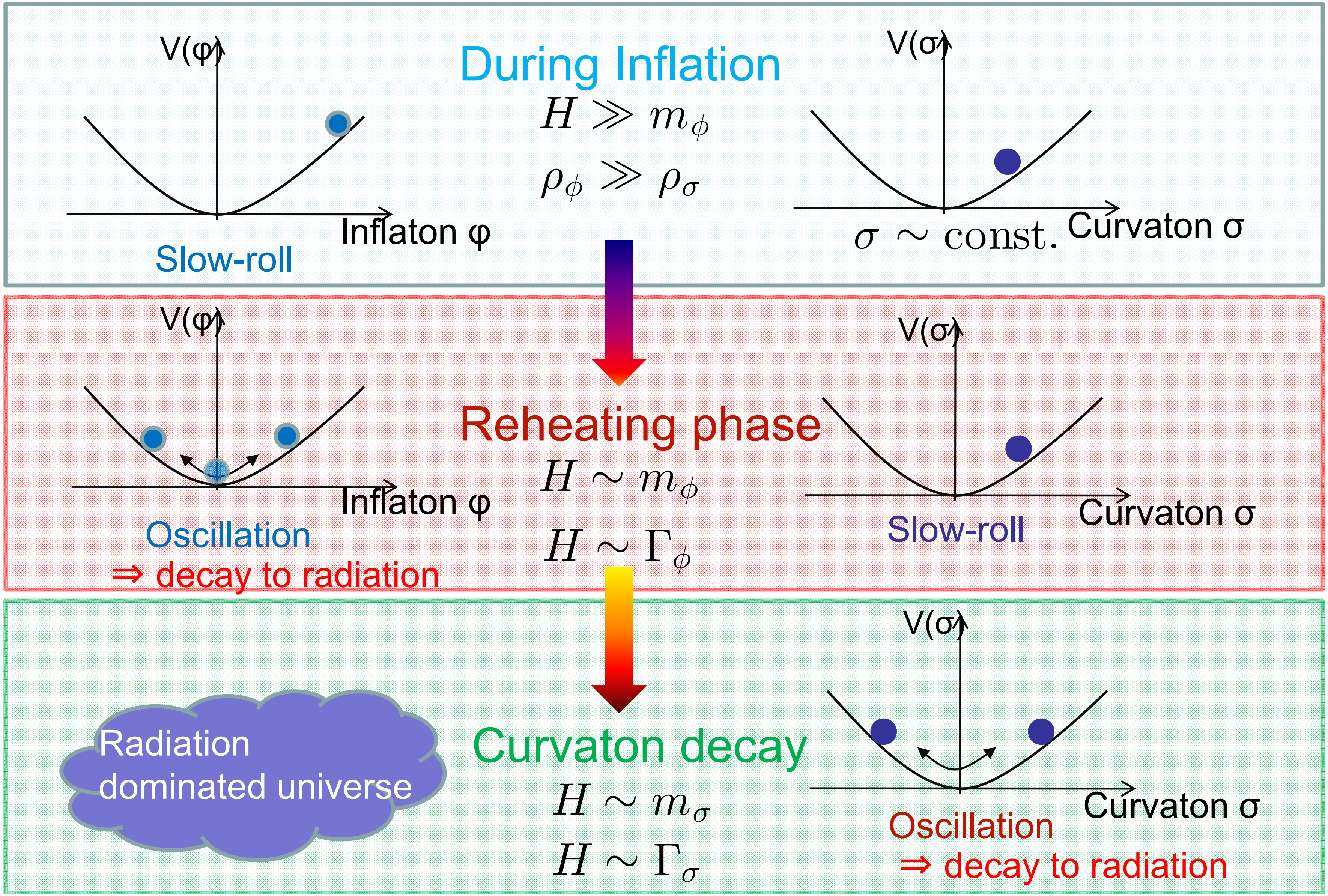
(Taruya et al.(2008), Sefusatti and Komatsu(2007), LoVerde et al.(2008), Kamionkowski et al.(2009), …)

Curvaton and
Modulated Reheating scenarios

Curvaton scenario

(Lyth and Wands, Moroi and Takahashi(2003), ……)

Two scalar fields (inflaton : Φ + light scalar field (curvaton) : σ) $m_\phi \gg m_\sigma$



Curvaton scenario

Perturbations

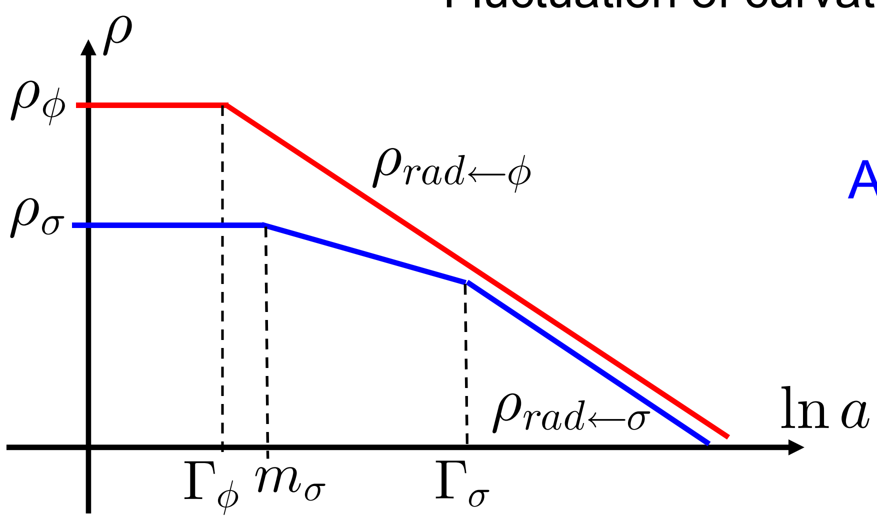
$\delta\phi$ ⇒ $\left.\frac{\delta\rho_\gamma}{\rho}\right|_{in} \sim \zeta_{in}$ Adiabatic curvature perturbation

Fluctuation of inflaton

$\delta\sigma$ ⇗ decay $\zeta \sim (1-f)\zeta_{in} + f\zeta_\sigma$

Fluctuation of curvaton

(large ζ_σ)



$$f \simeq \left.\frac{\rho_\sigma}{\rho_{tot}}\right|_{t=t_{decay}}$$

Amplitude;(neglect the inflaton fluctuation)

$$\zeta \simeq f\frac{\delta\sigma}{\sigma}$$

Non-Gaussianity;

$$f_{NL} \simeq \frac{1}{f}$$

Large non-Gaussianity ➔ small f ➔ large $\frac{\delta\sigma}{\sigma}$ ➔ small σ

Modulated reheating scenario (Dvali et al. (2004), ……)

Two scalar fields (inflaton : Φ + light scalar field : σ)

Reheating occurs

$$H = \Gamma_\phi$$

If $\Gamma_\phi = \Gamma_\phi(\sigma)$; σ is a light scalar field

then,

$$\zeta = N_\phi \delta\phi + N_\sigma \delta\sigma + \cdots$$

$$N_\sigma = \frac{\partial N}{\partial \Gamma_\phi} \Gamma'_\phi(\sigma) \qquad \Gamma'_\phi(\sigma) = \frac{\partial \Gamma_\phi(\sigma)}{\partial \sigma}$$

The spectra (amplitude and non-Gaussianity) depend on the dependence of the decay rate on the light field.

Efficient diagrammatic method

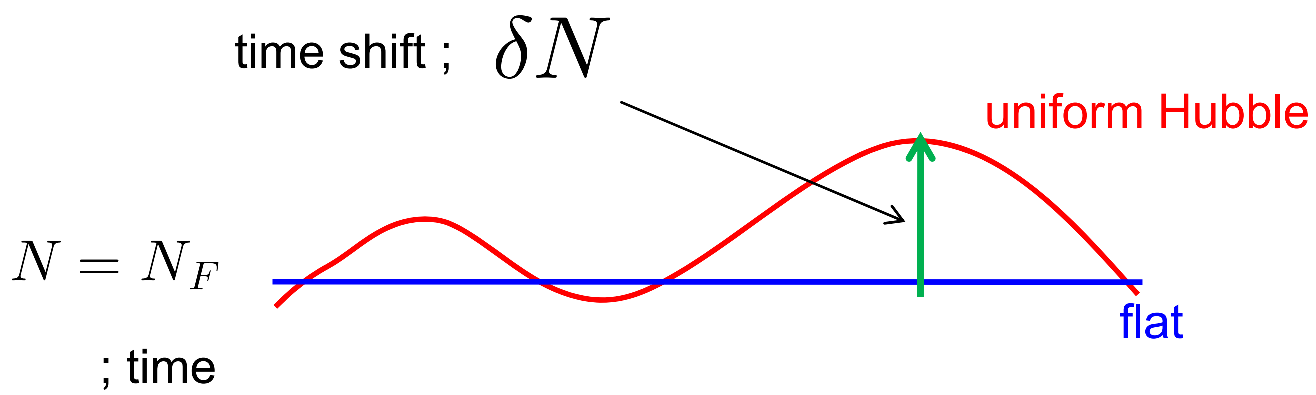
- Diagrammatic method – (SY, T. Suyama and T. Tanaka(2009))

The curvature perturbation is independent of the initial time. So we can shift the time at which the hypersurface is taken to be flat one, to the final time and then we have,..

$$\zeta(N_F) = \sum \frac{1}{n!} N^F_{a_1 a_2 \ldots a_n} \delta\varphi_F^{a_1} \delta\varphi_F^{a_2} \cdots \delta\varphi_F^{a_n} \,,$$

$\varphi^a = (\phi^I, \dot{\phi}^I)$; phase space variable

written by local quantities at the final time



Based on this expression, instead of considering the evolution of $N_{ab}, N_{abc}, \cdots$

We need to calculate the non-linear evolution of $\delta\varphi^a$

$\delta\varphi^a_F$ is no longer Gaussian statistics.

- Diagrammatic method -

Evolution equation ; (perturbed background eq. ←→ δN formalism)

$$\frac{d}{dN}\delta\varphi^a(N) = P^a_b\delta\varphi^b(N) + \frac{1}{2}Q^a_{(3)bc}(N)\delta\varphi^b(N)\delta\varphi^c(N) + \cdots + \frac{1}{(\ell-1)!}Q^a_{(\ell)b_1b_2\cdots b_{\ell-1}}(N)\delta\varphi^{b_1}(N)\delta\varphi^{b_2}(N)\cdots\delta\varphi^{b_{\ell-1}}(N) + \cdots,$$

As a time coordinate, we choose e-folding number.

P^a_b , $Q^a_{(\ell)b_1b_2\cdots b_{\ell-1}}$;decided by the background quantities

As a solution, we have

$\delta\varphi^a_F = \sum_{m=1}^{n-1} \delta\overset{(m)}{\varphi}{}^a_F,$;expansion w.r.t. $\delta\varphi^a_*$

$\delta\overset{(m)}{\varphi}{}^a$ is composed of $\delta\varphi^{a_1}_* \cdots \delta\varphi^{a_m}_*$

Using this solutions, the curvature perturbation can be described as

$$\zeta(N_F) = \sum \frac{1}{n!} N^F_{a_1a_2\ldots a_n} \delta\varphi^{a_1}_F \delta\varphi^{a_2}_F \cdots \delta\varphi^{a_n}_F \ ,$$

written by local quantities at the final time $N = N_F$

- Diagrammatic method -

Focusing on $\overset{(m)}{\delta\varphi}{}^{a}_{F}$

The evolution can be described as tree-shaped diagram ➔

Procedure;

✓ start with ●

✓ attach a line downward

✓ attach an interaction vertex ⊗

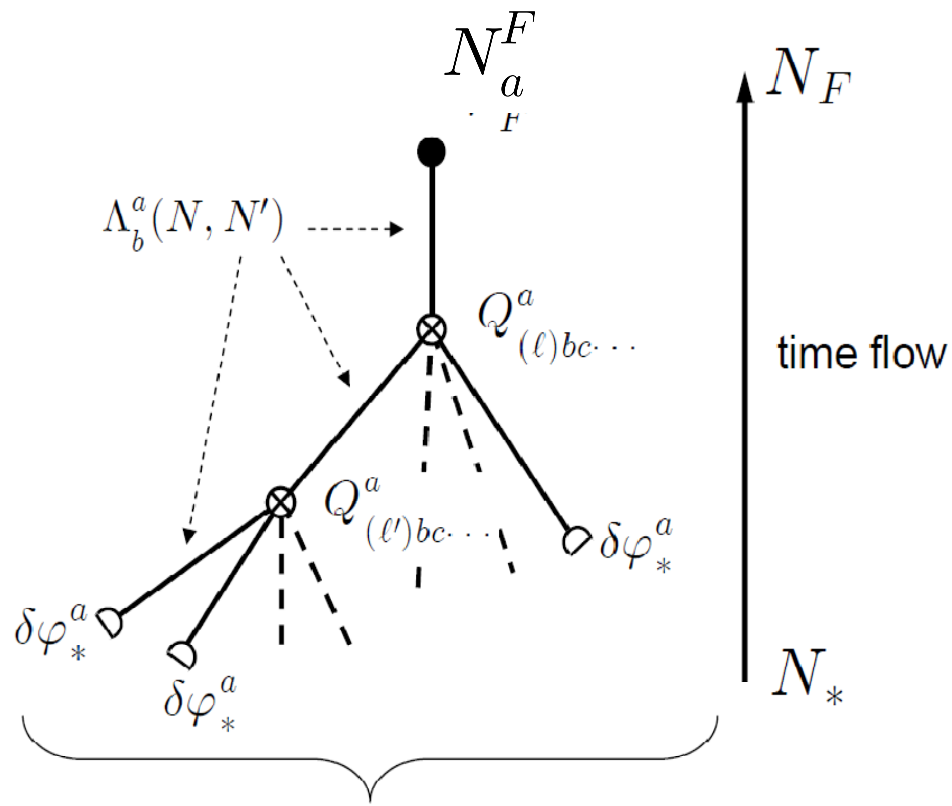
✓ repeat until all the end point are terminated by a half open circle.

● ; N_a^F

⊗ ; $Q^a_{(\ell)bc\cdots}$; interaction vertex

D ; $\delta\varphi^a_*$

— ; $\Lambda^a_b(N, N')$; propagator



The total number of $\delta\varphi^a_*$ is m

$(m \leq n-1)$

- Diagrammatic method -

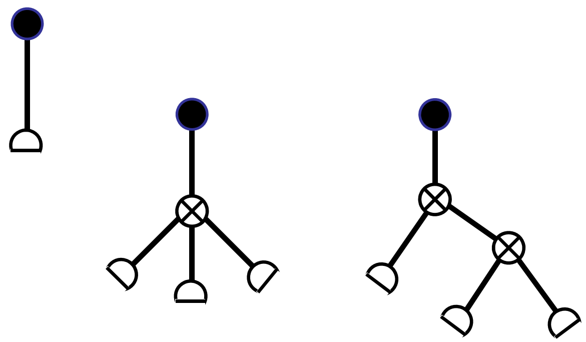
First, we consider $\zeta_F^{(\mathrm{lin})} = N_a^F \delta\varphi_F^a,$;linear term in terms of $\delta\varphi_F^a$

$$\delta\varphi_F^a = \sum_{m=1}^{n-1} \delta\overset{(m)}{\varphi}{}_F^a,$$

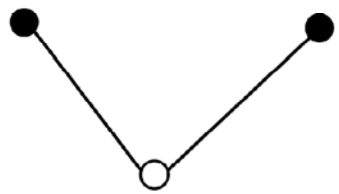
In order to obtain the higher order correlation func., We only need to consider the two-point correlation between half open circles in different tree-shaped diagram as

$$\langle \delta\varphi_*^a(k_1)\delta\varphi_*^b(k_2)\rangle \equiv A^{ab} P(k_1)\delta(\vec{k}_1 + \vec{k}_2)$$

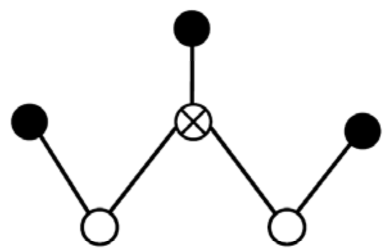
in order not to produce loop-diagram (higher-order in δ)



tree-shaped diagram



power spectrum



bi-spectrum

○ ; A^{ab} : contraction vertex

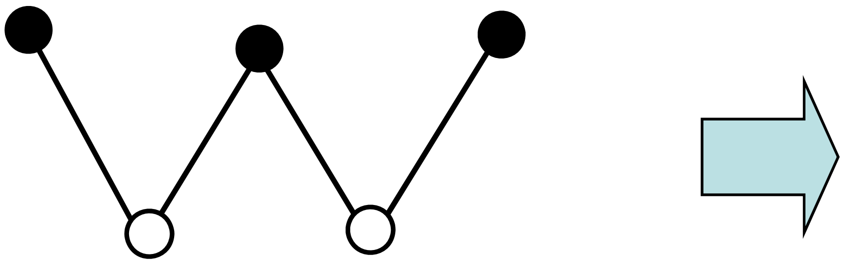
● ;n N_a^F

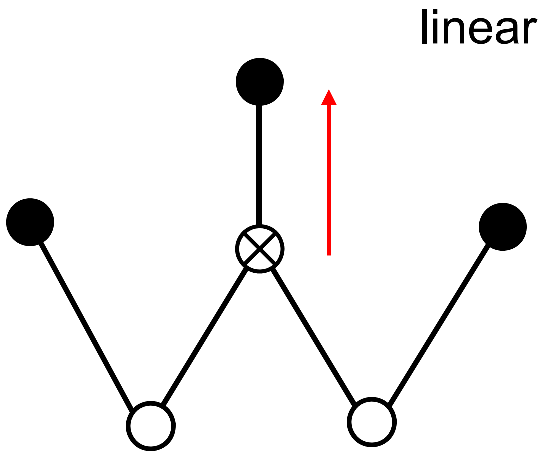
○ ;n − 1

We can associate n-point function with the diagram

- Diagrammatic method -

How to treat the non-linear term in terms of $\delta\varphi^{a}_{F}$?





Such term can be described as ;

We can consider this diagram as some limiting form of the previous diagram which corresponds to the linear contribution.

In the formulation procedure,

$$N^F_a \Lambda^a{}_c(N_F, N) Q^c_{(\ell)b_1\cdots b_{\ell-1}}(N) \longrightarrow$$

$$N^F_a \Lambda^a{}_c(N_F, N) \hat{Q}^c_{(\ell)b_1\cdots b_{\ell-1}}(N)$$
$$\equiv N^F_a \Lambda^a{}_c(N_F, N) Q^c_{(\ell)b_1\cdots b_{\ell-1}}(N) + N^F_{b_1\cdots b_{\ell-1}} \delta\left(N - (N_F - \varepsilon)\right),$$

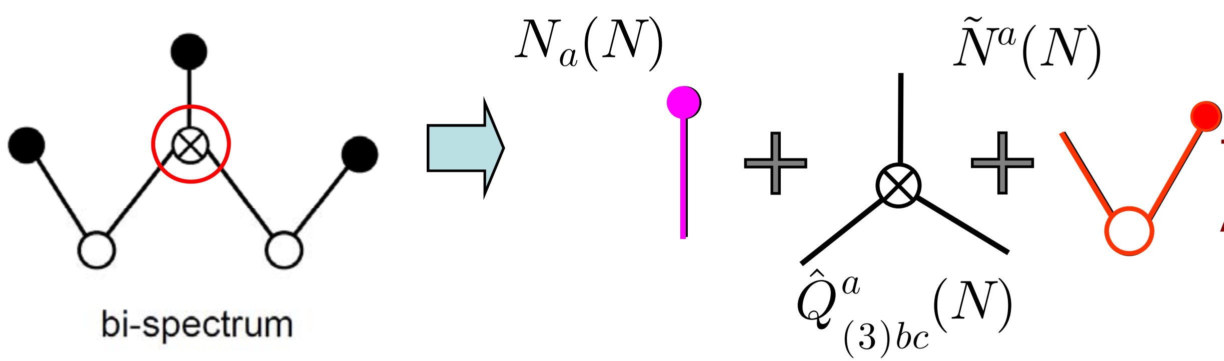
● 's nonlinearity

ε; small parameter

- Diagrammatic method -

As a result, we can describe the all contributions to n-point function as a diagram. Using this diagrammatic method, we can reduce to the problem of solving vector quantities.

For example, bi-spectrum is,..



To each sub-diagram, Assign the vector quantity

Focus on one vertex

Decompose to the sub-diagram

Each vector quantity is upward or downward.

downward ; $\frac{d}{dN}N_a(N) = -P^b{}_a(N)N_b(N),$ Solving backward in time with $N_a(N_F) = N_a^F$

upward; $\frac{d}{dN}\tilde{N}^a(N) = P^a_b(N)\tilde{N}^b(N)$, Solving forward in time with

$$\tilde{N}^a(N_*) = A^{ab}N_b(N_*)$$

- Diagrammatic method – bi-spectrum - -

Contracting all indices and integrating over the time, we have

$$\frac{6}{5} f_{NL} = W_*^{-2} \left[N^F_{ab} \tilde{N}^a(N_F) \tilde{N}^b(N_F) + \int_{N_*}^{N_F} dN N_a(N) Q^a_{(3)bc}(N) \tilde{N}^b(N) \tilde{N}^c(N) \right]$$

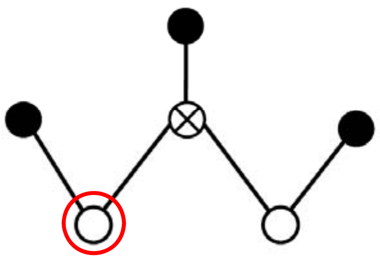
non-linear in terms of $\delta\varphi^a_F$

This recovers our previous work
T.Suyama and T.Tanaka(2008)

power spectrum; $\frac{P_\zeta}{P} = A^{ab} N_a(N_*) N_b(N_*) \equiv W_*,$

We can obtain the same result.

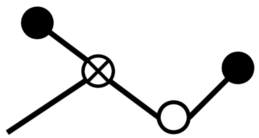
Of course, we can focus on another vertex, then,



bi-spectrum

$$\frac{6}{5} f_{NL} = W_*^{-2} A^{ab} N_a(N_*) \Omega_b(N_*)$$

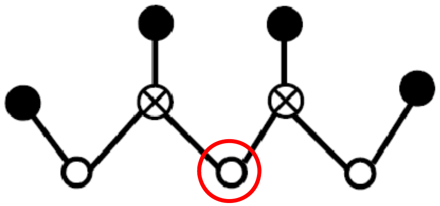
$$\frac{d}{dN} \Omega_a(N) = -\Omega_b(N) P^b{}_a(N) - N_b(N) Q^b_{(3)ac}(N) \tilde{N}^c(N)$$



$$\Omega_a(N_F) = N^F_{ab} \tilde{N}^b(N_F)$$

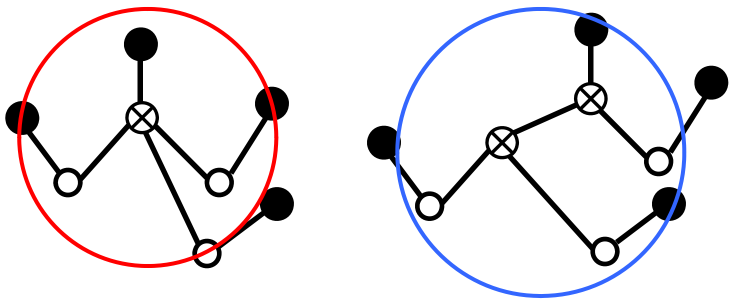
- Diagrammatic method – tri-spectrum - -

Trispectrum (2 parameters)



τ_{NL}

$$\tau_{NL} = W_*^{-3}\left[A^{ab}\Omega_a(N_*)\Omega_b(N_*)\right],$$



g_{NL} has two diagrams.

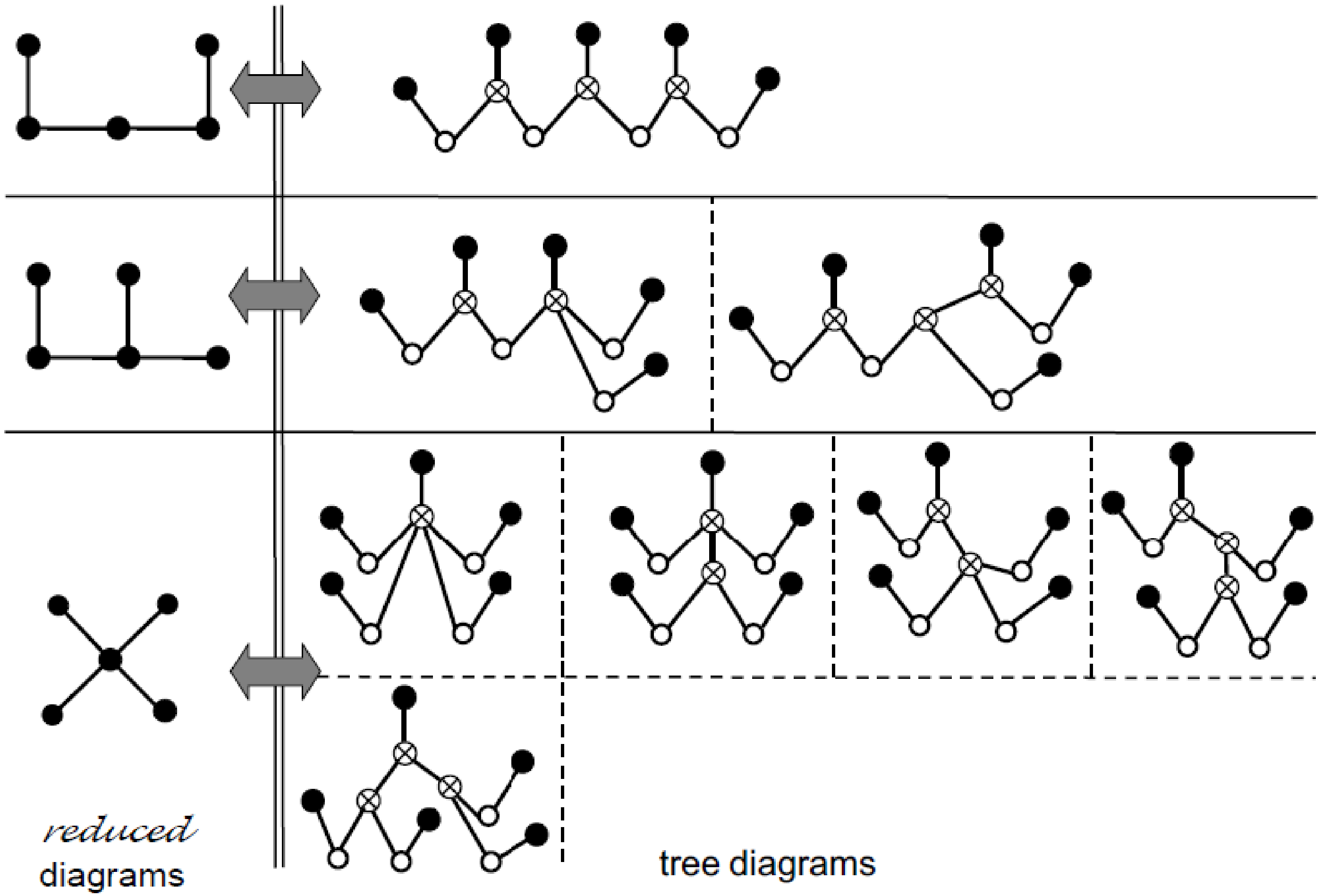
non-linearity of $\delta\varphi_F^a$

$$g_{NL} = \frac{25}{54}W_*^{-3}\left\{N^F_{abc}\tilde{N}^a(N_F)\tilde{N}^b(N_F)\tilde{N}^c(N_F) + \int_{N_*}^{N_F} dN\, N_a(N)Q^a_{(4)bcd}(N)\tilde{N}^b(N)\tilde{N}^c(N)\tilde{N}^d(N)\right.$$

$$\left. +3\int_{N_*}^{N_F} dN\,\Omega_a(N)Q^a_{(3)bc}(N)\tilde{N}^b(N)\tilde{N}^c(N)\right\}.$$

- Diagrammatic method – quad-spectrum - -

5-point function（quad-spectrum ）



- Diagrammatic method – quad-spectrum - -

5-point function（quad-spectrum)

$$\begin{aligned}\langle\zeta_{\mathbf{k}_1}\zeta_{\mathbf{k}_2}\zeta_{\mathbf{k}_3}\zeta_{\mathbf{k}_4}\zeta_{\mathbf{k}_5}\rangle_c \;=\; & u_{NL}^{(1)}\left(P_\zeta(k_1)P_\zeta(k_{12})P_\zeta(k_{45})P_\zeta(k_5) + 59 \text{ perms.}\right) \\ & + u_{NL}^{(2)}\left(P_\zeta(k_1)P_\zeta(k_{12})P_\zeta(k_4)P_\zeta(k_5) + 59 \text{ perms.}\right) \\ & + u_{NL}^{(3)}\left(P_\zeta(k_1)P_\zeta(k_2)P_\zeta(k_3)P_\zeta(k_4) + 4 \text{ perms.}\right),\end{aligned}$$

where

$$u_{NL}^{(1)} = W_*^{-4}\left\{\int_{N_*}^{N_F} dN N_a(N)\hat{Q}^a_{(3)bc}(N)\tilde{\Omega}^b(N)\tilde{\Omega}^c(N)\right\},$$

$$u_{NL}^{(2)} = W_*^{-4}\int_{N_*}^{N_F} dN\left\{N_a(N)\hat{Q}^a_{(4)bcd}(N)\tilde{\Omega}^b(N)\tilde{N}^c(N)\tilde{N}^d(N) + 3\Omega_a(N)\hat{Q}^a_{(3)bcd}(N)\tilde{\Omega}^b(N)\tilde{N}^c(N)\right\},$$

$$\begin{aligned}u_{NL}^{(3)} = W_*^{-4}\int_{N_*}^{N_F} dN\Big\{ & N_a(N)\hat{Q}^a_{(5)bcde}(N)\tilde{N}^b(N)\tilde{N}^c(N)\tilde{N}^d(N)\tilde{N}^e(N) + 6\Phi_a(N)\hat{Q}^a_{(3)bc}(N)\tilde{N}^b(N)\tilde{N}^c(N) \\ & +4\Omega_a(N)\hat{Q}^a_{(4)bcd}(N)\tilde{N}^b(N)\tilde{N}^c(N)\tilde{N}^d(N) + 12\Psi_a(N)\hat{Q}^a_{(3)bc}(N)\tilde{\Omega}^b(N)\tilde{N}^c(N) \\ & +3N_a(N)\hat{Q}^a_{(3)bc}(N)\tilde{\Pi}^b(N)\tilde{\Pi}^c(N)\Big\}. \qquad (\Gamma\end{aligned}$$

– the number of parameters for n-point functions --

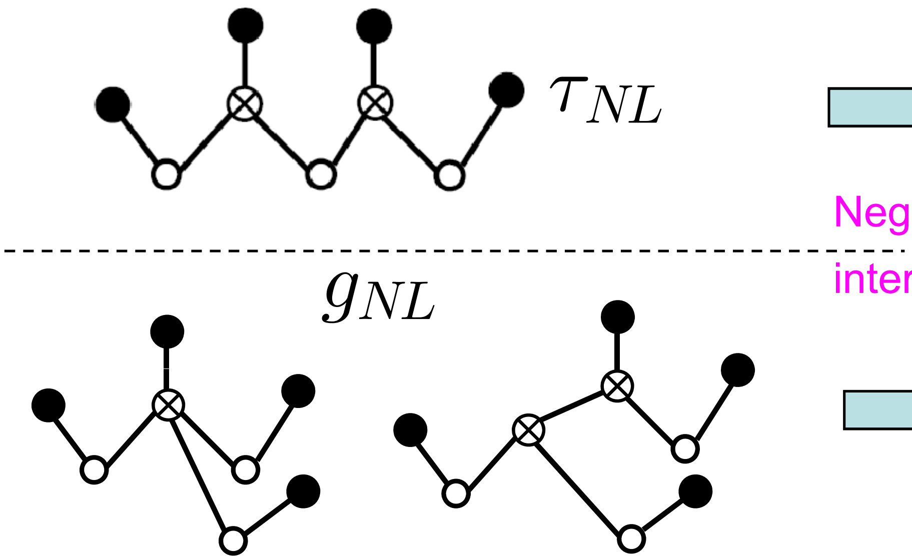
For trispectrum, we have

$$\langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\zeta(\mathbf{k}_4)\rangle = \Big[\underline{\tau_{NL}\,(P(k_{13})P(k_3)P(k_4)} + (11\mathrm{perms}))$$

$$+\underline{\frac{54}{25}g_{NL}\,(P(k_2)P(k_3)P(k_4)} + (3\mathrm{perms}))\Big]\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3+\mathbf{k}_4)$$

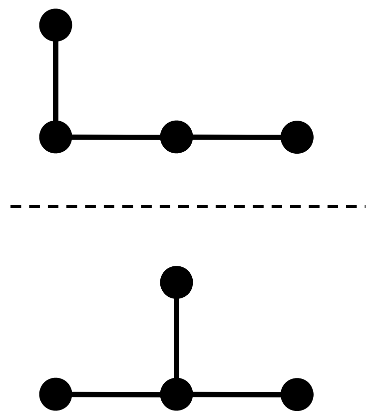
Then, we need 2 parameters which have different wave number dependence.

In the diagram description,



Neglect the interaction vertex

White circle ←→ line



reduced diagram

The number of the reduced diagrams which have mutually different topology are corresponding to the number of parameters for n-point functions.

(Byrnes et al (2007), SY, T.Suyama and T.Tanaka (2009))

- Loop correction (higher order in δ) -

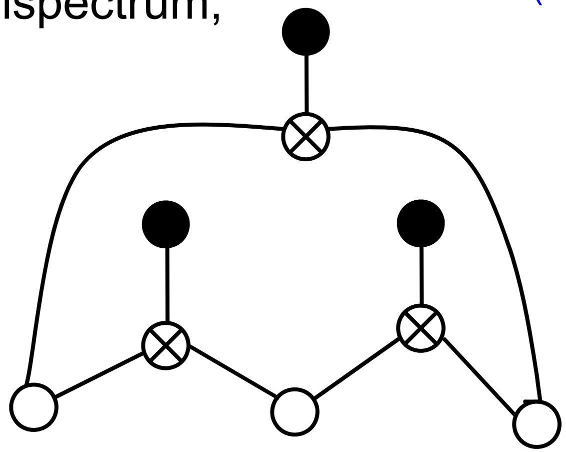
One-loop contributions

and the NG generated on sub-horizon scales (equilateral, non-local type)

One –loop (higher order in δ)

Bispectrum;

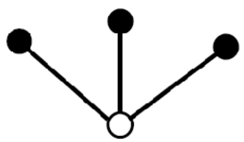
(Byrnes, et al.(2007), Rodriguez and Valenzuela-Toledo(2008))



For this diagram, we can not reduce the problem to that of solving the vector quantities.

NG generated on sub-horizon scales

(Seery and Lidsey (2005,2007))



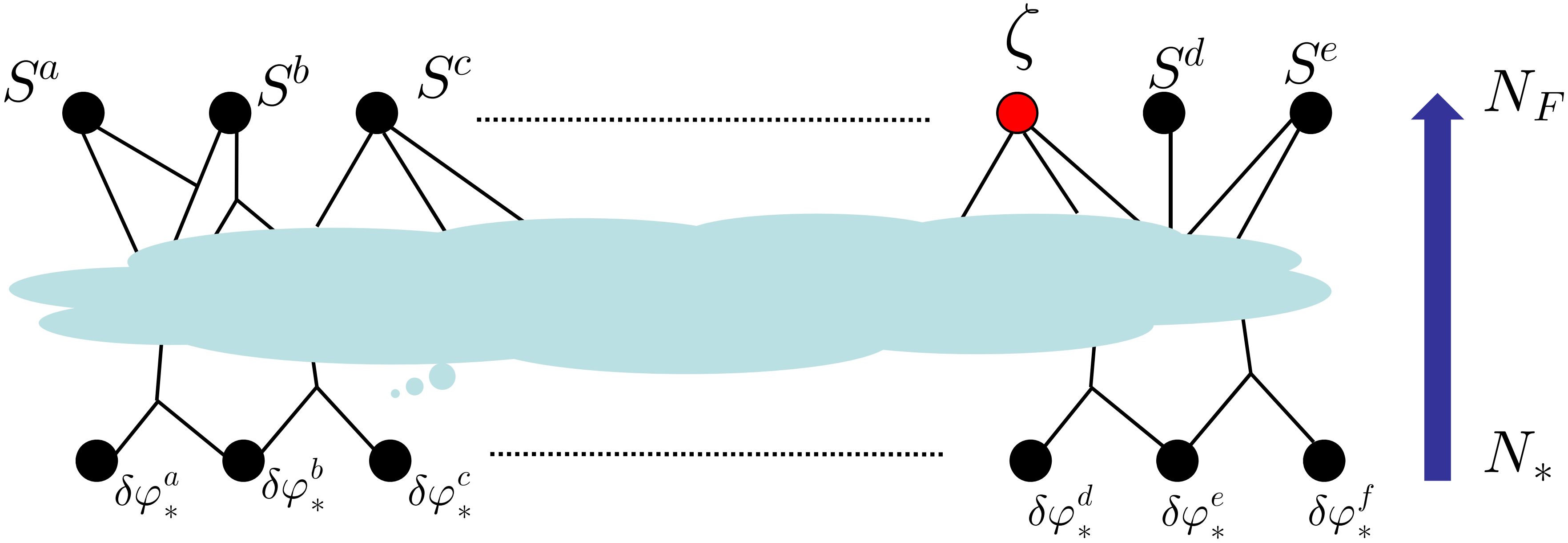
B^{abc}

⟺ $N^*_a N^*_b N^*_c B^{abc}$

$$\langle \delta\varphi^a_* \delta\varphi^b_* \delta\varphi^c_* \rangle \equiv B^{abc}(k_1, k_2, k_3)\delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) \ ,$$

– Why can the calculation be reduced ? -

If straightforward, …



ζ ; curvature perturbation on final hypersurface

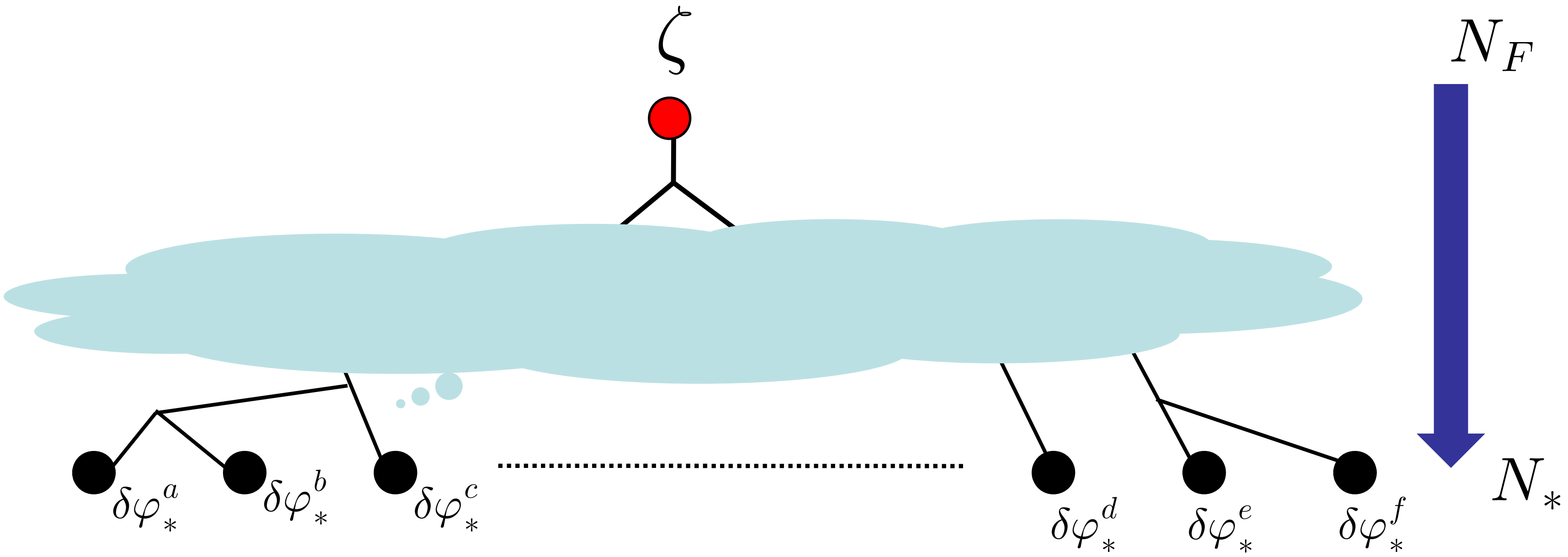
S^a ; iso-curvature perturbation on final hypersurface

$\delta\varphi_*^a$; field perturbation on initial flat hypersurface

We have to solve all the components in order to calculate the final curvature perturbation.

– Why can the calculation be reduced ? -

If our formula, …



- What we need is only the curvature perturbation on final hypersurface.
- We do not need to know the evolution of all the components of multi-scalar.
- We consider only the part of the perturbation that contributes to the final curvature perturbation.