ANISOTROPIC FRW UNIVERSE FROM NONLINEAR MASSIVE GRAVITY

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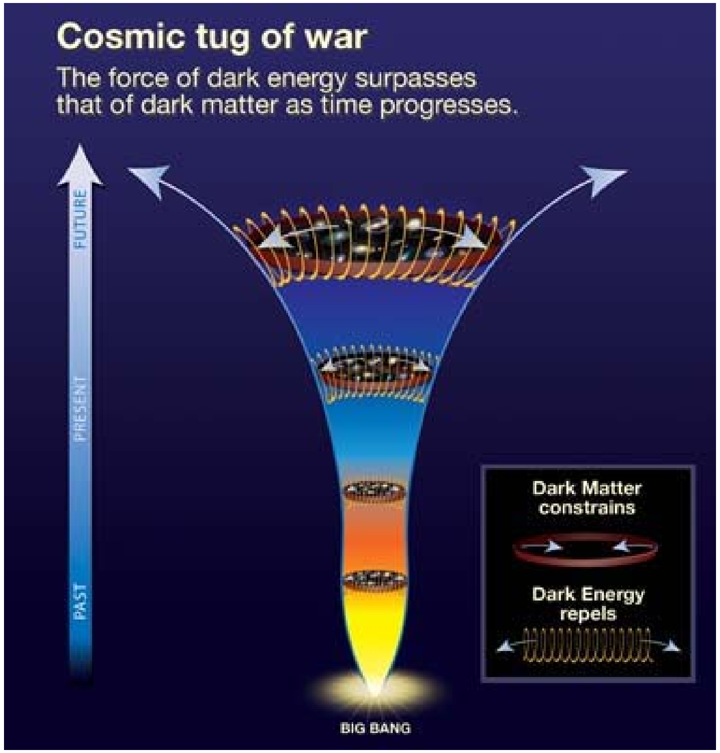
INTRODUCTION



Cosmic acceleration

INTRODUCTION

- Can we give graviton a mass?
  - Fierz and Pauli 1939

$$\mathcal{L}_{FP} = f^4 \left(h_{\mu\nu}h_{\mu\nu} - h^2\right)$$

van Dam–Veltman–Zakharov discontinuity

$$T^\mu_\nu h^\nu_\mu = T^\mu_\nu(\hat{h}^\nu_\mu + m_g^2\delta^\nu_\mu\phi) = T^\mu_\nu\hat{h}^\nu_\mu + \frac{1}{M_{\rm Pl}}T\phi^c$$

  - Vainshtein 1972 non-linear interactions
  - Boulware–Deser (BD) ghost 1972

Lack of Hamiltonian constrain and momentum constrain

↓

6 degrees of freedom

Helicity ±2, ±1, 0 ⟶ 5 dof ?

6th dof is BD ghost!

INTRODUCTION

- Whether there exist a nonlinear model without ghost?
  - N. Arkani–Hamed et al 2002
  - P. Creminelli et al., ghost free up 4th order, 2005
  - C. de Rham and G. Gabadadze 2010

$$\mathcal{L} = M_{\rm Pl}^2\sqrt{-g}R - \frac{M_{\rm Pl}^2 m^2}{4}\sqrt{-g}(U_2(g,H) + U_3(g,H) + U_4(g,H) + U_5(g,H)\cdots),$$

where U_i denotes the interaction term at ith order in $H_{\mu\nu}$,

$$U_2(g,H) = H_{\mu\nu}^2 - H^2,$$

$$U_3(g,H) = c_1 H_{\mu\nu}^3 + c_2 H H_{\mu\nu}^2 + c_3 H^3,$$

$$U_4(g,H) = d_1 H_{\mu\nu}^4 + d_2 H H_{\mu\nu}^3 + d_3 H_{\mu\nu}^2 H_{\alpha\beta}^2 + d_4 H^2 H_{\mu\nu}^2 + d_5 H^4,$$

$$U_5(g,H) = f_1 H_{\mu\nu}^5 + f_2 H H_{\mu\nu}^4 + f_3 H^2 H_{\mu\nu}^3 + f_4 H_{\alpha\beta}^2 H_{\mu\nu}^3 + f_5 H (H_{\mu\nu}^2)^2 + f_6 H^3 H_{\mu\nu}^2 + f_7 H^5.$$

$$g_{\mu\nu} = \eta_{\mu\nu} + \frac{h_{\mu\nu}}{M_{\rm Pl}} = H_{\mu\nu} + \eta_{ab}\partial_\mu\phi^a\partial_\nu\phi^b,$$

$$H_{\mu\nu} = \frac{h_{\mu\nu}}{M_{\rm Pl}} + \partial_\mu\pi_\nu + \partial_\nu\pi_\mu - \eta_{\alpha\beta}\partial_\mu\pi^\alpha\partial_\nu\pi^\beta.$$

INTRODUCTION

- C. de Rham, G. Gabadadze and A. Tolly 2011

$$I_g = M_{Pl}^2 \int d^4x \sqrt{-g} \left[\frac{R}{2} + m_g^2 (\mathcal{L}_2 + \alpha_3 \mathcal{L}_3 + \alpha_4 \mathcal{L}_4) \right]$$

$$\mathcal{L}_2 = \frac{1}{2} \left([\mathcal{K}]^2 - [\mathcal{K}^2] \right) ,$$

$$\mathcal{L}_3 = \frac{1}{6} \left([\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3] \right) ,$$

$$\mathcal{L}_4 = \frac{1}{24} \left([\mathcal{K}]^4 - 6[\mathcal{K}]^2[\mathcal{K}^2] + 3[\mathcal{K}^2]^2 + 8[\mathcal{K}][\mathcal{K}^3] - 6[\mathcal{K}^4] \right) ,$$

$$\mathcal{K}^{\mu}_{\nu}(g, H) = \delta^{\mu}_{\nu} - \sqrt{\delta^{\mu}_{\nu} - H^{\mu}_{\nu}} = -\sum_{n=1}^{\infty} \bar{d}_n (H^n)^{\mu}_{\nu}, \qquad \bar{d}_n = \frac{(2n)!}{(1-2n)(n!)^2 4^n}.$$

Automatically produce the "appropriate coefficients" to eliminate BD ghost!

Stukelberg fields

$$\langle f^{\mu}_{\mu} \rangle \neq 0$$

Source of MASS

$$\mathcal{K}^{\mu}_{\nu} = \delta^{\mu}_{\nu} - \left(\sqrt{g^{-1} f} \right)^{\mu}_{\nu}$$

$$f_{\mu\nu} \equiv \eta_{ab} \partial_{\mu} \phi^a \partial_{\nu} \phi^b$$

It is often called fiducial metric

INTRODUCTION

- No go result for flat FRW solution with Minkowski fiducial metric (G. D'Amico et al 2011 Aug.)
- It does not extend to open FRW universe

(E.Gumrukcuoglu, C. Lin, S. Mukohyama: 1109.3845)

$$3H^2 - \frac{3|K|}{a^2} = \rho_m + c_\pm m_g^2,$$

- Vanishing kinetic terms in scalar and vector sector

Tensor modes receive a modification

(E.Gumrukcuoglu, C. Lin, S. Mukohyama: 1111.4107)

- GHOST found

A. De Felice, E. Gumrukcuoglu, S. Mukohyama:1206.2080

vanishing of the kinetic terms is the consequence of the FRW symmetry

1) break FRW symmetry 2) turn to some extended theory

SETUP

- Action

$$I = \frac{M_p^2}{2}\int d^4x\sqrt{-g}[R-2\,\Lambda+m_g^2(\mathcal{L}_2+\alpha_3\mathcal{L}_3+\alpha_4\mathcal{L}_4)]$$

Consider the simplest anisotropic extension to FRW ansatz

$$g^{(0)}_{\mu\nu}dx^\mu dx^\nu = -N^2dt^2 + a^2[e^{4\sigma}dx^2 + e^{-2\sigma}\delta_{ij}dy^i dy^j],$$

As for the fiducial metric

$$f_{\mu\nu} = -n^2\partial_\mu\phi^0\partial_\nu\phi^0 + \alpha^2(\partial_\mu\phi^1\,\partial_\nu\phi^1 + \delta_{ij}\partial_\mu\phi^i\partial_\nu\phi^j),$$

$$H_f \equiv \dot{\alpha}/\alpha n = \text{constant}.$$

Varying the stuckelburg scalars $\phi^a = x^a + \pi^a$

$$I = I^{(0)} + M_{Pl}^2 m_g^2\int d^4x N a^3 n\pi^0\mathcal{E}_\phi + O[(\pi^a)^2],$$

SETUP

EoM of Stuckelburg scalar

$$\mathcal{E}_\phi \equiv J_\phi^{(x)} \left(H + 2\,\Sigma - H_f\, e^{-2\,\sigma}\, X\right) + 2\, J_\phi^{(y)} \left(H - \Sigma - H_f\, e^{\sigma}\, X\right) = 0\,,$$

Where

$$J_\phi^{(x)} \equiv \gamma_1 - 2\,\gamma_2\, e^{\sigma}\, X + \gamma_3\, e^{2\sigma}\, X^2\,,$$
$$J_\phi^{(y)} \equiv \gamma_1 - \gamma_2\,(e^{-2\sigma} + e^{\sigma})\, X + \gamma_3\, e^{-\sigma}\, X^2\,,$$

$$\gamma_1 \equiv 3 + 3\,\alpha_3 + \alpha_4,\ \gamma_2 \equiv 1 + 2\,\alpha_3 + \alpha_4,\ \gamma_3 \equiv \alpha_3 + \alpha_4,$$

$$H \equiv \frac{\dot{a}}{a\,N},\ \Sigma \equiv \frac{\dot{\sigma}}{N} \text{ and } X \equiv \frac{\alpha}{a}.$$

- H_f ⟶ invariants of fields metric, is independent of the background value of ϕ^a
- Algebraic equation for X, instead of a differential equation.

SETUP

- Einstein equations

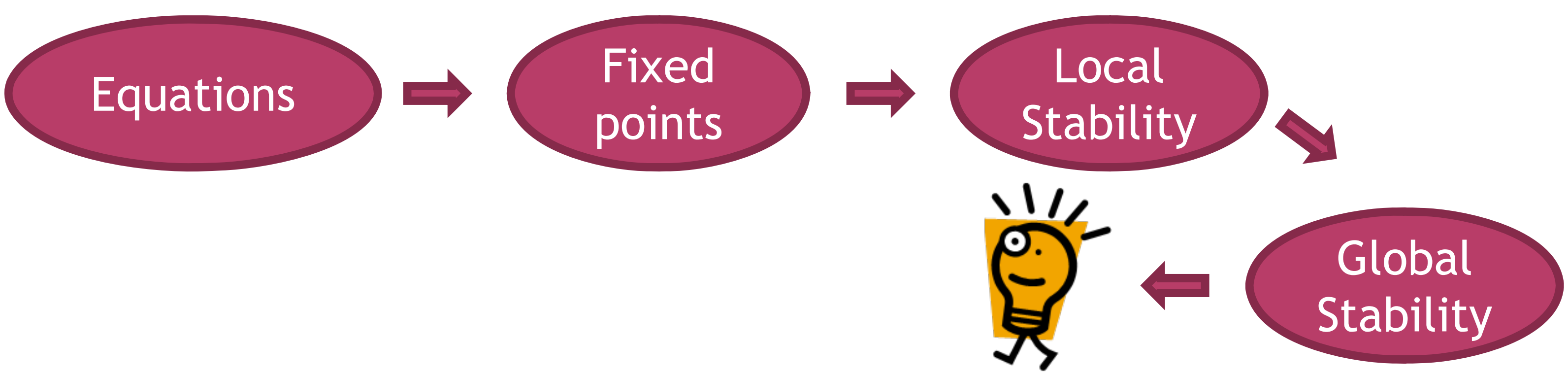
$$3\,(H^2-\Sigma^2)-\Lambda = m_g^2\left[-(3\,\gamma_1-3\,\gamma_2+\gamma_3)+\gamma_1\,(2\,e^\sigma+e^{-2\sigma})X-\gamma_2(e^{2\sigma}+2\,e^{-\sigma})\,X^2+\gamma_3\,X^3\right],$$

$$\frac{\dot\Sigma}{N}+3H\Sigma=\frac{m_g^2}{3}(e^{-2\,\sigma}-e^{\sigma})X\left[\gamma_1-\gamma_2(e^\sigma+r)X+\gamma_3\,re^\sigma X^2\right],$$

where $r\equiv \frac{n\,a}{N\,\alpha}=\frac{1}{X\,H_f}\left(\frac{\dot X}{N\,X}+H\right).$

Additionally

The above 2 eqns + EoM of stuckelburg scalars = equation for $\dot H$



FIXED POINTS

- Seek the solutions with $\dot{H} = \Sigma = \dot{X} = 0$

Einstein eqns + EoM for stuckelburg scalars become

$$\begin{aligned} 3\lambda - (3\gamma_1 - 3\gamma_2 + \gamma_3) + \gamma_1(2e^{\sigma} + e^{-2\sigma})X - \left[\gamma_2(2e^{-\sigma} + e^{2\sigma}) + 3\,r^2\mu^{-2}\right]X^2 + \gamma_3 X^3 &= 0, \\ (e^{\sigma} - 1)\left[\gamma_1 - \gamma_2(r + e^{\sigma})X + \gamma_3 e^{\sigma} r X^2\right] &= 0, \\ \gamma_1(3r - 2e^{\sigma} - e^{-2\sigma}) - 2\gamma_2\left[(2e^{\sigma} + e^{-2\sigma})r - (e^{2\sigma} + 2e^{-\sigma})\right]X + \gamma_3\left[(e^{2\sigma} + 2e^{-\sigma})r - 3\right]X^2 &= 0, \end{aligned}$$

where $\lambda \equiv \frac{\Lambda}{3m_g^2}$ and $\mu \equiv \frac{m_g}{H_f}$ are dimensionless parameters

2nd + 3rd eqns

$$(\gamma_1 - \gamma_2 X e^{\sigma})(e^{\sigma} - r)(re^{2\sigma} - 1) = 0$$

There are 3 solutions

$$e^{\sigma} = \left\{ \frac{\gamma_1}{\gamma_2 X}\,,\ r\,,\ r^{-1/2} \right\}$$

FIXED POINTS

$$e^{\sigma} = \left\{ \frac{\gamma_1}{\gamma_2 X}\, , \; r\, , \; r^{-1/2} \right\}$$

Case I
It is isotropic

Case II
Requires fine tuning parameters

Case III
Non-trivial anisotropic solution

Substitute into 1st and 2nd eqns

$$(3\lambda - 3\gamma_1 + 3\gamma_2 - \gamma_3) + \gamma_1(e^{-2\sigma} + 2e^{\sigma})X$$
$$-[\gamma_2(2e^{-\sigma} + e^{2\sigma}) + 3\, e^{-4\sigma}\, \mu^{-2}]X^2 + \gamma_3 X^3 = 0$$
$$\gamma_1 e^{\sigma} - \gamma_2(e^{2\sigma} + e^{-\sigma})X + \gamma_3 X^2 = 0$$

c_0, c_1, c_2, c_3 are functions of $\gamma_{1,2,3}$ λ and μ.

Given $(\alpha_3, \alpha_4, \lambda, \mu)$:

$e^{3\sigma}$ ← $$c_0 + c_1 e^{3\sigma} + c_2 e^{6\sigma} + c_3 e^{9\sigma} = 0,$$

X ← $$X = \frac{3\gamma_1 + [\gamma_1\gamma_2 - \gamma_3^2 + 3\gamma_3(\gamma_2 - \gamma_1 + \lambda)]\mu^2 e^{3\sigma}}{(e^{\sigma} + e^{-2\sigma})\,[3\gamma_2 + (\gamma_2^2 - \gamma_1\gamma_3)]\mu^2 e^{3\sigma}}$$

LOCAL STABILITY

- The homogeneous perturbations around fixed pointes

$$H = H_f[r_0 X_0 + \epsilon\, h_1(t) + O(\epsilon^2)],$$
$$\sigma = \sigma_0 + \epsilon\, \sigma_1(t) + O(\epsilon^2),$$
$$X = X_0 + \epsilon\, X_1(t) + O(\epsilon^2),$$

At linear order, EoM for σ_1

$$\frac{d^2\sigma_1}{d\tau^2} + 3X_0 e^{-2\sigma}\frac{d\sigma_1}{d\tau} + M^2\sigma_1 = 0$$

> 0

$$d\tau = H_f N dt$$
$$M^2 = \frac{X_0^2\mu^2 e^{-4\sigma_0}}{2}\left(\frac{d_1\,(3\,d_1 - d_2)(6 + d_1\,\mu^2)}{2\,d_2 - {d_1}^2\,\mu^2}\right)$$
$$d_1 \equiv (e^{3\,\sigma_0} - 1)\,\left[\gamma_2 - \gamma_3 e^{\sigma_0}\,X_0\right]\,,$$
$$d_2 \equiv (e^{3\,\sigma_0} - 1)\,\left[\gamma_2(3 + 2\,e^{3\,\sigma_0}) - 5\,\gamma_3 e^{\sigma_0}\,X_0\right]$$

$M^2 > 0$

Local Stability Condition!

GLOBAL STABILITY

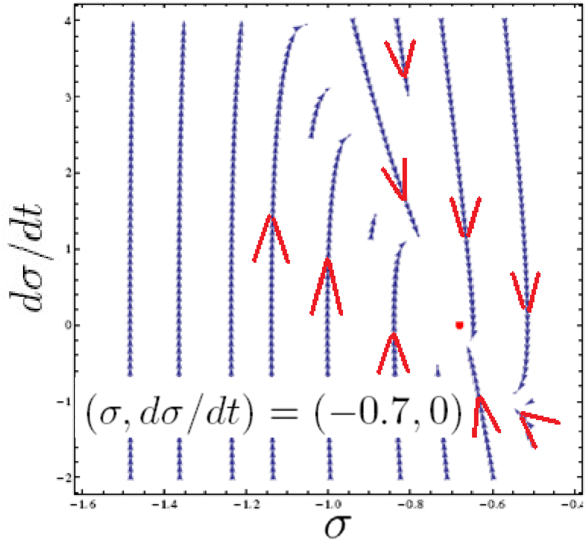
- We consider an example with

$$\lambda = 0\,, \quad \mu = 20\,, \quad \alpha_3 = -1/20\,, \quad \alpha_4 = 1$$

For which local stability condition is satisfied.

There is only one set of solution to 3 eqns of fixed point:

$$X \simeq 4\,, \quad e^{\sigma} \simeq \tfrac{1}{2}\,, \quad r \simeq 4$$



Remarks:

- Anisotropy differs from GR;
- Ghost free around fixed point;
- Coordinates redefinition

$$\frac{y}{x} = const \longrightarrow 1 = \text{isotropic universe}$$

- Anisotropy $\longrightarrow$ cosmological perturbation spectrum

CONCLUSION

- Graviton mass greatly changes the behaviors of anisotropy;
- We find an attractor solution, we call it anisotropic FRW universe
  - Ghost free
  - Identical to isotropic universe at background level
  - Anisotropy is shifted to cosmological backgroud

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THANK YOU!