A Uniqueness Theorem for Dipole Rings

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Plan

- *Introduction*
- *Uniqueness for D=4, 5 black holes*
- *Proof of uniqueness*
  - *Non-linear σ-model in D=5 SUGRA* *(by Bouchareb-Clement,-Chen-Gal'tsov-Scherbluk-Wolf,)*
  - *Boundary conditions & sketch of proof*
  - *Theorem*

Einstein-Maxwell-Chern-Simons Theory

- *Action*

$$S = \frac{1}{16\pi G}\left[\int \sqrt{-g}\left(R - \frac{1}{4}F^2\right) d^5x + \frac{\lambda}{3\sqrt{3}}\int F \wedge F \wedge A\right]$$

- *EOM*

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{1}{2}\left(F_{\mu}^{\lambda}F_{\nu\lambda} - \frac{1}{4}F^2 g_{\mu\nu}\right)$$

$$d * F + \frac{\lambda}{\sqrt{3}}F \wedge F = 0$$

Einstein-Maxwell-Chern-Simons black holes

- *Action*

$$S = \frac{1}{16\pi G}\left[\int \sqrt{-g}\left(R - \frac{1}{4}F^2\right) d^5x + \frac{\lambda}{3\sqrt{3}}\int F\wedge F\wedge A\right]$$

  - $\lambda=0$ *[5D Einstein-Maxwell theory]*

    No exact solution has been found

    Numerical solutions (Kunz & Navarro-Lerida & Petersen '05)

  - $\lambda=1$ *[Minimal SUGRA]*

    The only found exact solution (Chong-Cvetic-Lu-Pope '05)

  - $\lambda>1$, $0<\lambda<1$

    No exact solution has been found

    Numerical solutions (Kunz & Navarro-Lerida '06)

    In $\lambda>\lambda_0$, for same (M, $J_1=J_2$, Q), there exist at least two kinds of black holes with S^3 horizon

Dipole rings

- Elvang-Emparan-Figueras (‘ 05)

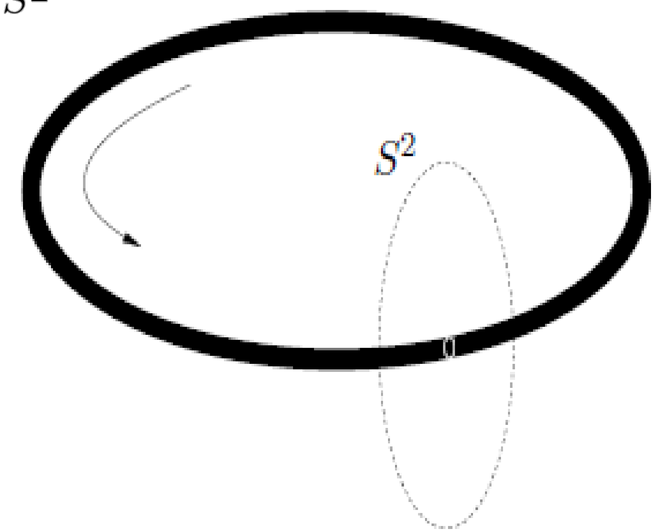
- D=5 Minimal SUGRA
- The solution has its mass, two angular momenta, charge and dipole charge
- $q=J_\varphi/Q \Rightarrow$ not general

$$q := \frac{1}{2\pi}\int_{S^2} F$$

cf)

✓ Emparan (‘ 04)

- 3 form field coupled with scalar field
- Dipole charge $\Rightarrow$ non-unique



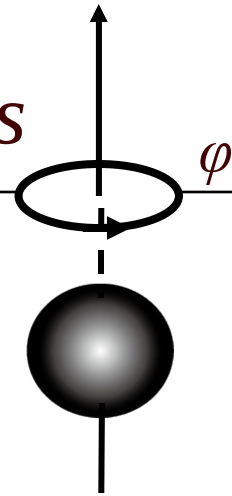
✓ Yazadijev (‘ 06)

- D=5 EM(CS) theory

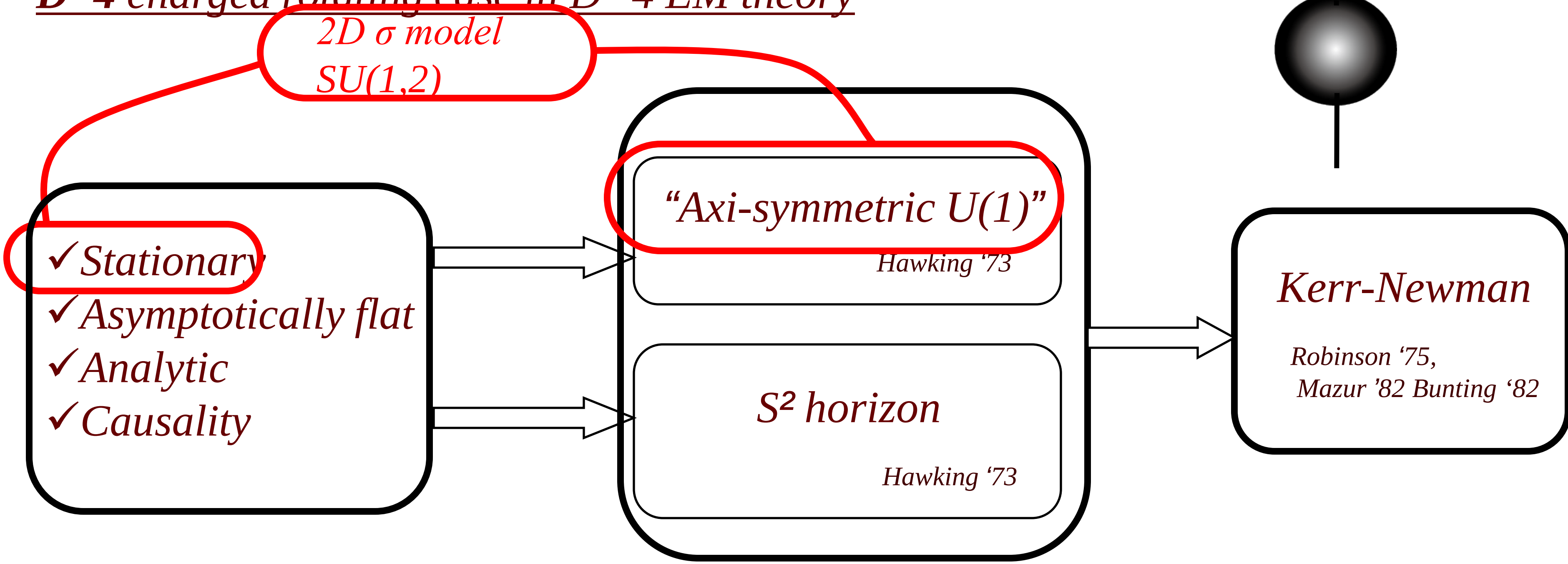
- *unique, or not ?*
- *If not, what is its origin ?*

Uniqueness for D=4, 5 black holes

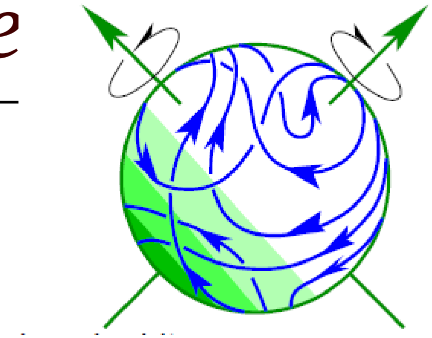
Uniqueness for Kerr-Newman black holes



D=4 *charged rotating case in D=4 EM theory*



Uniqueness for rotating black hole



__D=5 vacuum rotating case__

2D σ model
SL(3,R)

✓Stationary
✓Asymptotically flat
✓Analytic
✓Causality

Axi-symmetric
U(1) × U(1)

Hollands-Ishibashi-Wald ('04)

✓S^3 horizon

✓$S^1 \times S^2$ horizon
✓L(p,q) horizon

Cai-Galloway '01
Gallowy-Schen '06
Heflgotto-Oz-Yanay '06

Myers-Perry

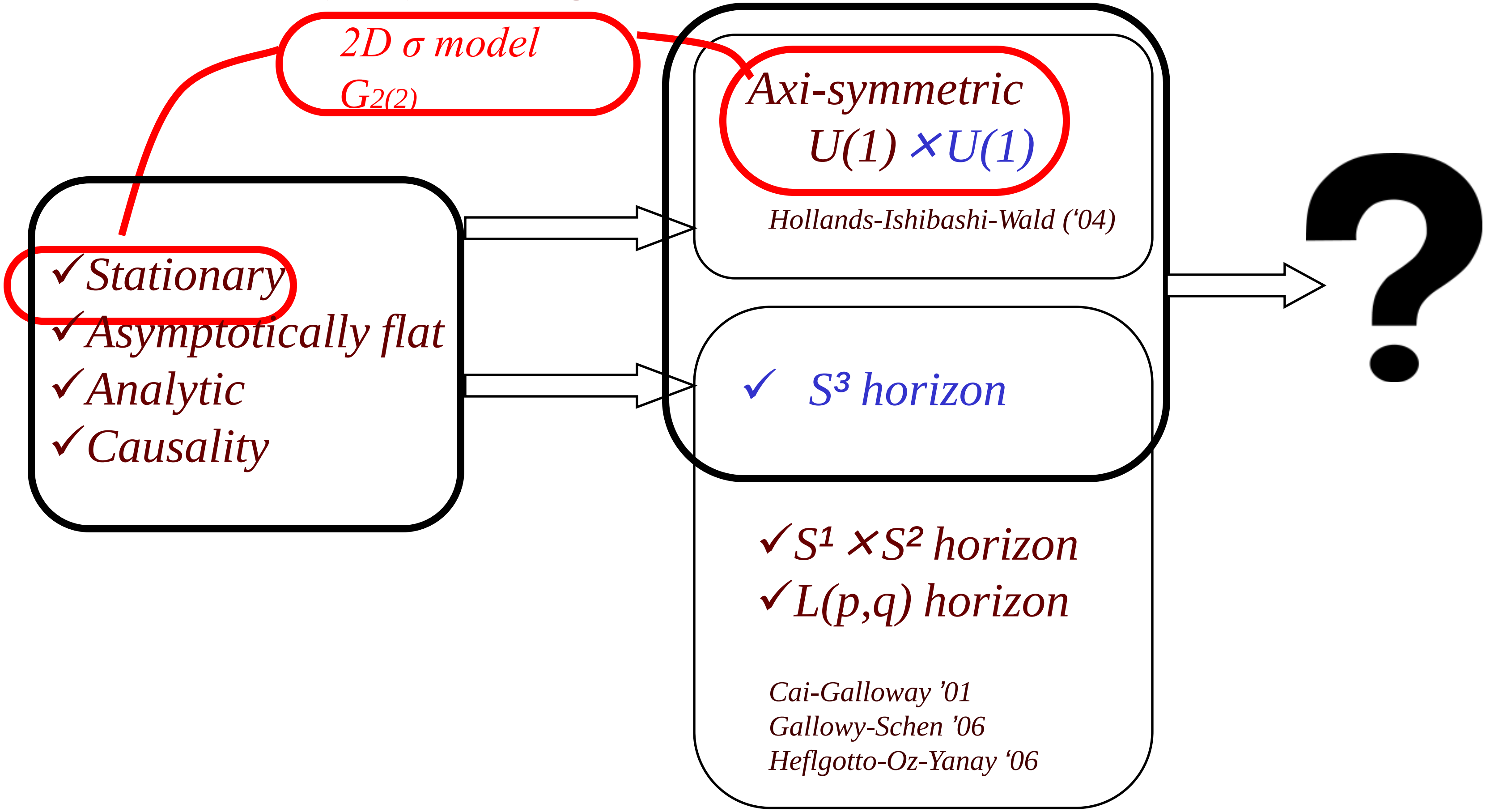
Morisawa-Ida ('04)

Pomeransky-Sen'kov

Hollands-Yadzajev'08
Morisawa-Tomizawa-Yasui '08

Uniqueness for rotating black holes

D=5 SUGRA (λ=1) rotating case



Proof of uniqueness theorem

-Rotaing case-

Basis idea in rotating case

Non-linear σ-model approach

- *Under a certain symmetry assumptions, theory can be reduced to 2D non-linear σ-model*
- *Consider as "Boundary value problem" of scalar fields*

- *D=4*
  - *Kerr (Robinson '74)*
  - *Kerr-Newman (Mazur '82; Bunting '82)*
- *D=5*
  - *Myers-Perry (Morisawa-Ida '04)*
  - *Pomerasky-Sen'kov (Morisawa-Tomizawa-Yasui '08; Hollands-Yazajiev '08)*

Basis idea in rotating case

- *Non-linear σ-model approach*
  - *Under a certain symmetry assumptions, theory can be reduced to 2D non-linear σ-model*
  - *Consider as "Boundary value problem" of scalar fields*

theory	*target space*
D=4 Einstein	*SU(1,1)*
D=4 Einstein-Maxwell	*SU(1,2)*
D=5 Einstein	*SL(3,R)*
D=5 Minimal SUGRA	$G2_{(2)}$

2D σ-model in SUGRA

2-Killing system in D=5 Einstein gravity (Maison '79)

Assume existence of 2 commuting Killing vectors

$$\xi_\phi = \frac{\partial}{\partial \phi}, \ \xi_\psi = \frac{\partial}{\partial \psi}$$

- *Inner products* $(\lambda_{\phi\phi}, \lambda_{\psi\psi}, \lambda_{\phi\psi})$

$$\lambda_{\phi\phi} = g(\xi_\phi, \xi_\phi) \ \ \lambda_{\psi\psi} = g(\xi_\psi, \xi_\psi) \ \ \lambda_{\phi\psi} = g(\xi_\phi, \xi_\psi)$$

- *Twist potentials* $(\omega_\phi, \omega_\psi)$

$$d\omega_\phi = *(\xi_\phi \wedge \xi_\psi \wedge d\xi_\phi)$$

$$d\omega_\psi = *(\xi_\phi \wedge \xi_\psi \wedge d\xi_\psi)$$

2-Killing system in D=5 Minimal SUGRA (Bouchareb-Clement-Chen-Gal'tsov-Scherbluk-Wolf '07)

Assume existence of 2 commuting Killing vectors

$$\xi_\phi = \frac{\partial}{\partial \phi}, \ \xi_\psi = \frac{\partial}{\partial \psi}$$

- *Inner products* $(\lambda_{\phi\phi}, \lambda_{\psi\psi}, \lambda_{\phi\psi})$

$$\lambda_{\phi\phi} = g(\xi_\phi, \xi_\phi) \ \ \lambda_{\psi\psi} = g(\xi_\psi, \xi_\psi) \ \ \ \lambda_{\phi\psi} = g(\xi_\phi, \xi_\psi)$$

- *Twist potentials* $(\omega_\phi, \omega_\psi)$

$$d\omega_\phi = *(\xi_\phi \wedge \xi_\psi \wedge d\xi_\phi) - \psi_\phi (3d\mu + \psi_\phi d\psi_\psi - \psi_\psi d\psi_\phi)$$

$$d\omega_\psi = *(\xi_\phi \wedge \xi_\psi \wedge d\xi_\psi) - \psi_\psi (3d\mu + \psi_\phi d\psi_\psi - \psi_\psi d\psi_\phi)$$

- *Electromagnetic potentials* $(\psi_\phi, \psi_\psi, \mu)$

$$d\psi_\phi = -i_{\xi_\phi} F \quad \ d\psi_\psi = -i_{\xi_\psi} F$$

$$d\mu = *(\xi_\phi \wedge \xi_\psi \wedge F) + \psi_\phi d\psi_\psi - \psi_\psi d\psi_\phi$$

3-Killing system in 5D EMCS

- *Assume 3rd Killing vector*

$$\xi_3 = \frac{\partial}{\partial t}$$

- *Metric can be written in Weyl-Papapetrou form*

$$\begin{aligned} ds^2 &= \lambda_{\phi\phi}(d\phi + a^{\phi}{}_t dt)^2 + 2\lambda_{\phi\psi}(\phi + a^{\phi}{}_t dt)(d\psi + a^{\psi}{}_t dt) \\ &\quad + \lambda_{\psi\psi}(d\psi + a^{\psi}{}_t dt)^2 + |\tau|^{-1}\left[-\rho^2 dt^2 + e^{2\sigma}(d\rho^2 + dz^2)\right] \end{aligned}$$

determined by $\Phi^A = (\lambda_{ab},\ \omega_a,\ \psi_a,\ \mu)$

- *Gauge potential can be written as*

$$A = A_t dt + \sqrt{3}(\psi_\phi d\phi + \psi_\psi d\psi)$$

determined by $\Phi^A = (\lambda_{ab},\ \omega_a,\ \psi_a,\ \mu)$

Non-linear σ-model action

- *EOMs of the scalar fields are derived from G2 invariant σ-model action:*

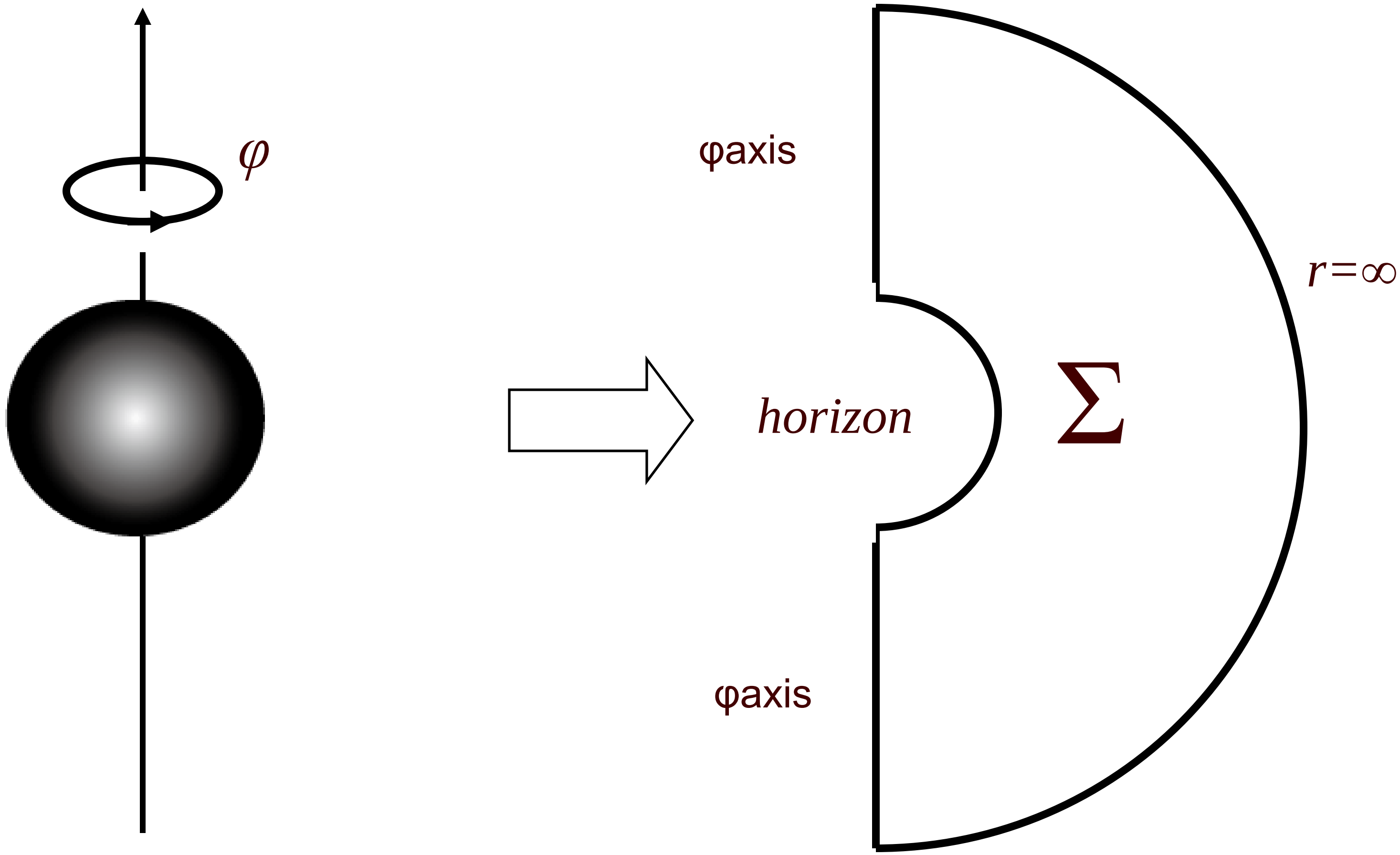
  (Bouchareb-Clement-Chen-Gal'tsov-Scherbluk-Wolf '07)

$$
\begin{aligned}
S &= \int d\rho dz \rho \Big[\frac{1}{4}\mathrm{Tr}(\lambda^{-1}\partial\lambda\lambda^{-1}\partial\lambda) + \frac{1}{4}\tau^{-2}\partial\tau^{2} + \frac{3}{2}\partial\psi^{T}\lambda^{-1}\partial\psi \\
&\quad -\frac{1}{2}\tau^{-1}v^{T}\lambda^{-1}v - \frac{3}{2}(\partial\mu + \epsilon^{ab}\psi_a\partial\psi_b)^2\Big] \\
&=: \int d\rho dz \rho \; G_{AB}\Phi^{A}_{,i}\Phi^{B,i}
\end{aligned}
$$

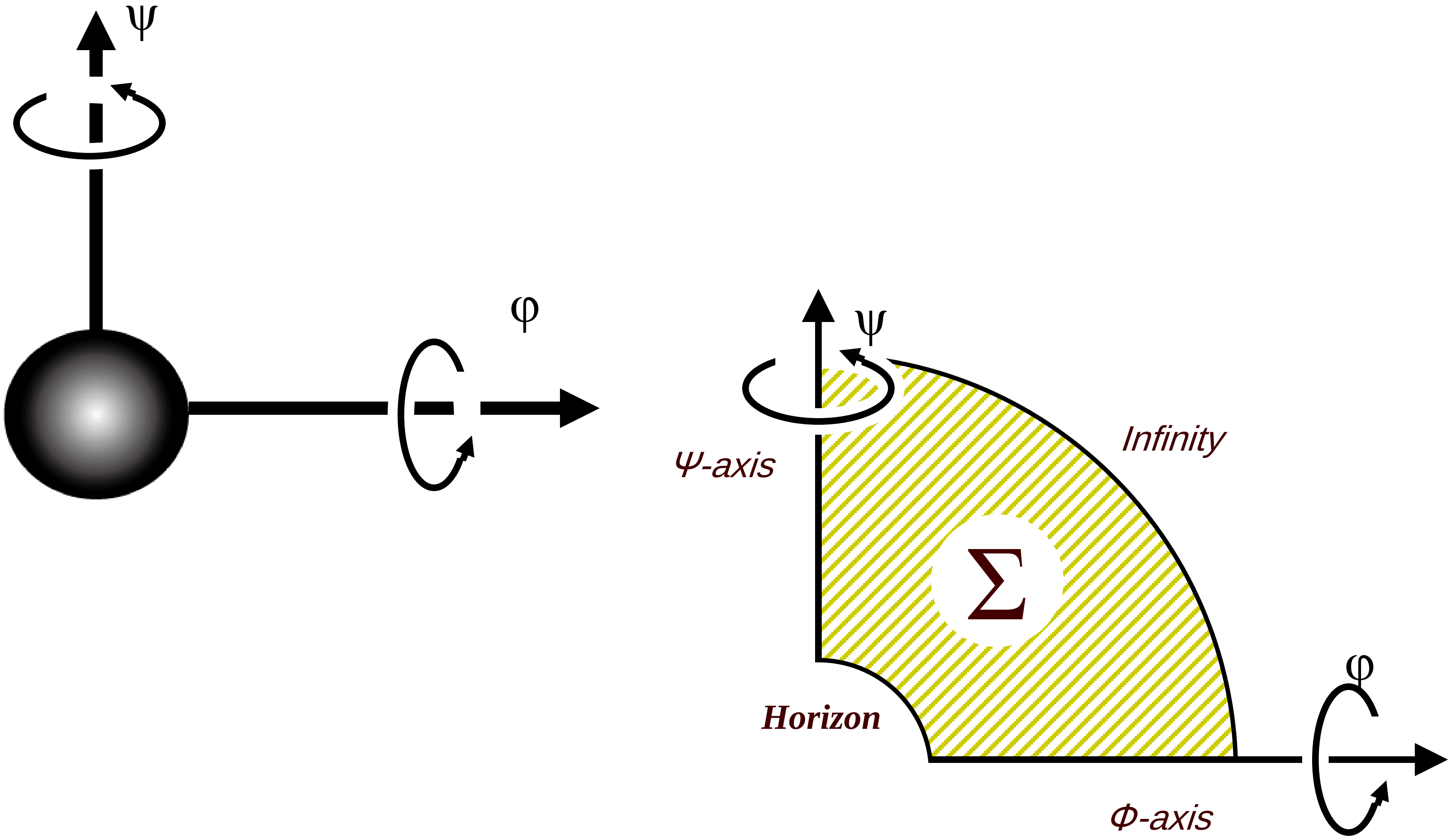
- *Base space: 2D region* Σ={(ρ,z)|ρ≧0}
- *Target space*: $\Phi^A = (\lambda_{\phi\phi}, \lambda_{\phi\psi}, \lambda_{\psi\psi}, \omega_\phi, \omega_\psi, \psi_\phi, \psi_\psi, \mu) \in G_{2(2)}/SO(4)$

(Mizoguchi-Ohta '98)

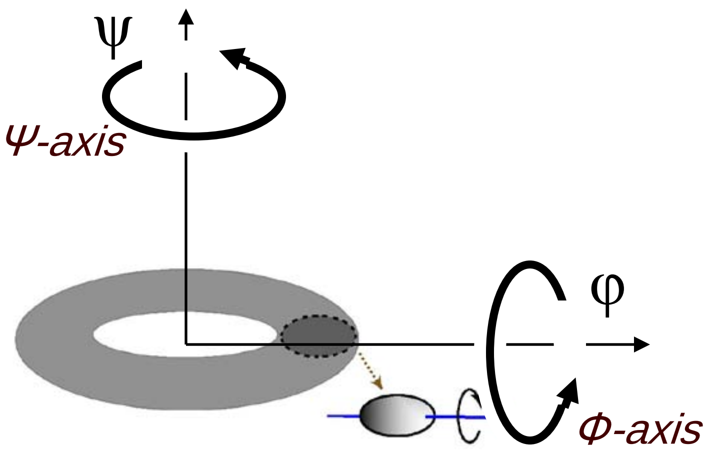
Base space: 2D region $\Sigma=\{(\rho,z)|\rho\geqq 0\}$

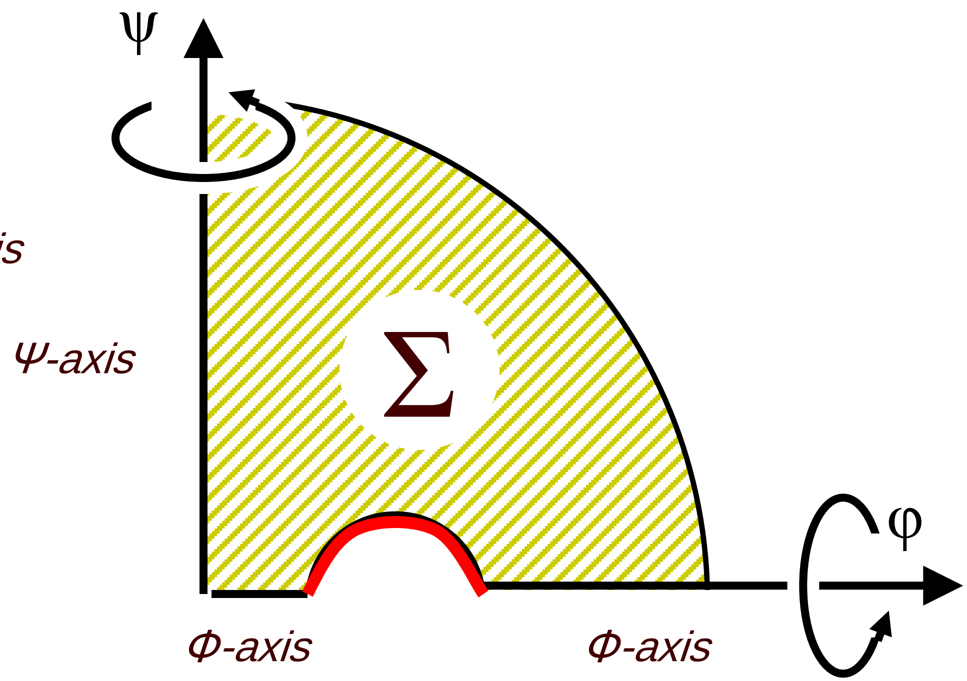


Base space: 2D region $\Sigma=\{(\rho,z)|\rho\geqq 0\}$



Base space: 2D region Σ={(ρ,z)|ρ≧0}





Deviation Matrix

- Introduce 7×7 coset matrix;

$$M = \begin{pmatrix} \hat{A} & \hat{B} & \sqrt{2}\hat{U} \\ \hat{B}^T & \hat{C} & \sqrt{2}\hat{V} \\ \sqrt{2}\hat{U}^T & \sqrt{2}\hat{V}^T & \hat{S} \end{pmatrix}$$

where

$$\hat{A} = \begin{pmatrix} [(1-y)\lambda + (2+x)\psi\psi^T - \tau^{-1}\tilde{\omega}\tilde{\omega}^T + \mu(\psi\psi^T\lambda^{-1}\tilde{J} - \tilde{J}\lambda^{-1}\psi\psi^T)] & \tau^{-1}\tilde{\omega} \\ \tau^{-1}\tilde{\omega}^T & -\tau^{-1} \end{pmatrix},$$

$$\hat{B} = \begin{pmatrix} (\psi\psi^T - \mu\tilde{J})\lambda^{-1} - \tau^{-1}\tilde{\omega}\psi^T\tilde{J} & [(-(1+y)\lambda\tilde{J} - (2+x)\mu + \psi^T\lambda^{-1}\tilde{\omega})\psi + (z - \mu\tilde{J}\lambda^{-1}\tilde{]}\omega] \\ \tau^{-1}\psi^T\tilde{J} & -z \end{pmatrix},$$

$$\hat{C} = \begin{pmatrix} (1+x)\lambda^{-1} - \lambda^{-1}\psi\psi^T\lambda^{-1} & \lambda^{-1}\tilde{\omega} - \tilde{J}(z - \mu\tilde{J}\lambda^{-1})\psi \\ \tilde{\omega}^T\lambda^{-1} + \psi^T(z + \mu\lambda^{-1}\tilde{J})\tilde{J} & [\tilde{\omega}^T\lambda^{-1}\tilde{\omega} - 2\mu\psi^T\lambda^{-1}\tilde{\omega} - \tau(1 + x - 2y - xy + z^2)] \end{pmatrix},$$

$$\hat{U} = \begin{pmatrix} (1 + x - \mu\tilde{J}\lambda^{-1})\psi - \mu\tau^{-1}\tilde{\omega} \\ \mu\tau^{-1} \end{pmatrix},$$

$$\hat{V} = \begin{pmatrix} (\lambda^{-1} + \mu\tau^{-1}\tilde{J})\psi \\ \psi^T\lambda^{-1}\tilde{\omega} - \mu(1 + x - z) \end{pmatrix},$$

$$\hat{S} = 1 + 2(x - y),$$

$$\tilde{\omega} = \omega - \mu\psi,$$

$$x = \psi^T\lambda^{-1}\psi, \quad y = \tau^{-1}\mu^2, \quad z = y - \tau^{-1}\psi^T\tilde{J}\tilde{\omega},$$

- Define deviation Matrix:

$$\Psi = (M_{[2]} - M_{[1]})M_{[1]}^{-1}$$

Steps of Proof

◇ *Mazur identity* ◇

$\mathcal{M} = g_2(D\Psi)g_1^{-1}$

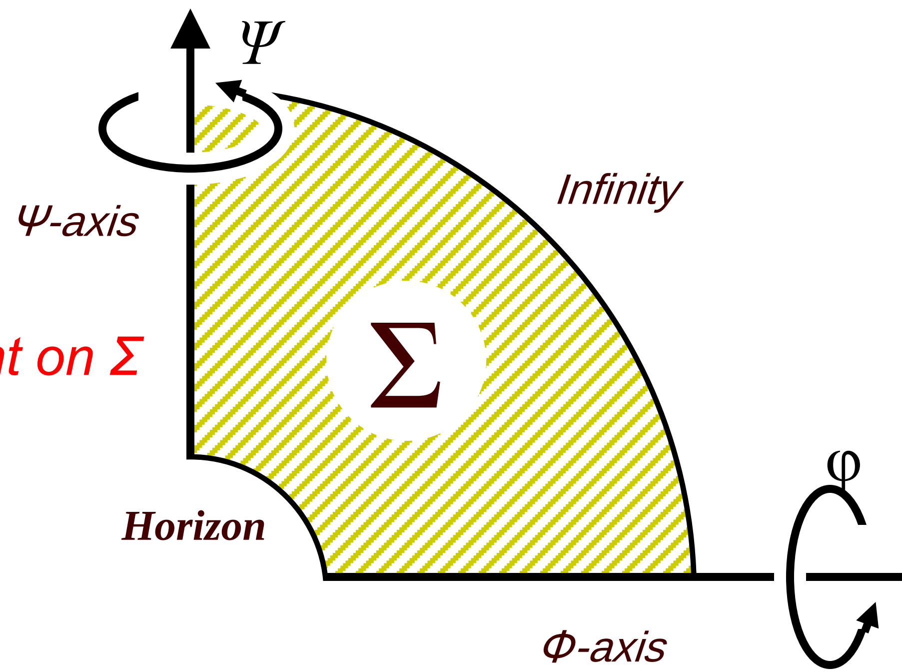
$$\oint_{\partial\Sigma} \rho \partial^a \mathrm{tr}\Psi dS_a = \int_{\Sigma} \rho \mathrm{tr}\left\{\mathcal{M}^t \mathcal{M}\right\} d\rho dz \geq 0$$

① *Show LHS =0 on ∂Σ*

- RHS=0 over Σ
- Ψ=constant over Σ

② *Show Ψ=0 (at least) at a single point on Σ*

- Ψ=0 over Σ
- M_1=M_2 over Σ



Boundary conditions & Sketch of proof

Infinity $r = \rho^2 + z^2 \to \infty$

- *Metric with asymptotic flatness*

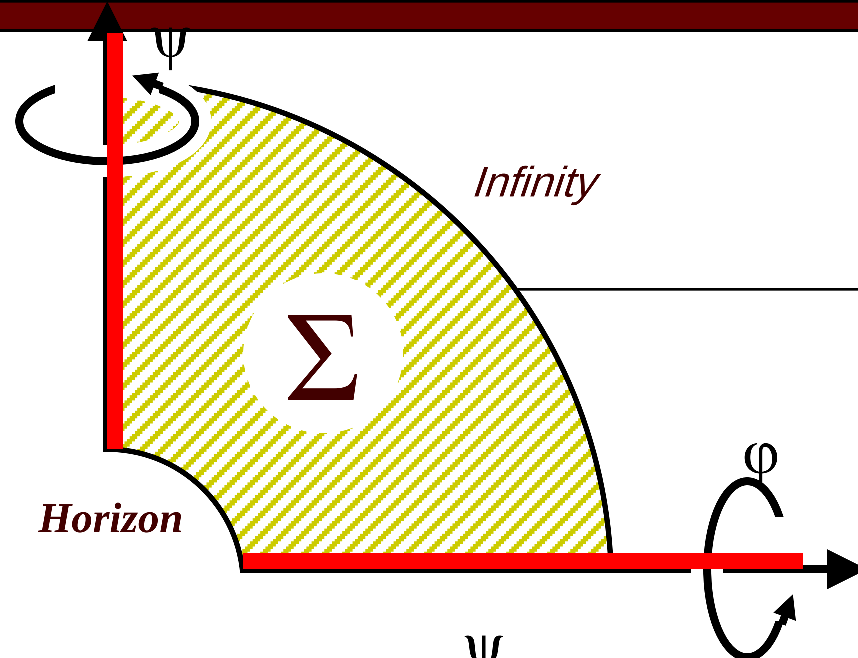
$$ds^2 \simeq \left(-1 + \frac{8M}{3\pi r^2}\right) dt^2 + \left(\frac{8J_\phi}{r^2} + \mathcal{O}(r^{-3})\right) \sin^2\theta dt d\phi + \left(\frac{8J_\psi}{r^2} + \mathcal{O}(r^{-3})\right) \cos^2\theta dt d\psi^2$$
$$+ r^2(d\theta^2 + \sin^2\theta d\phi^2 + \cos^2\theta d\psi^2)$$

- *Asymptotic behavior of gauge field*

$$A \simeq \left(\frac{4Q}{\pi r^2} + \mathcal{O}(r^{-3})\right) dt + \mathcal{O}(r^{-2}) d\phi + \mathcal{O}(r^{-2}) d\psi$$

$\lambda_{\varphi\varphi}$	$\lambda_{\psi\psi}$	$\lambda_{\varphi\psi}$	ω_φ	ω_ψ	ψ_φ	ψ_ψ	μ
$(\sqrt{\rho^2+z^2} - z)\left(1 + \frac{2(M+\eta)}{3\pi\sqrt{\rho^2+z^2}}\right)$	$(\sqrt{\rho^2+z^2} + z)\left(1 + \frac{2(M-\eta)}{3\pi\sqrt{\rho^2+z^2}}\right)$	$\mathcal{O}(1)$	$\frac{J_\phi}{\pi}\left(\frac{\rho^2}{\rho^2+z^2} - \frac{2z}{\sqrt{\rho^2+z^2}}\right)$	$\frac{J_\psi}{\pi}\left(\frac{\rho^2}{\rho^2+z^2} - \frac{2z}{\sqrt{\rho^2+z^2}}\right)$	$\mathcal{O}\left(\frac{1}{\sqrt{\rho^2+z^2}}\right)$	$\mathcal{O}\left(\frac{1}{\sqrt{\rho^2+z^2}}\right)$	$\frac{2Qz}{\pi\sqrt{\rho^2+z^2}}$

ψ (φ)-axis



(1) Gravitational potentials

$$\lambda_{\psi\psi} = (\xi_\psi|\xi_\psi) = 0$$
$$\lambda_{\phi\phi} \simeq \mathcal{O}(1)$$
$$\lambda_{\phi\psi} = (\xi_\phi|\xi_\psi) = 0$$

(2) Electric potentials

$$d\psi_\psi = F(\xi_\psi) = 0 \quad \Rightarrow \psi_\psi = const. = 0$$

$$d\psi_\phi = F(\xi_\phi) \neq 0 \quad \Rightarrow \psi_\phi = O(1)$$

(3) Magnetic potentials

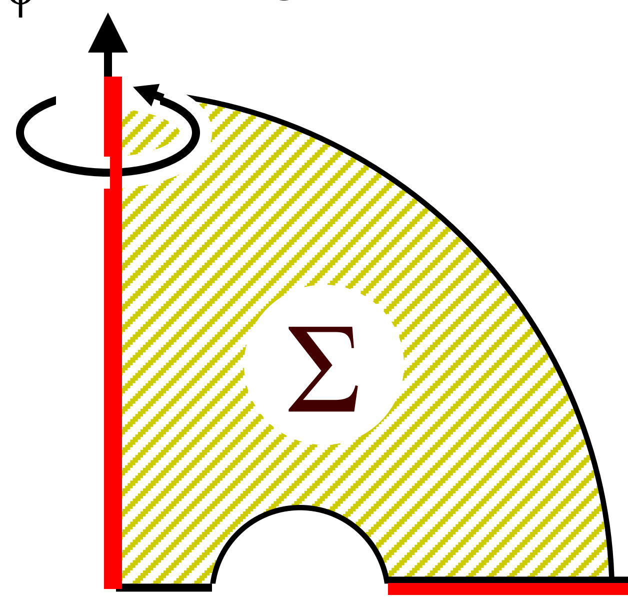
$$d\mu = *(\xi_\phi \wedge \xi_\psi \wedge F) - (\psi_\phi d\psi_\psi - \psi_\psi d\psi_\phi) = 0$$

$$\Rightarrow \mu = const. = Q$$

(4) Twist potentials

$$d\omega_a = *(\xi_\phi \wedge \xi_\psi \wedge d\xi_a) + (3 d\mu + \psi_\phi d\psi_\psi - \psi_\psi d\psi_\phi) = 0$$

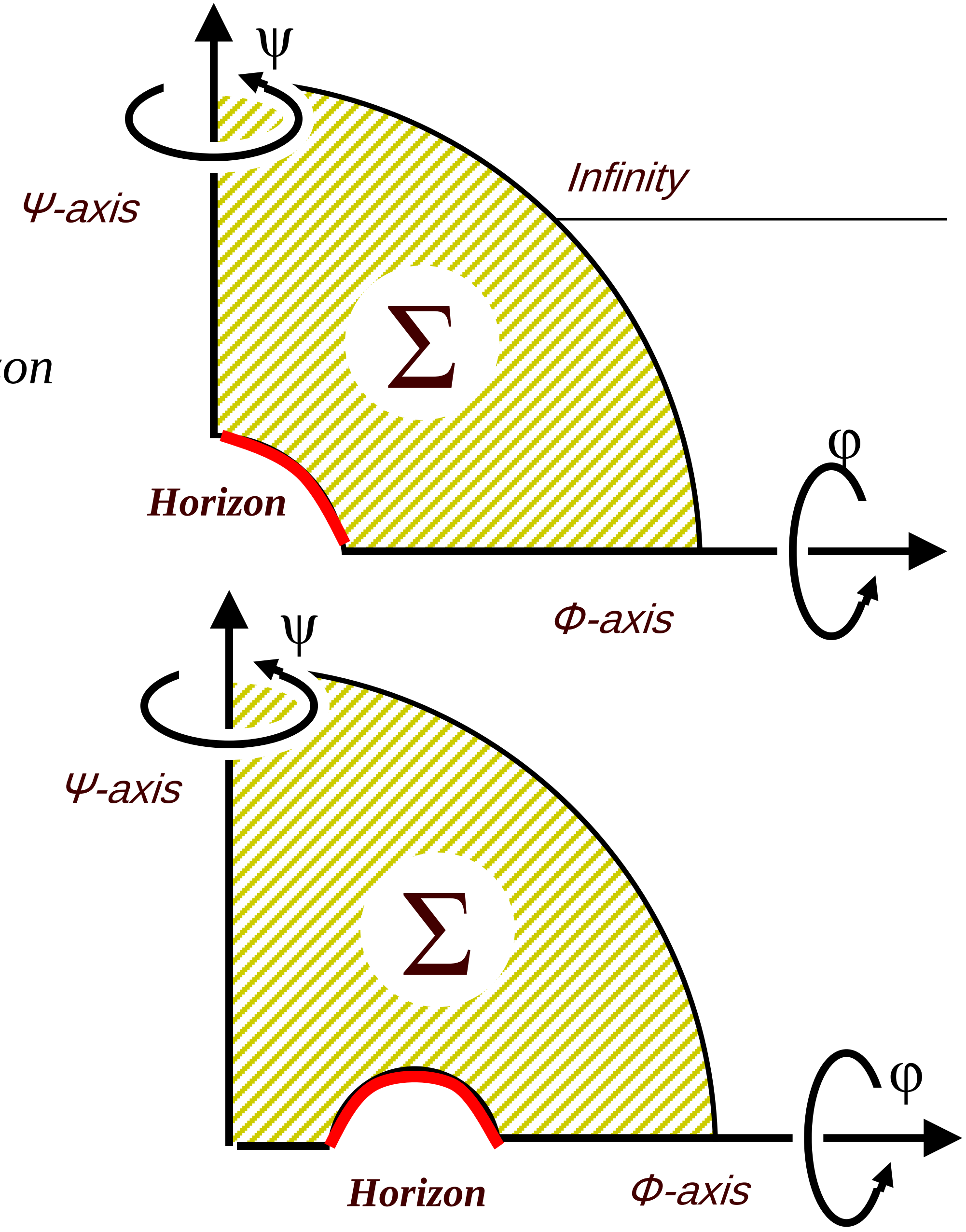
$$\Rightarrow \omega_a = const. = J_a$$



Horizon

- *Only regularity is required :*
  All scalar fields are finite on horizon

$$\Phi^A \simeq \mathcal{O}(1)$$
$$\Phi^A_{,i} \simeq \mathcal{O}(1)$$



Inner φ-axis

(1) Gravitational potentials

$$\lambda_{\phi\phi} = (\xi_\phi|\xi_\phi) = 0$$

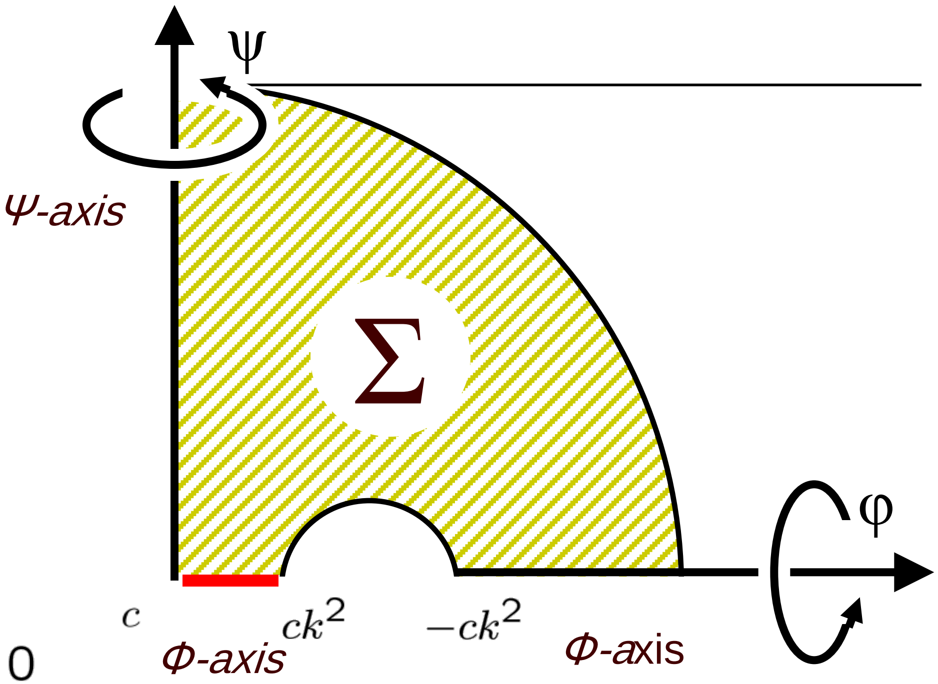
$$\lambda_{\psi\psi} \simeq \mathcal{O}(1)$$

$$\lambda_{\phi\psi} = (\xi_\phi|\xi_\psi) = 0$$

(2) Electric potentials

$$d\psi_\phi = F(\xi_\phi) = 0$$

$=0 \Rightarrow \psi_\phi = const. \neq 0$



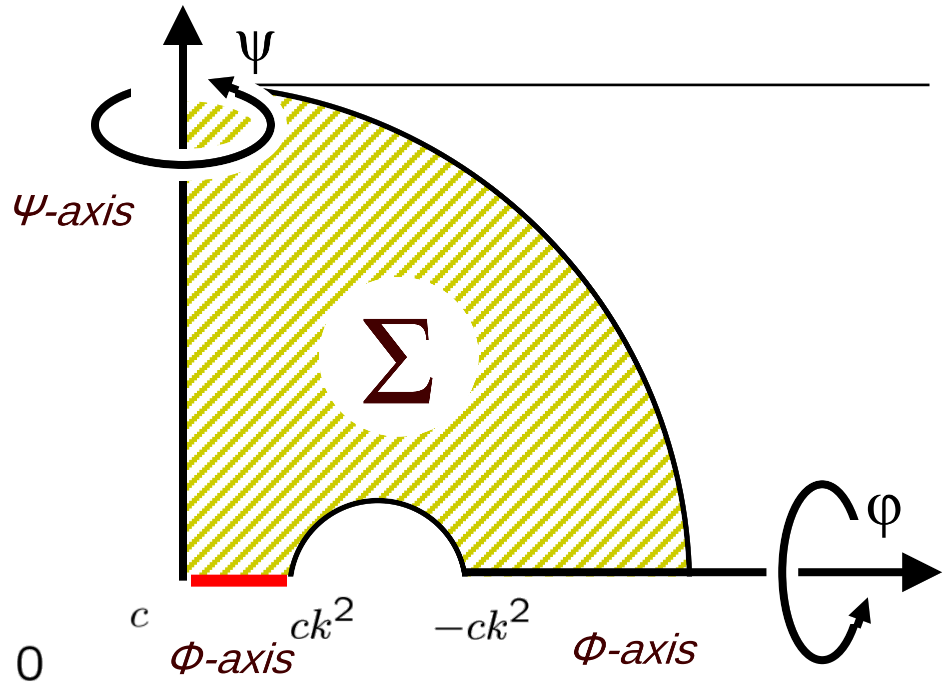
Inner φ-axis

(1) Gravitational potentials

$$\lambda_{\phi\phi} = (\xi_\phi|\xi_\phi) = 0$$

$$\lambda_{\psi\psi} \simeq \mathcal{O}(1)$$

$$\lambda_{\phi\psi} = (\xi_\phi|\xi_\psi) = 0$$



(2) Electric potentials

$$d\psi_\phi = F(\xi_\phi) = 0$$

$$\Rightarrow \psi_\phi = const. \neq 0$$

$$q := \frac{1}{2\pi}\int_{S^2} F = \psi_\phi(z = ck^2) - \psi_\phi(z = -ck^2) = \psi_\phi(z = ck^2)$$

=0

$$\boxed{\Rightarrow \psi_\phi = q}$$

$$d\psi_\psi = F(\xi_\psi) \neq 0$$

$$\Rightarrow \psi_\psi = O(1)$$

(3) Magnetic potentials

- *Definition*

$$d\mu = *(\xi_\phi \wedge \xi_\psi \wedge F) - (\psi_\phi d\psi_\psi - \psi_\psi d\psi_\phi)$$

ξ_ϕ =0, ψ_ϕ =q, $d\psi_\phi$ =0

- *1[st] term vanishes and* $\boldsymbol{\psi_\varphi}$*=q=const. on* $\boldsymbol{\varphi}$*-axis*

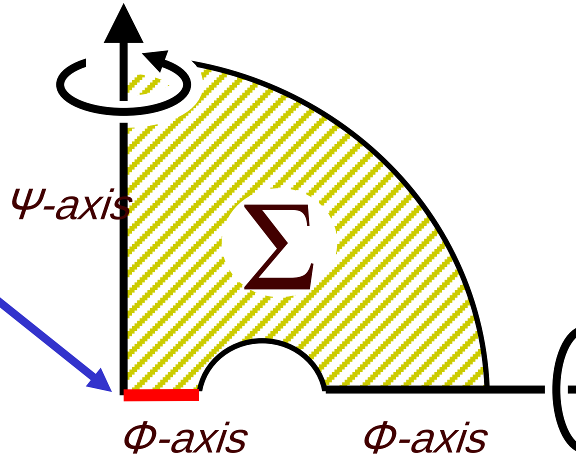
$$d\mu = -q d\psi_\psi \;\Rightarrow \mu = -q\psi_\psi + const.$$

- $\boldsymbol{\mu}$*= Q,* $\boldsymbol{\psi_\psi}$*=0 at the center of the ring*

$$\Rightarrow const. = Q$$

- *On inner* $\boldsymbol{\varphi}$*-axis,* $\boldsymbol{\mu}$ *has to behave as*

$$\mu = -q\psi_\psi + Q$$



(4) Twist potentials

- *Definition*

$$d\omega_a = *(\xi_\phi \wedge \xi_\psi \wedge d\xi_a) + \psi_a(3d\mu + \psi_\phi d\psi_\psi - \psi_\psi d\psi_\phi)$$

- *1st term vanishes on* **φ**-*axis*
  and boundary condition **ψ**$_\phi$ *=q,* **μ** *=-q* **ψ**$_\psi$ *+Q*

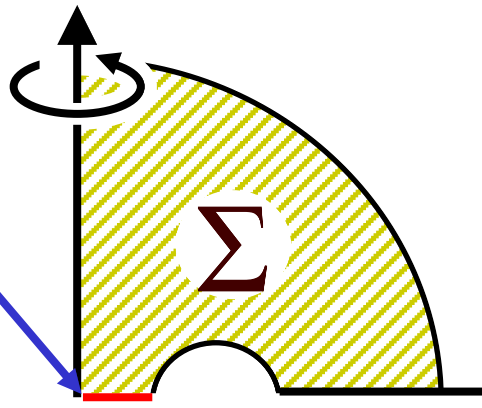
$$d\omega_a = -2q d\psi_\phi \quad \Rightarrow \omega_\phi = -2q^2\psi_\psi + const_1, \quad \omega_\psi = -q\psi_\psi^2 + const_2$$

- **ω**$_a$ *=* J_a*,* **ψ**$_\psi$ *=0 at the center of the ring*

$$\Rightarrow const_1 = J_\phi, \quad const_2 = J_\psi$$

- *On inner* **φ**-*axis,* **ω**$_a$ *has to behave as*

$$\omega_\phi = -2q^2\psi_\psi + J_\phi, \quad \omega_\psi = -q\psi_\psi^2 + J_\psi$$



Asymptotic behaviors of scalar fields

Regularity (ψ-axis, Inner φ-axis, outer φ-axis, horizon) — **Asymp flat** (infinity)

	ψ-axis	*Inner φ-axis*	*outerφ-axis*	*horizon*	infinity
$\lambda_{\varphi\varphi}$	*O(1)*	*0*	*0*	$\mathcal{O}(1)$	$(\sqrt{\rho^2+z^2}-z)\left(1+\frac{2(M+\eta)}{3\pi\sqrt{\rho^2+z^2}}\right)$
$\lambda_{\psi\psi}$	*0*	*O(1)*	*O(1)*	$\mathcal{O}(1)$	$(\sqrt{\rho^2+z^2}+z)\left(1+\frac{2(M-\eta)}{3\pi\sqrt{\rho^2+z^2}}\right)$
$\lambda_{\varphi\psi}$	*0*	*0*	*0*	$\mathcal{O}(1)$	$\zeta\frac{\rho^2}{\sqrt{\rho^2+z^2}^3}$
ω_φ	$-J_\phi$	$-2q^2\psi_\psi - J_\phi$	J_ϕ	$\mathcal{O}(1)$	$\frac{J_\phi}{\pi}\left(\frac{\rho^2}{\rho^2+z^2}-\frac{2z}{\sqrt{\rho^2+z^2}}\right)$
ω_ψ	$-J_\psi$	$-q\psi_\psi^2 - J_\psi$	J_ψ	$\mathcal{O}(1)$	$\frac{J_\psi}{\pi}\left(\frac{\rho^2}{\rho^2+z^2}-\frac{2z}{\sqrt{\rho^2+z^2}}\right)$
ψ_φ	*O(1)*	*q*	*0*	$\mathcal{O}(1)$	$\mathcal{O}((\rho^2+z^2)^{-1/2})$
ψ_ψ	*0*	*O(1)*	*O(1)*	$\mathcal{O}(1)$	$\mathcal{O}((\rho^2+z^2)^{-1/2})$
μ	*Q*	$-q\psi_\psi + Q$	*-Q*	$\mathcal{O}(1)$	$\frac{2Qz}{\pi\sqrt{\rho^2+z^2}}$
$\int_{\partial\Sigma}\rho\partial_a \mathrm{Tr}\Psi dS^a$					
Ψ					

Asymptotic behaviors of scalar fields

Regularity (ψ-axis, Inner φ-axis, outer φ-axis, horizon) ***Asymp flat*** (infinity)

	ψ-axis	*Inner φ-axis*	*outer φ-axis*	*horizon*	infinity
$\lambda_{\varphi\varphi}$	*O(1)*	*0*	*0*	$\mathcal{O}(1)$	$(\sqrt{\rho^2+z^2}-z)\left(1+\frac{2(M+\eta)}{3\pi\sqrt{\rho^2+z^2}}\right)$
$\lambda_{\psi\psi}$	*0*	*O(1)*	*O(1)*	$\mathcal{O}(1)$	$(\sqrt{\rho^2+z^2}+z)\left(1+\frac{2(M-\eta)}{3\pi\sqrt{\rho^2+z^2}}\right)$
$\lambda_{\varphi\psi}$	*0*	*0*	*0*	$\mathcal{O}(1)$	$\zeta\frac{\rho^2}{\sqrt{\rho^2+z^2}^3}$
ω_{φ}	$-J_\phi$	$-2q^2\psi_\psi - J_\phi$	J_ϕ	$\mathcal{O}(1)$	$\frac{J_\phi}{\pi}\left(\frac{\rho^2}{\rho^2+z^2}-\frac{2z}{\sqrt{\rho^2+z^2}}\right)$
ω_{ψ}	$-J_\psi$	$-q\psi_\psi^2 - J_\psi$	J_ψ	$\mathcal{O}(1)$	$\frac{J_\psi}{\pi}\left(\frac{\rho^2}{\rho^2+z^2}-\frac{2z}{\sqrt{\rho^2+z^2}}\right)$
ψ_{φ}	*O(1)*	*q*	*0*	$\mathcal{O}(1)$	$\mathcal{O}((\rho^2+z^2)^{-1/2})$
ψ_{ψ}	*0*	*O(1)*	*O(1)*	$\mathcal{O}(1)$	$\mathcal{O}((\rho^2+z^2)^{-1/2})$
μ	*Q*	$-q\psi_\psi + Q$	*-Q*	$\mathcal{O}(1)$	$\frac{2Qz}{\pi\sqrt{\rho^2+z^2}}$
$\int_{\partial\Sigma}\rho\partial_a \mathrm{Tr}\Psi dS^a$	*0*	*0*	*0*	*0*	*0*
Ψ					*0*

Theorem

- *Consider, in five-dimensional Einstein-Maxwell-Chern-Simons theory (5D minimal SUGRA), a stationary charged rotating black hole with finite temperature that is regular on and outside the event horizon and asymptotically flat.*
- *If the black hole spacetime admits, besides the stationary Killing vector field, two mutually commuting axial Killing vector fields so that the isometry group is* $\mathbb{R} \times U(1) \times U(1)$

Then

(1) *the black hole with horizon topology S^3 is uniquely characterized by its mass, electric charge, and two independent angular momenta, and hence must be isometric to the Chong-Cvetic-Lu-Pope solution.(cf Ida-Morisawa 05, Tomizawa-Yasui-Ishibashi '09)*

(2) *the black ring with horizon topology S^1 × S^2 is uniquely characterized by its mass, electric charge, and two independent angular momenta, the dipole charge, the ratio of S^1/S^2 and hence it must be unique if such a solution exist (Tomizawa-Yasui-Ishibashi '09)*