COSMOLOGY AND BLACK HOLES IN STRING THEORY

1. Introduction
2. Part I
   Dynamics of intersecting branes and cosmology
3. Part II
   String Frame : warning for a conformal transformation
4. Summary

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1. INTRODUCTION

1. Difficulties（or Mysteries）in Ordinary 4D cosmology

Inflation

Initial Singularity

Creation of the Universe

Dark Energy

HIGHER-DIMENSIONS ?

2. Fundamental Unified Theory predicts higher–dimensions

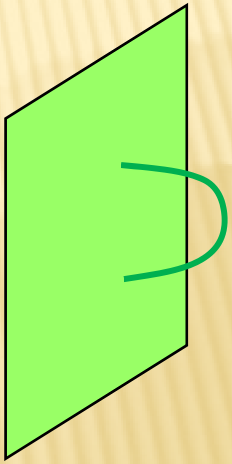
Supergravity
Superstring/M-theory

10D or 11D

How to find our present 4D universe ?

KEY 1　A brane: an interesting object in string theory

D3 brane : could be our universe



Some interesting cosmological senarios

Brane world

Ekpyrotic (or cyclic) universe

Brane inflation （Dvali-Tye , Rolling Tachyon , KKLMMT, ・・・）

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KEY 2 Higher-order curvature corrections

$$S = \int d^D x \sqrt{-g} \left[\frac{1}{2\kappa^2} R + c_1 \alpha' e^{-2\phi} L_2 + c_2 \alpha'^2 e^{-4\phi} L_3 + c_3 \alpha'^3 e^{-6\phi} L_4 + \cdots \right]$$

$$L_2 = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4 R_{\mu\nu} R^{\mu\nu} + R^2 \ (\text{Gauss – Bonnet term})$$

$$L_m = (\text{Lovelock})_m + (\text{higher derivative terms}) \ (m = 3, 4, \cdots)$$

- Inflation ?
- singularity avoidance ?
- new effects ?

theories	c_1	c_2	c_3
bosonic string	$\frac{1}{4}$	$\frac{1}{48}$	$\frac{1}{8}$
heterotic string	$\frac{1}{8}$	0	$\frac{1}{8}$
type II string	0	0	$\frac{1}{8}$

OVERVIEW OF HIGHER DIMENSIONAL COSMOLOGY

KALUZA-KLEIN COSMOLOGY

- Cosmological dimensional reduction

A. Chodos & S. Detweiler (1980)

5D Kasner solution

$$ds_5^2 = -dt^2 + a^2(t)d\mathrm{x}^2 + b^2(t)dy^2$$

$$a(t) \propto t^{1/2} \quad b(t) \propto t^{-1/2}$$

3 space : expanding, 5th space : contracting

⇒ **dynamically explain the large 3 space**

- supergravity (11D; N=1,10D) P.G.O. Freund (1982)

∃ $A_{\mu\nu\rho}$

$$a(t) \propto t \quad b(t) \propto t^{1/7} \qquad k_3 < 0, \quad k_7 = 0$$

$$a(t) \propto \cos \alpha t \quad b(t) = \text{constant} \qquad k_3 < 0, \quad k_7 > 0$$

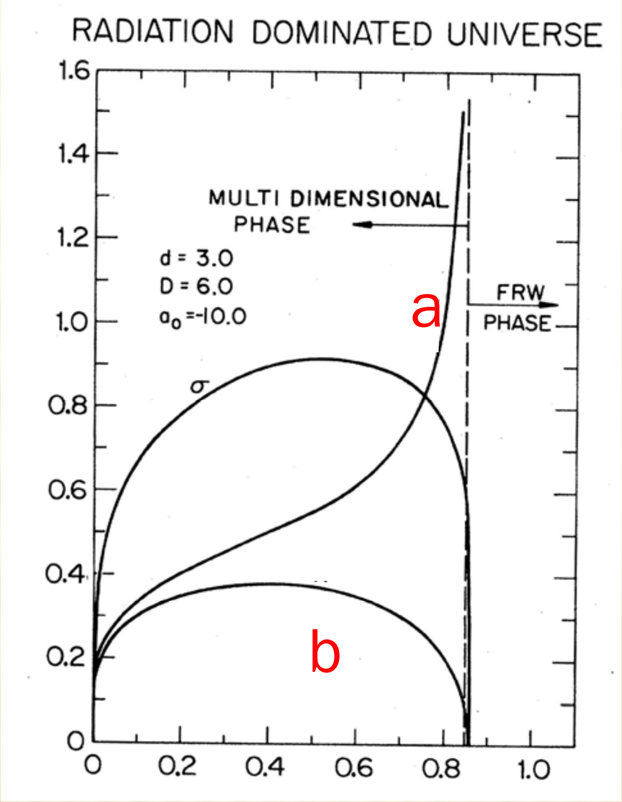
(AdS [anti de Sitter])

Kaluza-Klein inflation

D. Sahdev (1983)

perfect fluid in D-dimensions $P = w\rho$

$$k_3 = 0, \quad k_7 > 0$$



pole inflation

$$a \to \infty \quad b \to 0$$

at a finite time

However,
this point is a singularity

How to exit from inflation
and go beyond

b (volume modulus) : **time dependent** ➤ ***time dependent G_N***

observational constraint

$$\left|\frac{\dot{G}_N}{G_N}\right| \leq (0.2 \pm 0.4) \times 10^{-11} \text{years}^{-1}$$

Viking Project (1983)

$$\leq (-0.06 \pm 0.2) \times 10^{-11} \text{years}^{-1}$$

binary pulsar (1996)

⇒ ***Stabilization of volume modulus***

N=2, D=6 Kaluza-Klein supergravity

KM & Nishino 1985

compactification $M_4 \times S^2$ ← internal space

↑ our world

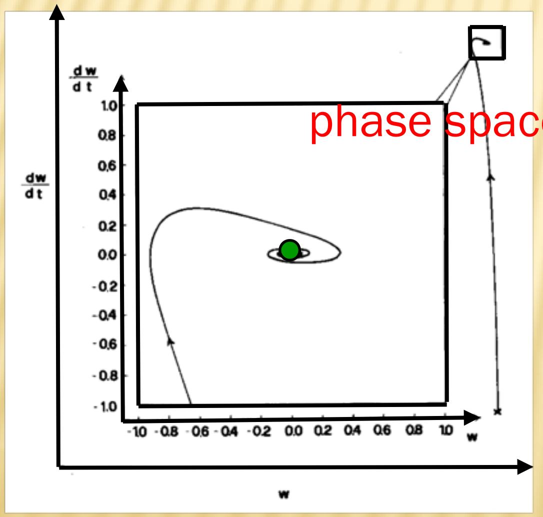
Size b(t) : small & "stabilize"

scale factor: a(t) large & inflation

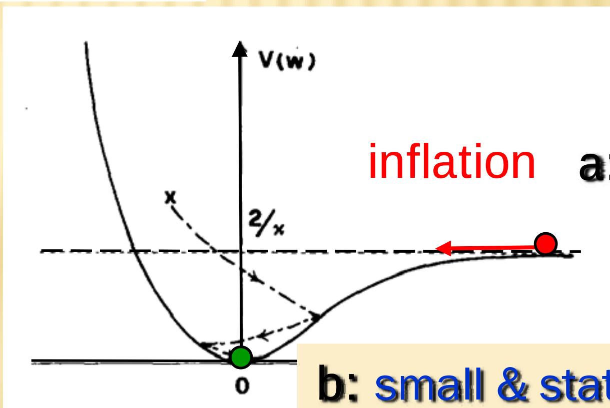
scalar field in 4D spacetime $\phi = \ln b$

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi)\right]$$

effective potential



phase space



inflation

a: extremely large

b: small & static

$\Lambda = 0$

transient inflation to standard Big Bang

Our universe is obtained as an attractor !

The similar analysis possible for many KK type universes

K. M., Class. Quant. Grav. 3(1986)233;651

K. M., Phys. Lett. B 166(1986) 59

4D effective equations

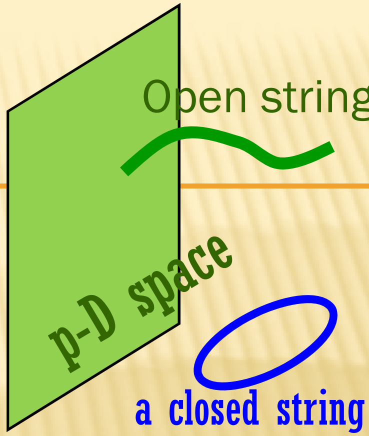
Using the effective potential,
we can analyze stability of our present universe.

10D Einstein + dilaton + Gauss-Bonnet + Form field

↓ Calabi-Yau compactification

4D FRW universe

BRANE WORLD



gravity

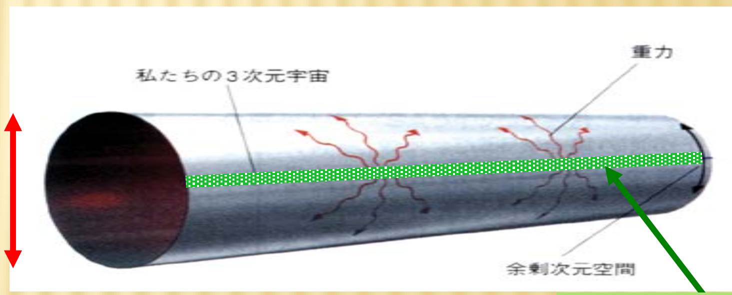
Dp brane **p-dimensional (soliton like) object**

Polchinsky (95)

Matter field (gauge field) is confined on Dp brane.

Large Extra Dimensions N. Arkani-Hamed, S. Dimopoulos, G. Dvali (98)

R < 0.1mm



extra dimensions could be large

d < 10^{-17} cm

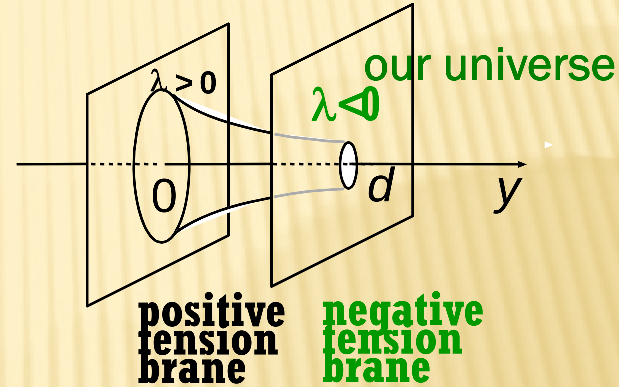
Gravity: Kaluza-Klein type

Simple toy models [5D Einstein gravity +$\Lambda(<0)$]

Randall-Sundrum model I

two-brane model

hierarchy problem

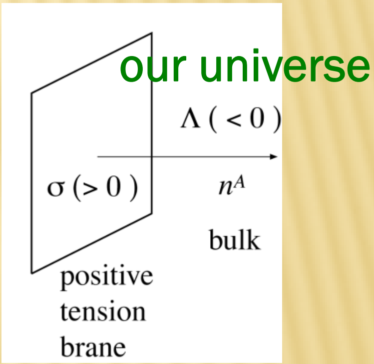


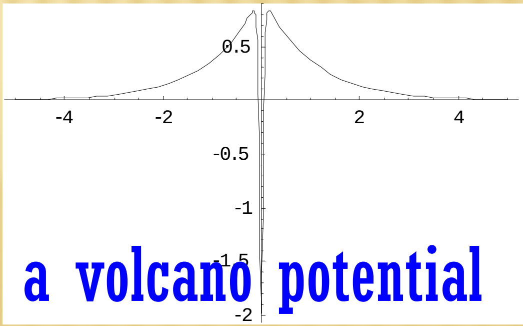
Randall-Sundrum model II

one-brane model

non-compact compactification

massless gravitons are confined in a brane





4D gravity is modified

BRANE COSMOLOGY

5D spacetime (codimension one)

- **FIVE-DIMENSIONAL APPROACH**

P. Binetruy et al (00), C. Csaki et al (99)
J. Clineet al (99), E. Flanaganet al (00)

5D Einstein eqs. with Israel's junction condition

- **EFFECTIVE FOUR-DIMENSIONAL APPROACH**

T.Shiromizu-KM-M.Sasaki (00)

4D effective Einstein eqs. By use of Gauss-Codacci eqs.

- **DOMAIN WALL APPROACH**

A domain wall motion in 5D Schwarzschild-AdS

P. Kraus

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{\kappa^2}{3}\rho + \frac{\kappa_5^4}{36}\rho^2 + \frac{\mu}{a^4}$$

- dark radiation (μ/a^4)
- brane quintessence
- inflation
- dark energy
- singularity avoidance   **U(1) or Non-abelian field**
- creation of the universe
- density perturbation

◆ codimension two (or higher)

◆ induced gravity on the brane (GDP)

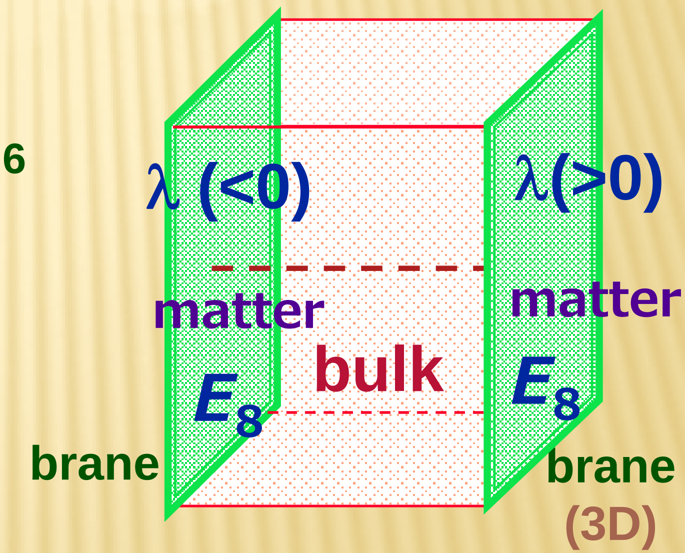
⇨ Cosmology based on fundamental physics

MORE "REALISTIC" MODELS

- **HORAVA-WITTEN** (1996)

11-D M theory compactified on S^1/Z_2 ↔ id. ↔ 10-D $E_8 X E_8$ **heterotic string**

⇒ $M^4 X\ S^1/Z_2 X(\text{Calabi-Yau})^6$



HW model -> effective 5D theory

A. Lukas, B. Ovrut, K. Stelle, D. Waldram (99)

Cosmological solution

$$ds_5^2 = b^{-1}(t)H^{1/2}(y)\left(-dt^2 + a^2(t)d\mathbf{x}^2\right) + b^2(t)H^2(y)dy^2$$

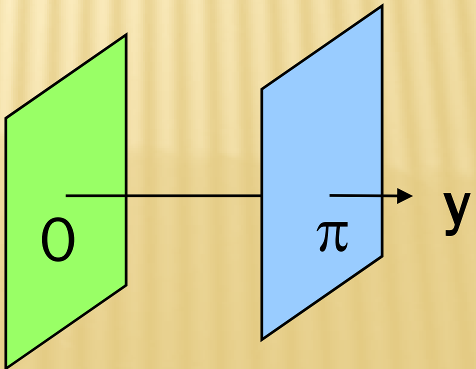
4D Einstein frame

$$H = \frac{\sqrt{2}}{3}\alpha|y| + h_0$$

$$a \propto t^p \qquad b \propto t^q \qquad \phi \propto \frac{1}{6}\ln V \sim \frac{q}{6}\ln t$$

$$p = p_\pm \equiv \frac{3}{11}\left[1 \pm \frac{4\sqrt{3}}{9}\right] \qquad q = q_\pm \equiv \frac{2}{11}\left[1 \mp 2\sqrt{3}\right]$$

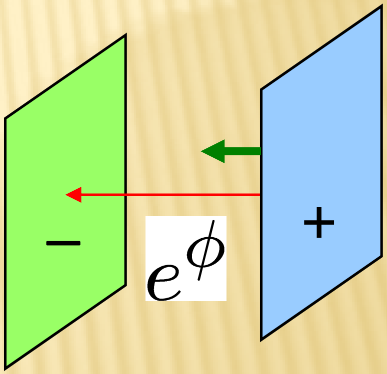
(0.48, 0.06) (–0.45, 0.81)



New idea: A brane collision

- **Ekpyrotic or cyclic universe** J. Khoury, P.Steinhardt, N.Turok

The alternative to inflation ?



- **collision of D brane & $\bar{\mathrm{D}}$ brane**

Brane inflation

Dvali-Tye , Rolling Tachyon , KKLMMT, ・・・

+ test branes

KKLT : stable Calabi-Yau space by flux

Effective 4D theory

inflaton (~distance) ⇨ expansion of the Universe

4D Effective Theories with Warped Compactification

IIB, HW model H. Kodama, K. Uzawa (06)

Some solutions are not allowed in Higher dimensions

10D $$ds_{10}^2 = h^{-1/2}(x,y)ds_4^2(x) + h^{1/2}(x,y)ds_6^2(y)$$

$$h(x,y) = h_0(x) + h_1(y)$$

$$R_{\mu\nu}(x) = 0 \qquad R_{ab}(y) = \lambda g_{ab}(y)$$

$$D_\mu D_\nu h_0 = \lambda g_{\mu\nu}(x) \qquad \Delta_y h_1 = -\frac{g_s}{2}(G_3 \cdot \bar{G}_3)_y$$

4D effective theory

$$R_{\mu\nu}(x) = H^{-1}\left[D_\mu D_\nu H - \lambda g_{\mu\nu}(x)\right]$$

$$\Delta_x H = 4\lambda \qquad H = h_0(x) + V_6^{-1}\int_{Y_6} d\Omega_6 h_1(y)$$

⇨ **Careful analysis when extra dimension is time dependent**

Part I

Dynamics of intersecting branes and cosmology

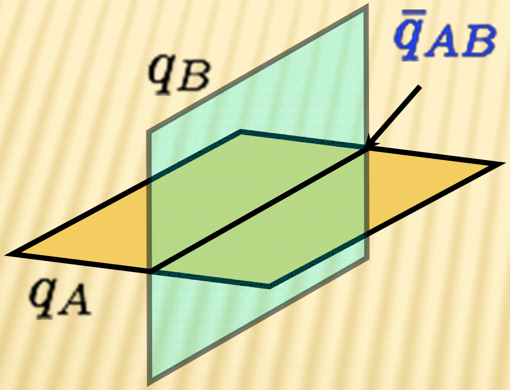
TIME DEPENDENT INTERSECTING BRANES

Higher dimensional cosmology with branes

- microscopic description of BH by branes

S. R. Das ('96),
M. Cvetic and C. M. Hull ('88)

branes in some dimensions ⟶ gravitational sources



BHs (Black objects) in 4 or 5 dim

branes ~ charges

⇕

area of horizon (BH entropy)

⇨ **cosmology ?**

D-dimensional effective action

P. Binetruy, M. Sasaki , K. Uzawa (07)
KM, N. Ohta, K. Uzawa (in preparation)

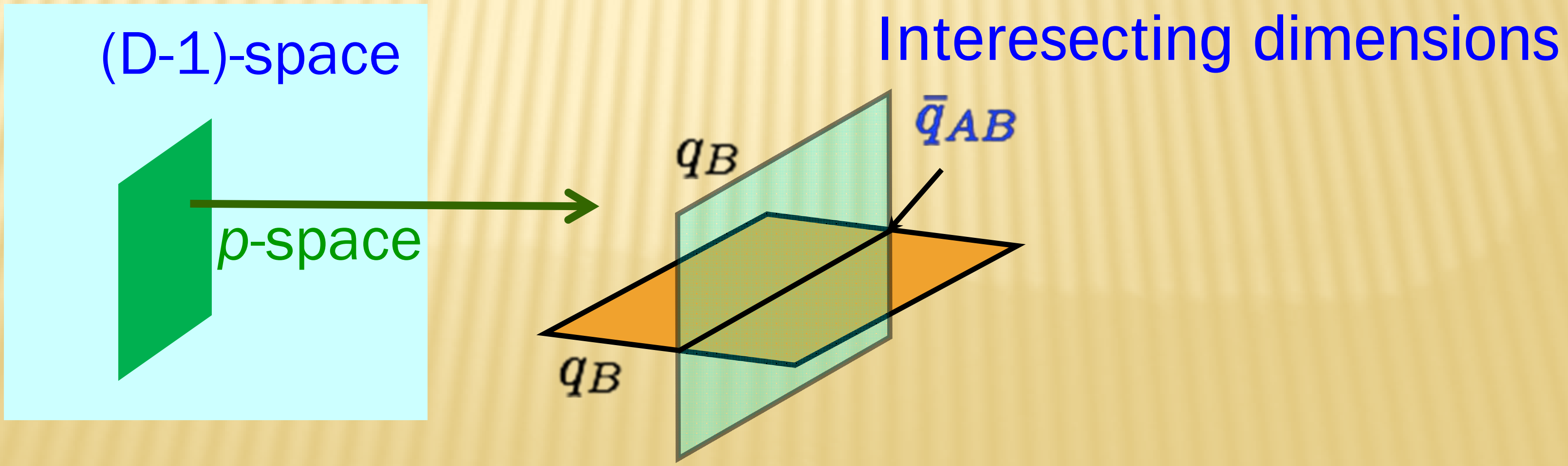
$$S = \frac{1}{2\kappa^2}\int d^D X \sqrt{-g}\left[R - \frac{1}{2}(\nabla\phi)^2 - \sum_A \frac{1}{2\cdot n_A!} e^{a_A \phi} F_{n_A}^2\right]$$

ϕ : **dilaton** F_{n_A} **:** n_A **form fields**

A: type of branes (2-brane, 5-brane etc)

Ansatz:

Source: Several types of branes in *p*-dim space



time dependence

branes

$$ds^2 = -\prod_A h_A^{-\frac{D-q_A-3}{D-2}}(t,z)dt^2 + \sum_{\alpha=1}^{p}\prod_A h_A^{\frac{\delta_A^\alpha}{D-2}}(t,z)(dx^\alpha)^2(X)$$

$$+\prod_A h_A^{\frac{q_A+1}{D-2}}(t,y)u_{ij}(Z)dy^i dy^j$$

Forms

$$F_{(q_A+2)} = d(h_A^{-1}) \wedge \Omega(X_A)$$

We classify all possible configuration

One brane ($\tilde{I}$) can be time dependent

$$h_{\tilde{I}} = At + B + H_{\tilde{I}} \qquad \Delta_Z h_{\tilde{I}} = 0$$

$$h_I = H_I \quad (I \neq \tilde{I}) \qquad \Delta_Z h_I = 0$$

$$R_{ij}(Z) = 0$$

To find our 4D universe, we need compactification

Our universe : isotropic and homogeneous 3D space

Two intersecting branes

	0	1	2	3	4	5	6	7	8	9	10
M5	○	○	○	○	○	○					
M5	○	○	○	○			○	○			

X

	0	1	2	3	4	5	6	7	8	9	10
M2	○	○	○								
M5	○	○		○	○	○	○				

X

	0	1	2	3	4	5	6	7	8	9	10
M2	○	○	○								
M2				○	○						

$$ds^2 = (h_5 h_{\tilde{5}})^{-1/3}\left[\eta_{\mu\nu}dx^\mu dx^\nu + h_5\gamma_{ij}(Y_2)dy^i dy^j + h_{\tilde{5}} s_{mn}(Y_2')dy^m dy^n + h_5 h_{\tilde{5}} u_{ab}(Z_3)dz^a dz^b\right]$$

$$h_{\tilde{5}} = At + B + H_{\tilde{5}}(z) \qquad h_5 = H_5(z)$$

Harmonics (q-dim: compactification)

$$H_5 = \sum_k M_k |z - z_k|^{q-1} \quad \left(\text{or } \sum_k M_k \ln|z - z_k| \text{ for } q = 1\right)$$

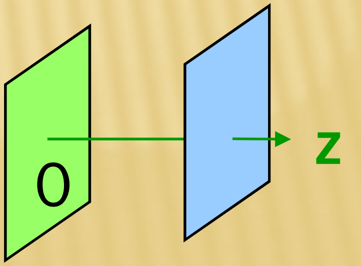
$$ds^2_{4\text{Einstein}} = (h_5 h_{\tilde{5}})^{q/3}\eta_{\mu\nu}dx^\mu dx^\nu = -d\tau^2 + \tau^{\frac{2q}{q+6}}d\mathbf{x}^2$$

$q = 0$: Minkowski

$q = 2 : a \propto \tau^{1/4}$

	0	1	2	3	4	5	6	7	8	9	10
M5	○	○	○	○	○	○			$q = 2$		
M5	○	○	○	○			○	○			

x (1–3)   compactification (4–9)



Time dependent BH ?

	0	1	2	3	4	5	6	7	8	9	10
	t	x	y	w^1	w^2	w^3	w^4	z^1	z^2	z^3	z^4
M2	○	○	○								
M5	○	○		○	○	○	○				
W	○										

compactification

$$ds^2 = h_2^{-2/3} h_5^{-1/3} \Big[-h^{-1} dt^2 + h \left(dx + (h^{-1} - 1) dt \right)^2 + h_5 dy^2$$
$$+ h_2 \gamma_{ij}(Y) dw^i dw^j + h_2 h_5 u_{ab}(Z) dz^a dz^b \Big]$$

$$F_{(4)} = dh_2^{-1} \wedge dt \wedge dy \wedge dx^2 + *_Z (dh_5) \wedge dx^2$$

$$h_2 = At + B + H_2(z), h_5 = H_5(z), h_W = H_W(z)$$
$$h_2 = H_2(z), h_5 = At + B + H_5(z), h_W = H_W(z)$$

$$ds^2_{5\text{Einstein}} = -(h_2 h_5 h)^{-2/3} dt^2 + (h_2 h_5 h)^{1/3} u_{ab} dz^a dz^b$$

$$h_2 h_5 h = \left(At + B + \frac{Q_2}{z^2}\right)\left(1 + \frac{Q_5}{z^2}\right)\left(1 + \frac{Q_W}{z^2}\right)$$

$$h_2 h_5 h = \left(1 + \frac{Q_2}{z^2}\right)\left(At + B + \frac{Q_5}{z^2}\right)\left(1 + \frac{Q_W}{z^2}\right)$$

Time dependent : O

Asymptotically flat : X

z=0 : horizon

$$ds^2_{5\text{Einstein}} = -(\tilde{h}_2 h_5 h)^{-2/3} d\tilde{t}^2 + \left(\frac{\tilde{t}}{\tilde{t}_0}\right)^{1/2} (\tilde{h}_2 h_5 h)^{1/3} u_{ab} dz^a dz^b$$

$$\tilde{h}_2 = \left(1 + \frac{\tilde{Q}_2 \tilde{t}^{3/2}}{z^2}\right)$$

↳ stiff matter (P=ρ) in 5D

Spacetime structure ?

BH with time dependent mass in expanding universe ?

compactification with infinitely periodic BHs

→ Asymptotically 4D BH

or

	0	1	2	3	4	5	6	7	8	9	10
	t	x^1	x^2	y^1	y^2	w^1	w^2	w^3	z^1	z^2	z^3
M2	○	○	○								
M2	○			○	○						
M5	○	○		○		○	○	○			
M5	○		○		○	○	○	○			

compactification

Two time dependent branes ?

KM. N. Ohta, M. Tanabe, R, Wakebe

$D = d + (\tilde{d} + 2)$ d : brane spacetime

null coordinate *u-dependent* ⟺ supersymmetry

$$ds_D^2 = e^{2\Xi(u,r)}\left[-2dudv + K(u, y^\alpha, r)du^2\right] + \sum_{\alpha=1}^{d-2} e^{2Z_\alpha(u,r)}(dy^\alpha)^2$$

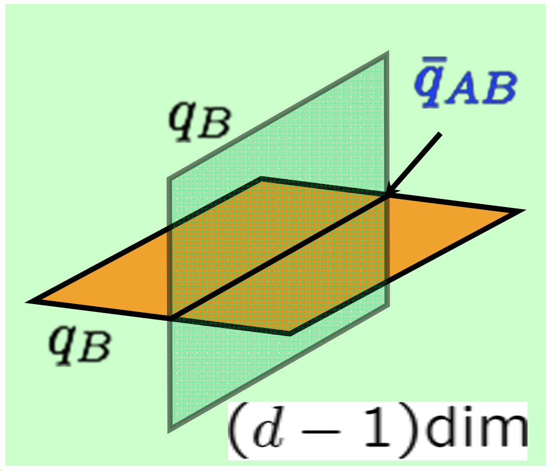
$$+e^{2B(u,r)}\left(dr^2 + r^2 d\Omega_{\tilde{d}+1}^2\right)$$

Spherically symmetric

$$F_{n_A} = E'_A(u, r)\, du \wedge dv \wedge dy^{\alpha_1} \wedge \cdots \wedge dy^{\alpha_{q_A - 1}} \wedge dr$$

Intersection rule

$$\bar{q}_{AB} = \frac{(q_A + 1)(q_B + 1)}{D - 2} - 1 - \frac{1}{2}\epsilon_A a_A \epsilon_B a_B$$



$$H_A = \sqrt{\frac{2(D-2)}{\Delta_A}\frac{1}{\tilde{E}_A}}$$

$$\tilde{E}_A \equiv e^{-U} E_A$$

$$U \equiv 2\Xi + \sum_{\alpha=1}^{d-2} Z_\alpha + \tilde{d}B$$

$$\Delta_A = (D - q_A - 3)(q_A + 1) + \frac{D-2}{2} a_A^2$$

$$H_A = h_A(u) + \frac{Q_A}{r^{\tilde{d}}}$$

$$\Xi = -\sum_A \frac{D - q_A - 3}{\Delta_A} \ln H_A + \xi(u)$$

$$B = \sum_A \frac{q_A + 1}{\Delta_A} \ln H_A + \beta(u)$$

$$Z_\alpha = -\sum_A \frac{\delta_A^{(\alpha)}}{\Delta_A} \ln H_A + \zeta_\alpha(u)$$

$$\Phi = \sum_A \epsilon_A a_A \frac{D-2}{\Delta_A} \ln H_A + \phi(u)$$

Constraint equations for $\xi(u), \beta(u), \zeta_\alpha(u), \phi(u)$:

- $$\epsilon_A a_A \phi + 2 \sum_{\alpha \notin q_A} \zeta_\alpha + 2\tilde{d}\beta = 0$$
- $$W(u,r) + V(u) + \frac{1}{2} \sum_{\alpha=1}^{d-2} e^{2(\Xi - Z_\alpha)} \partial_\alpha^2 K + \frac{1}{2} e^{2(\Xi - B)} r^{-(\tilde{d}+1)} \left(r^{(\tilde{d}+1)} K' \right)' = 0$$

$$W(u,r) \equiv \sum_{A,B} \frac{(D-2)^2}{\Delta_A \Delta_B} (M_{AB}+2) (\ln H_A)^{\cdot} (\ln H_B)^{\cdot} + 2 \sum_A \frac{D-2}{\Delta_A} (\ln H_A)^{\cdot\cdot} + 4(D-2)(\dot{\beta} - \dot{\xi}) \sum_A \frac{(\ln H_A)^{\cdot}}{\Delta_A}$$

$$V(u) \equiv \sum_{\alpha=1}^{d-2} \left(\ddot{\zeta}_\alpha + \dot{\zeta}_\alpha^2 \right) + (\tilde{d}+2) \left(\ddot{\beta} + \dot{\beta}^2 \right) - 2\dot{\xi} \left[\sum_{\alpha=1}^{d-2} \dot{\zeta}_\alpha + (\tilde{d}+2)\dot{\beta} \right] + \frac{1}{2} (\dot{\phi})^2$$

Assume

$$K = e^{-2\xi(u)} k(u, y^\alpha) + \frac{m(u, y^\alpha)}{r^{\tilde{d}}}$$

$$k(u, y^\alpha) = k_0(u) + \sum_{\alpha=1}^{d-2} k_\alpha(u) y^\alpha$$

$$+ \sum_{\alpha,\beta=1(\alpha\neq\beta)}^{d-2} k_{\alpha\beta}(u) y^\alpha y^\beta + \sum_{\alpha\in^\forall q_A} e^{2\zeta_\alpha(u)} h_{\alpha\alpha}(u)(y^\alpha)^2$$

$$m(u, y^\alpha) = m_0(u) + \sum_{\alpha=1}^{d-2} m_\alpha(u) y^\alpha$$

solution

$$ds_D^2 = \prod_A H_A^{2\frac{q_A+1}{\Delta_A}} \Big[e^{2\xi(u)} \prod_A H_A^{-2\frac{D-2}{\Delta_A}} \left(-2dudv + K(u, r, y^\alpha) du^2\right)$$

$$+ \sum_{\alpha=1}^{d-2} \prod_A H_A^{-2\frac{\gamma_A^{(\alpha)}}{\Delta_A}} e^{2\zeta_\alpha(u)} (dy^\alpha)^2 + e^{2\beta(u)} \left(dr^2 + r^2 d\Omega_{\tilde{d}+1}^2\right) \Big]$$

$$E_A = \sqrt{\frac{2(D-2)}{\Delta_A} H_A^{-1}} \qquad \Phi = \sum_A \epsilon_A a_A \frac{D-2}{\Delta_A} \ln H_A + \phi(u)$$

$$H_A = h_A(u) + \frac{Q_A}{r^{\tilde{d}}}$$

$$(1)\dot{H}_A = 0$$

- single D3 brane
- D1D5 brane with pp wave

$$ds^2 = H_1^{-\frac{3}{4}} H_5^{-\frac{1}{4}} \left[-2dudv + K(u, y^\alpha)du^2\right] + \left(\frac{H_1}{H_5}\right)^{\frac{1}{4}} \sum_{\alpha=1}^{4} dy_\alpha^2 + H_1^{\frac{1}{4}} H_5^{\frac{3}{4}} \left(dr^2 + r^2 d\Omega_3^2\right)$$

$$H_A = h_A + \frac{Q_A}{r^2}$$

$$K = e^{-2\xi(u)} \left(k_0(u) + \sum_{\alpha=1}^{4} k_\alpha(u) y^\alpha\right)$$

$$\Phi = \ln\left(\frac{H_1}{H_5}\right)^{\frac{1}{2}} + \phi(u)$$

$(2)\dot{H}_A \neq 0$

- D1D5 brane

$$ds^2 = -2H_1^{-\frac{3}{4}}H_5^{-\frac{1}{4}}dudv + \left(\frac{H_1}{H_5}\right)^{\frac{1}{4}} \sum_{\alpha=1}^{4} dy_\alpha^2 + H_1^{\frac{1}{4}}H_5^{\frac{3}{4}}\left(dr^2 + r^2 d\Omega_3^2\right)$$

$$\Phi = \ln\left(\frac{H_1}{H_5}\right)^{\frac{1}{2}}$$

two branes : time dependent

$$H_1(u,r) = h_1(u) + \frac{Q_1}{r^2}, \quad H_5(u,r) = h_5(u) + \frac{Q_5}{r^2}$$

$$h_1(u) = \frac{Q_1}{2}[au+b+f(u)], \quad h_5(u) = \frac{Q_5}{2}[au+b-f(u)],$$

$$au + b = \sqrt{f(f-c_1)} + c_1 \ln(\sqrt{f} + \sqrt{f-c_1}) + c_2$$

D2D6 brane

$$ds^2 = H_2^{-\frac{5}{8}} H_6^{-\frac{1}{8}} \left(-2e^{2\xi(u)} du dv + e^{2\zeta_1(u)} (dy^1)^2\right)$$
$$+ H_2^{\frac{3}{8}} H_6^{-\frac{1}{8}} \sum_{\alpha=2}^{5} e^{2\zeta_\alpha(u)} dy_\alpha^2 + H_2^{\frac{3}{8}} H_6^{\frac{7}{8}} e^{2\beta(u)} \left(dr^2 + r^2 d\Omega_2^2\right)$$

$$\Phi = \frac{1}{4} \ln H_2 - \frac{3}{4} \ln H_6 + \phi(u)$$

two branes : time dependent

$$H_2(u,r) = h_2(u) + \frac{Q_2}{r}, \quad H_6(u,r) = h_6(u) + \frac{Q_6}{r}$$

$$h_2(u) = \frac{Q_2}{2}[F(f(u)) + f(u)], \quad h_6(u) = \frac{Q_6}{2}[F(f(u)) - f(u)]$$

$$F(f) \equiv \sqrt{f(f-c_1)} + c_1 \ln\left(\sqrt{f} + \sqrt{f-c_1}\right) + c_2$$

$$\sum_{\alpha=2}^{5} \zeta_\alpha = -\phi = \frac{4}{3}\beta \qquad \beta - \xi = -\frac{1}{2} \ln\left[\frac{\sqrt{f}}{\sqrt{f-c_1}} \dot{f}\right] + c_3$$

$$\ddot{\left(\zeta_1 + \frac{5}{3}\beta\right)} - 2\dot{\xi}\dot{\left(\zeta_1 + \frac{5}{3}\beta\right)} + \dot{\zeta}_1^2 + \frac{17}{3}\dot{\beta}^2 - \sum_{\alpha,\beta=2}^{5} \dot{\zeta}_\alpha \dot{\zeta}_\beta (1 - \delta_{\alpha\beta}) = 0$$

4 arbitrary functions

Some properties

(1) singularity $r \to 0$

- single brane

$$\mathcal{R} \propto r^{\frac{2(q_A+1)(7-q_A)}{\Delta_A}-2} \qquad \mathcal{R}_{\mu\nu\rho\sigma}\mathcal{R}^{\mu\nu\rho\sigma} \propto r^{-\frac{(q_A-3)^2}{4}}$$

D3: non-singular

$$\mathcal{R}_{\mu\nu\rho\sigma}\mathcal{R}^{\mu\nu\rho\sigma} = \frac{80Q_3^2\left(Q_3^2+12r^8h_3^2\right)}{\left(Q_3+r^4h_3\right)^5}$$

- two branes

$$\mathcal{R} \propto r^{2[7+\bar{q}-(q_A+q_B)]\left[\frac{q_A+1}{\Delta_A}+\frac{q_B+1}{\Delta_B}\right]-2}$$

D1D5 $$\mathcal{R}_{\mu\nu\rho\sigma}\mathcal{R}^{\mu\nu\rho\sigma} = \frac{24Q_1^4Q_5^4+\mathcal{O}(r^2)}{(Q_1+r^2h_1)^{9/2}(Q_5+r^2h_5)^{11/2}}$$

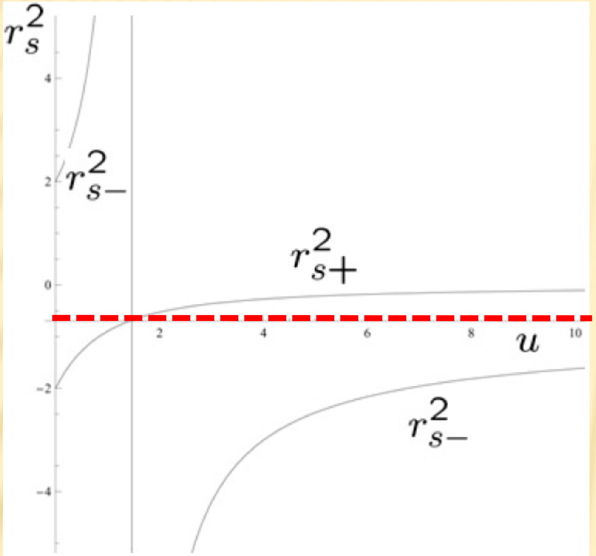
D2D4 $$\mathcal{R}_{\mu\nu\rho\sigma}\mathcal{R}^{\mu\nu\rho\sigma} = \frac{24Q_2^4Q_4^4+\mathcal{O}(r^2)}{(Q_2+r^2h_2)^{19/4}(Q_4+r^2h_4)^{21/4}}$$

D3D3 $$\mathcal{R}_{\mu\nu\rho\sigma}\mathcal{R}^{\mu\nu\rho\sigma} = \frac{24Q_3^4Q_{\tilde{3}}^4+\mathcal{O}(r^2)}{(Q_3+r^2h_3)^5(Q_{\tilde{3}}+r^2h_{\tilde{3}})^5}$$

singularity of D1D5 brane system

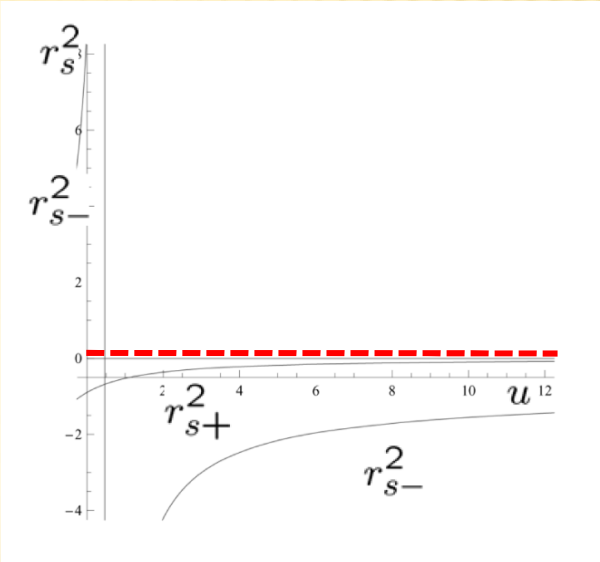
$$r^2 = r_{s\pm}^2 \equiv -\frac{2}{au + b \pm f(u)}$$

$$au + b = \sqrt{f(f - c_1)} + c_1 \ln(\sqrt{f} + \sqrt{f - c_1}) + c_2$$



$a = 1, b - c_2 = 0, c_1 = 1$

a singularity hits the brane



$a = 1, b - c_2 = 1, c_1 = 1$

spacetime structure near the branes

D3 $z^2 = Q_3/r^2$

$$ds_{10}^2 = Q_3^{1/2}\Big[\frac{1}{z^2}\left(-2dudv + dz^2\right) + d\Omega_5^2\Big] + \sum_{\alpha=1}^{2} e^{2\zeta_\alpha(u)}(dy^\alpha)^2$$

$AdS_3 \times S^5 \times \tilde{E}^2$

$z^2 = Q_A Q_B/r^2$

D1D5 $$ds_{10}^2 = Q_1^{1/4}Q_5^{3/4}\Big[\frac{1}{z^2}\left(-2dudv + dz^2\right) + d\Omega_3^2\Big] + \left(\frac{Q_1}{Q_5}\right)^{1/4}\sum_{\alpha=1}^{4}(dy^\alpha)^2$$

D2D4 $$ds_{10}^2 = Q_2^{3/8}Q_4^{5/8}\Big[\frac{1}{z^2}\left(-2dudv + dz^2\right) + d\Omega_3^2\Big] + \left(\frac{Q_4}{Q_2}\right)^{5/8}(dy^1)^2 + \left(\frac{Q_2}{Q_4}\right)^{3/8}\sum_{\alpha=2}^{4}(dy^\alpha)^2$$

D3D3 $$ds_{10}^2 = Q_3^{1/2}Q_{\tilde{3}}^{1/2}\Big[\frac{1}{z^2}\left(-2dudv + dz^2\right) + d\Omega_3^2\Big] + \left(\frac{Q_{\tilde{3}}}{Q_3}\right)^{1/2}\sum_{\alpha=1}^{2}(dy^\alpha)^2 + \left(\frac{Q_3}{Q_{\tilde{3}}}\right)^{1/2}\sum_{\alpha=3}^{4}(dy^\alpha)^2$$

$AdS_3 \times S^3 \times E^4$ static

+KK $\longrightarrow$ time dependent BH ?

Asymptotic structure ($r \to \infty$)

$$ds_D^2 = -2dudv + \sum_{\alpha=1}^{d-2} f_\alpha(u)(dy^\alpha)^2 + g(u)d\vec{r}_{\tilde{d}+2}^{\,2}$$

$$f_\alpha(u) \equiv \prod_A h_A^{-\frac{2\delta_A^{(\alpha)}}{\Delta_A}} e^{2\zeta_\alpha} \qquad g(u) \equiv \prod_A h_A^{\frac{2(q_A+1)}{\Delta_A}} e^{2\beta}$$

compactification ⇏ FRW universe ?

Part II

String Frame : warning for a conformal transformation

COSMOLOGY WITH HIGHER CURVATURE TERMS

Higher curvature terms

KK type

Type II (or M) quartic correction terms

$$S = \frac{1}{2\kappa_{11}^2} \int \sqrt{-g}\,[R + \alpha_4 E_8 + \gamma J_0]$$

$$E_8 = -\frac{1}{2^5 \times 3} \epsilon^{\alpha_1\alpha_2\alpha_3\rho_1\sigma_1\cdots\rho_4\sigma_4} \epsilon_{\alpha_1\alpha_2\alpha_3\mu_1\nu_1\cdots\mu_4\nu_4} \times R^{\mu_1\nu_1}{}_{\rho_1\sigma_1} R^{\mu_2\nu_2}{}_{\rho_2\sigma_2} R^{\mu_3\nu_3}{}_{\rho_3\sigma_3} R^{\mu_4\nu_4}{}_{\rho_4\sigma_4}$$

$$J_0 = C^{\lambda\mu\nu\kappa} C_{\alpha\mu\nu\beta} C_\lambda{}^{\alpha\rho\sigma} C^\beta{}_{\rho\sigma\kappa} + \frac{1}{2} C^{\lambda\kappa\mu\nu} C_{\alpha\beta\mu\nu} C_\lambda{}^{\rho\sigma\alpha} C^\beta{}_{\rho\sigma\kappa}$$

$$\alpha_4 = \frac{\kappa_{11}^2 T_2}{3^2 \times 2^9 \times (2\pi)^4} \qquad \gamma = \frac{\kappa_{11}^2 T_2}{3 \times 2^4 \times (2\pi)^4}$$

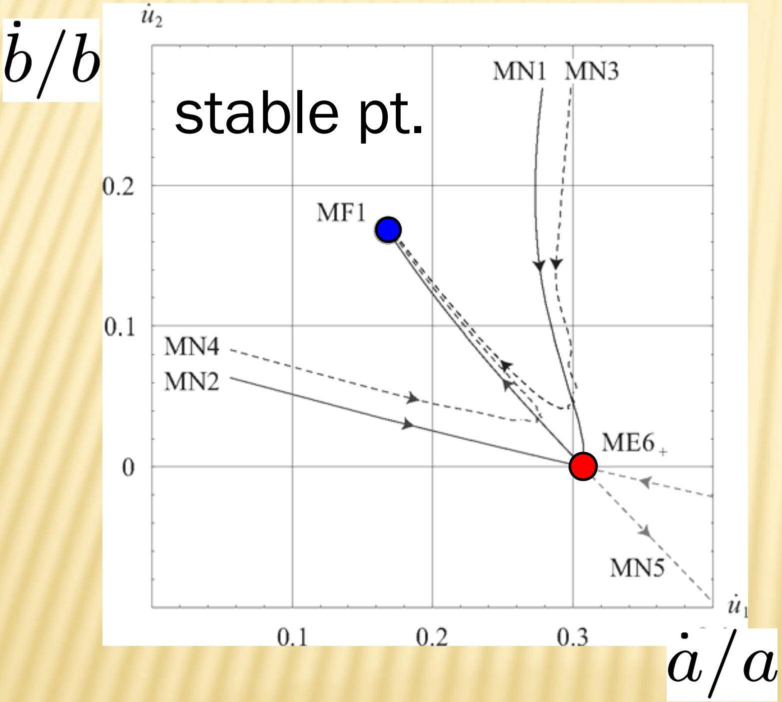
$$T_2 = \left(2\pi^2/\kappa_{11}^2\right)^{1/3}$$ membrane tension

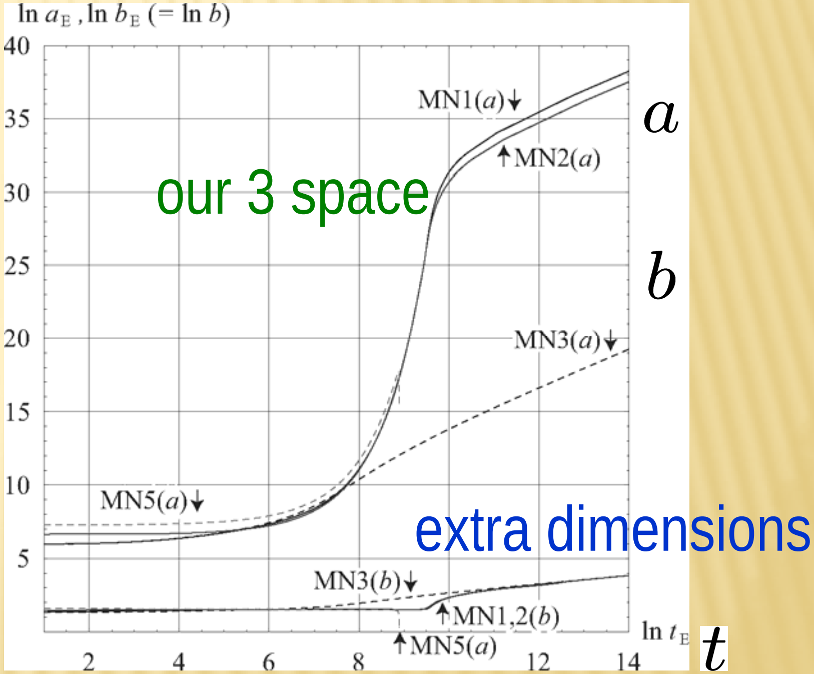
Cosmology with higher curvature

KM, N. Ohta, PLB (04), PRD (05)
K. Akune, KM, N. Ohta, PRD (06)

$$ds^2 = -dt^2 + a^2 \sum_{i=1}^{3} \left(dx^i\right)^2 + b^2 \sum_{\alpha=5}^{11} (dy^\alpha)^2$$

de Sitter : transient attractor





inflationary phase

Heterotic type

Einstein-Gauss-Bonnet + dilaton K. Bamba, Z.-K. Guo, N. Ohta (07)

KK type inflation (pole inflation) : attractor

A singularity appears at a finite time

---

common problems

- graceful exit ?
- reheating ?
- density perturbations ?

Importance of analysis in string frame

String frame

$$S = \frac{1}{2\kappa_D^2} \int d^D x \sqrt{-\hat{g}} e^{-2\phi} \left[R(\hat{g}) + 4(\hat{\nabla}\phi)^2 + \alpha_2 R^2_{GB}(\hat{g}) \right]$$

$$\hat{g}_{\mu\nu} = e^{\frac{4\phi}{D-2}} g_{\mu\nu}$$

Einstein frame

$$S = \frac{1}{2\kappa_D^2} \int d^D x \sqrt{-g} \left[R(g) - \frac{1}{2}(\nabla\phi)^2 + \alpha_2 e^{-\gamma\phi} R^2_{GB}(g) + \mathcal{F}(\nabla\phi, R) \right]$$

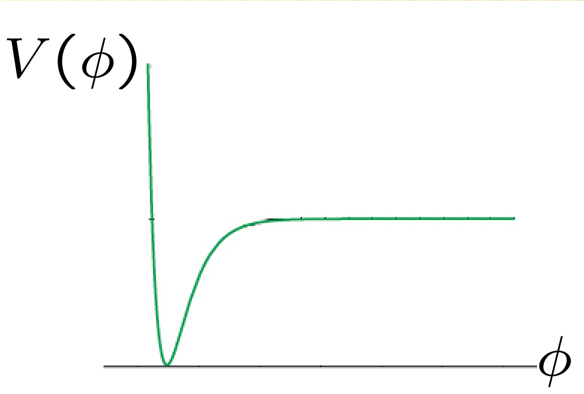
- usually ignored
- could be important

Conformal transformation is convenient

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} R + \alpha R^2 \right]$$

$$\hat{g}_{\mu\nu} = e^{\sqrt{2/3}\kappa\phi} g_{\mu\nu} \qquad \kappa\phi = \sqrt{\frac{3}{2}} \ln(1 + 4\kappa^2 \alpha R)$$

$$S = \int d^4x \sqrt{-\hat{g}} \left[\frac{1}{2\kappa^2} \hat{R} - \frac{1}{2} (\hat{\nabla}\phi)^2 - V(\phi) \right]$$



Be careful !

Attractor Universe in Scalar-Tensor Theory

KM, Y. Fujii: hep-th/0902.1221

MODEL

$$S_J = \int d^4x\sqrt{-g}\left[\frac{\xi}{2}\phi^2 R(g) - \frac{\epsilon}{2}(\nabla\phi)^2 - V_0\right] + \int d^4x\sqrt{-g}L_{\rm m}(\psi, g)$$

Non-minimal coupling (ξ) + cosmological constant (V_0)

BD parameter $\omega = 1/4\xi$

conformal transformation

$g \rightarrow g\exp(2\zeta\kappa\sigma)$ $\quad \zeta = \sqrt{\xi/(\varepsilon+6\xi)}$

String theory $\zeta = \sqrt{1/2}$

Einstein theory (g) + scalar field σ $\quad V = V_0 \exp(-4\zeta\kappa\sigma)$

Dynamics without matter is well-known

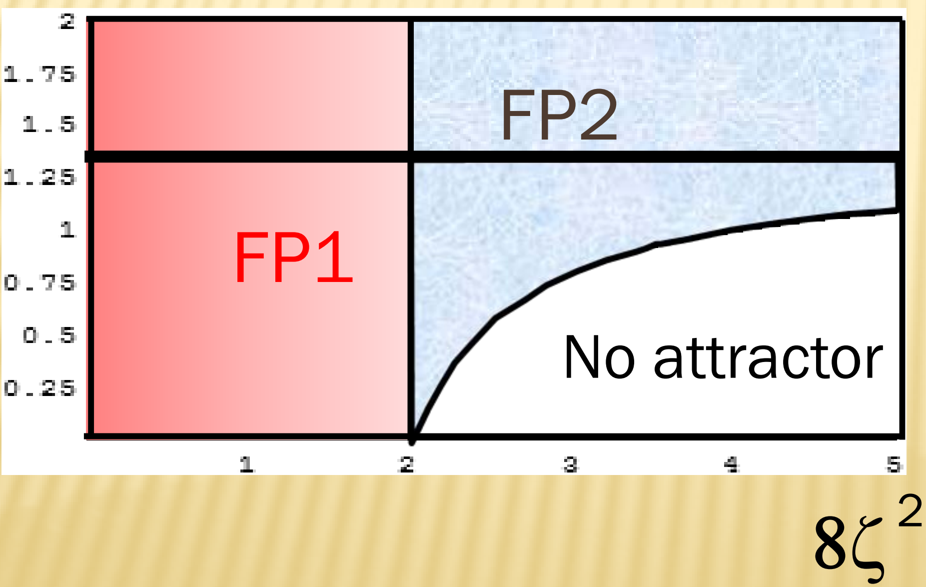
But, coupling with matter is important

$$H^2 + \frac{k}{a^2} = \frac{\kappa^2}{3}\left[\frac{1}{2}\left(\frac{d\sigma}{dt}\right)^2 + V + \rho\right]$$

$$\frac{d^2\sigma}{dt^2} + 3H\frac{d\sigma}{dt} + \frac{\partial V}{\partial \sigma} = \zeta\kappa(\rho - 3P)$$

$$\frac{d\rho}{dt} + 3\gamma H\rho = -\zeta\kappa(4-3\gamma)\frac{d\sigma}{dt}\rho$$

$$P = (\gamma - 1)\rho$$



Two fixed points

FP1 Scalar field dominant

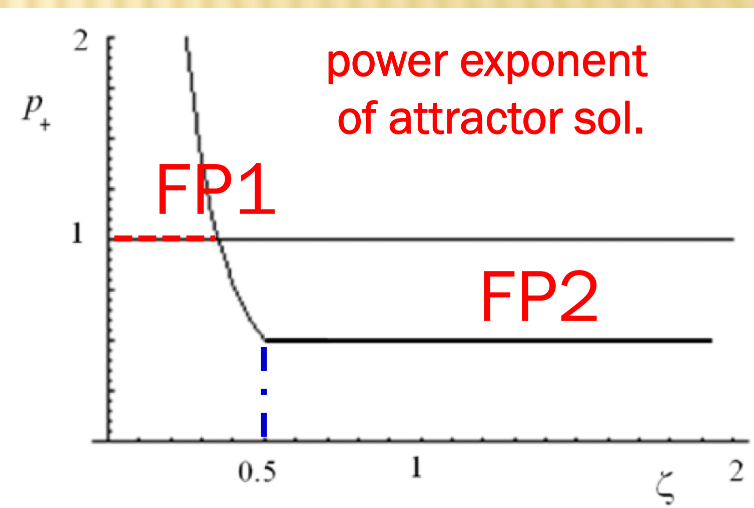
$$a \propto t^{\frac{1}{8\zeta^2}} \qquad \kappa\sigma = \frac{1}{2\zeta}\ln t + \text{const}$$

FP2 Scaling solution

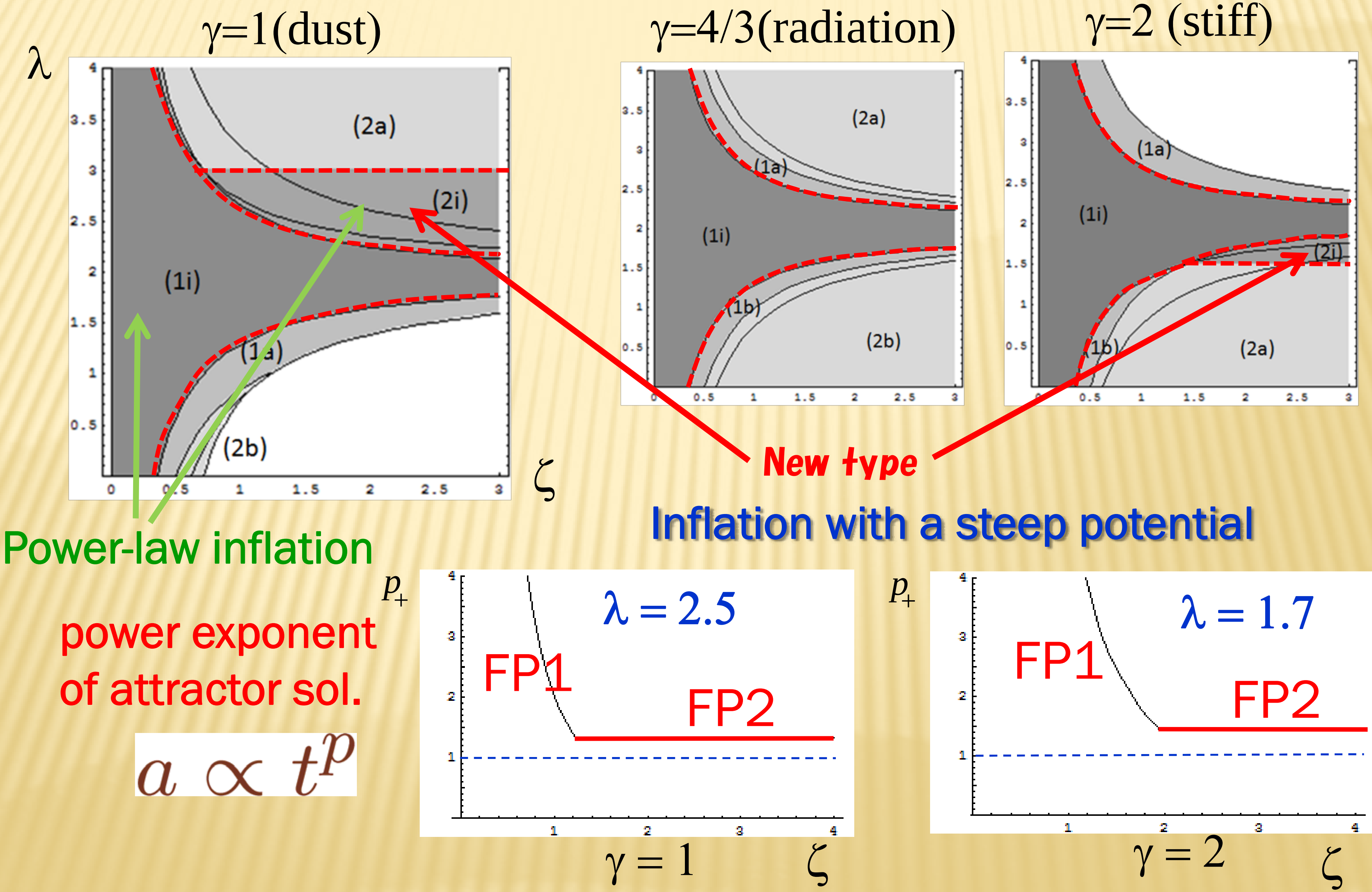
$$\left(\frac{\rho}{V}\right)_2 = \frac{2(4\zeta^2-1)}{2-\gamma-2(4-3\gamma)\zeta^2} \quad \text{const}$$

$$a \propto t^{\frac{1}{2}} \qquad \kappa\sigma = \frac{1}{2\zeta}\ln t + \text{const}$$

Minkowski in Jordan frame

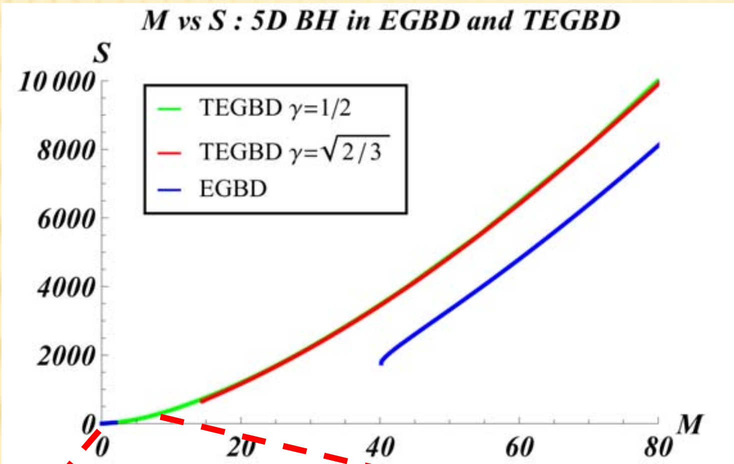


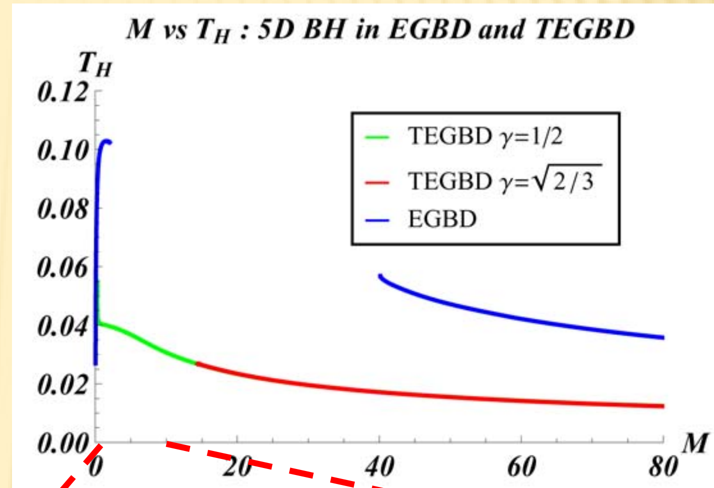
power-law potential $V_0 \to (\kappa^2\phi^2)^\lambda V_0$

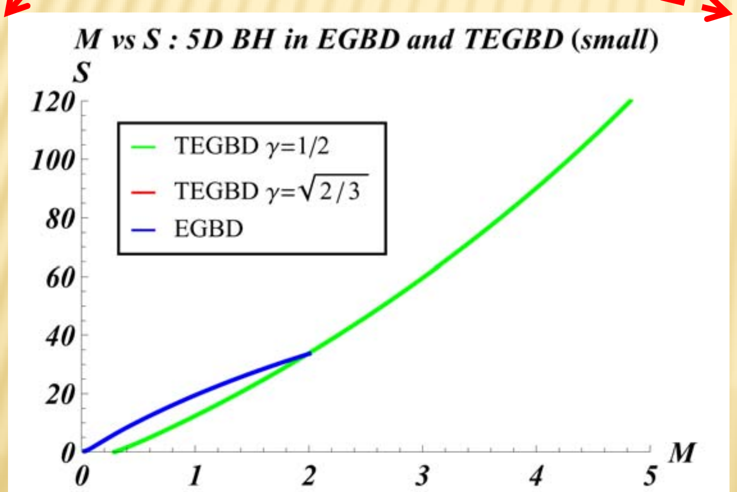


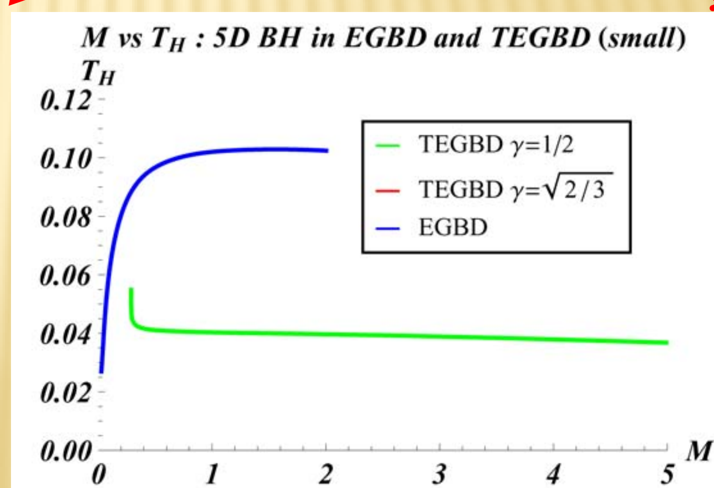
DILATONIC EINSTEIN GAUSS-BONNET BH

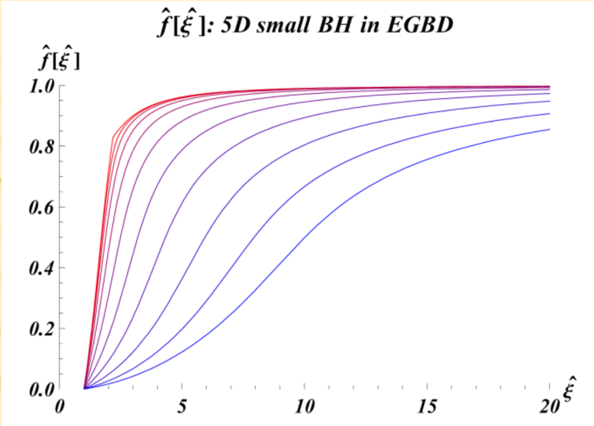
KM, N.Ohta, Y. Sasagawa

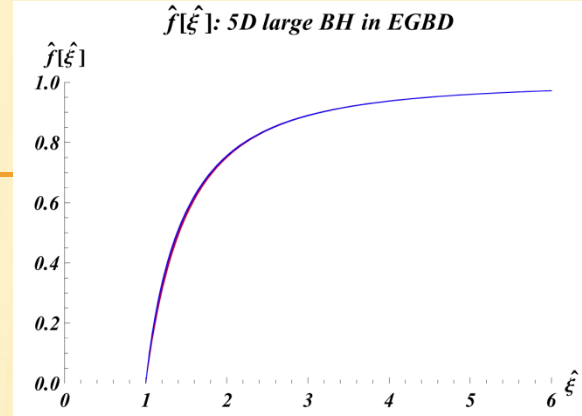


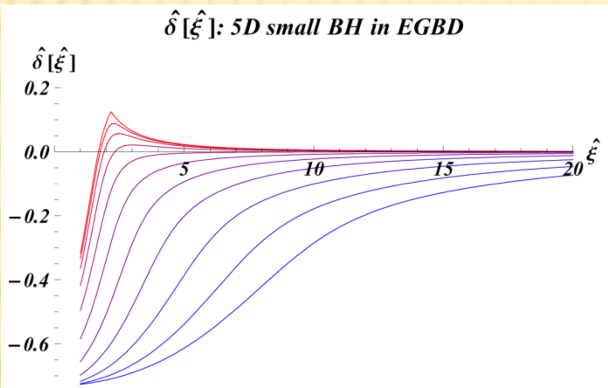


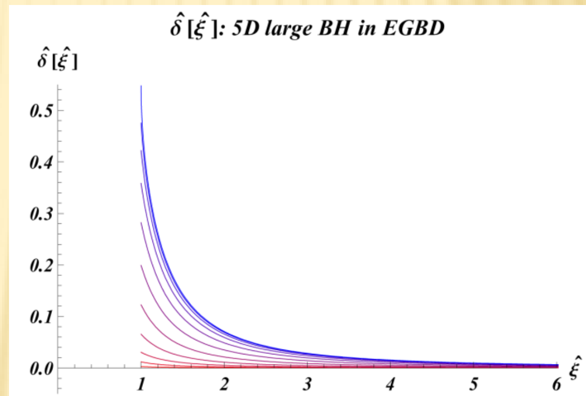


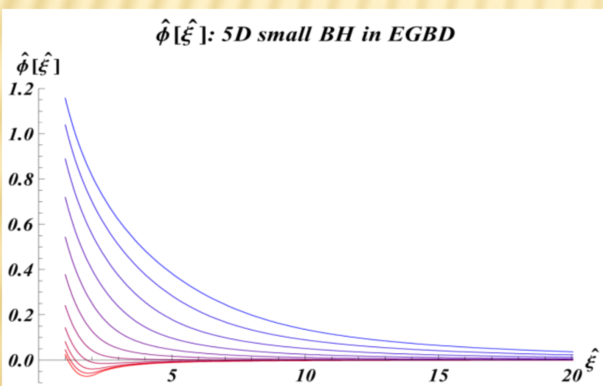


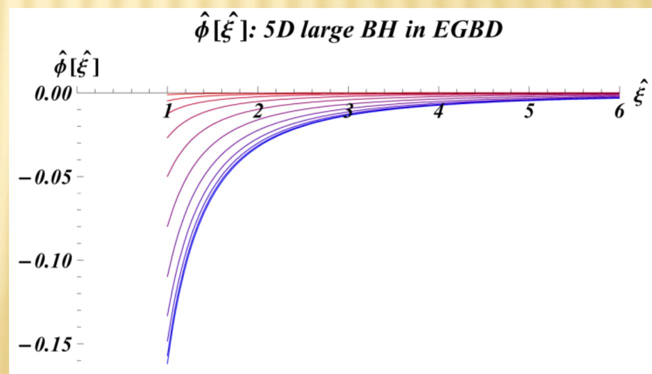












$$ds_D^2 = -fe^{-2\delta}dt^2 + f^{-1}dr^2 + r^2 d\Omega_{D-2}^2$$

SUMMARY

- We study a time-dependent spacetime with intersecting branes in M/superstring theory
- We study cosmology with higher curvature corrections
- We give a warning of study in the Einstein frame