重力/流体対応を用いた
高次元ブラックホール安定性の理解

Umpei Miyamoto
（Hebrew U. ➔ Rikkyo U.）

0803.3037

0804.1723

0811.2305

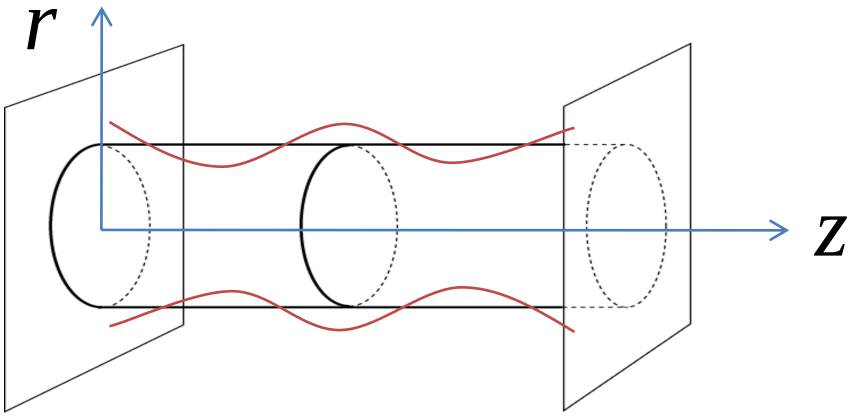
Plan

- **Introduction**
  - Gregory-Laflamme instability
  - Rayleigh-Plateau instability
- **Black-hole black-string phase transitions via fluid/gravity corresp.**
- **Conclusion & Future prospects**

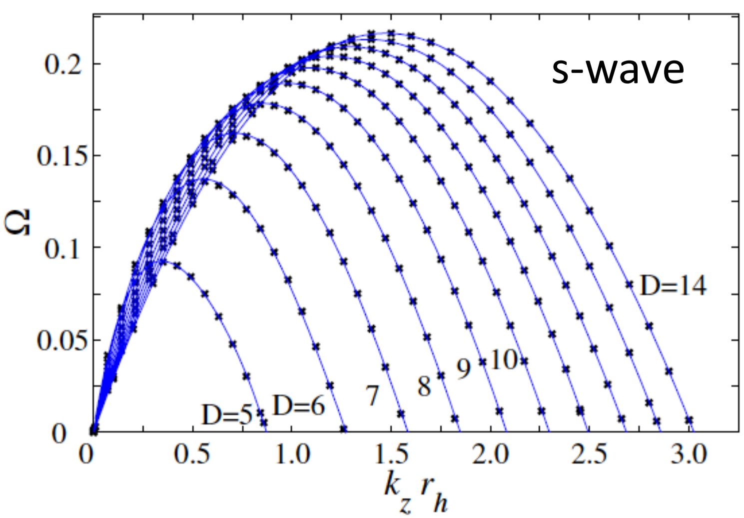
Gregory-Laflamme instability

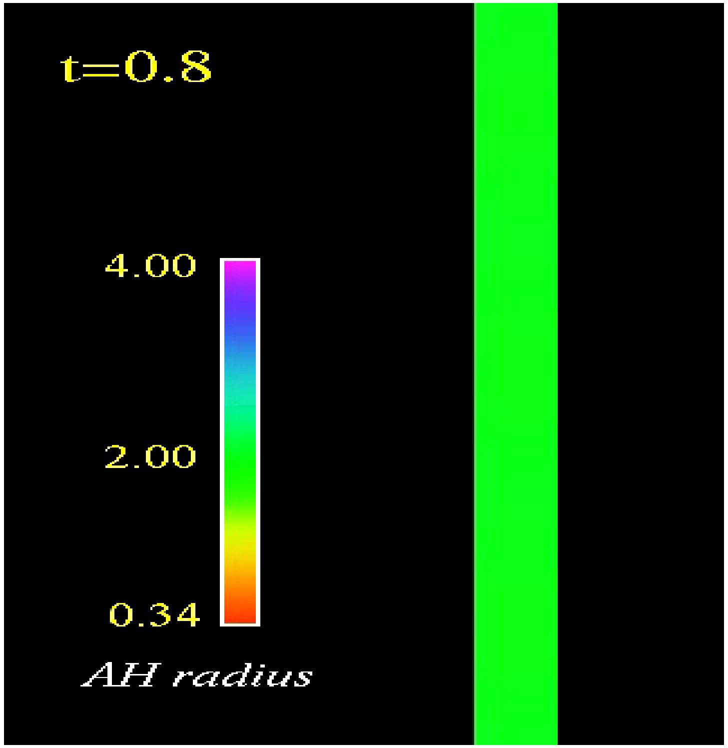
Gregory, Laflamme 93
Horowitz, Maeda 01
Reall, Choptuik et al

d-dim. Black String $\mathrm{Sch}_{d-1} \times S^1$



$$\delta g_{ab} = e^{\Omega t + ikz} f_{ab}(r) Y(\Omega)$$

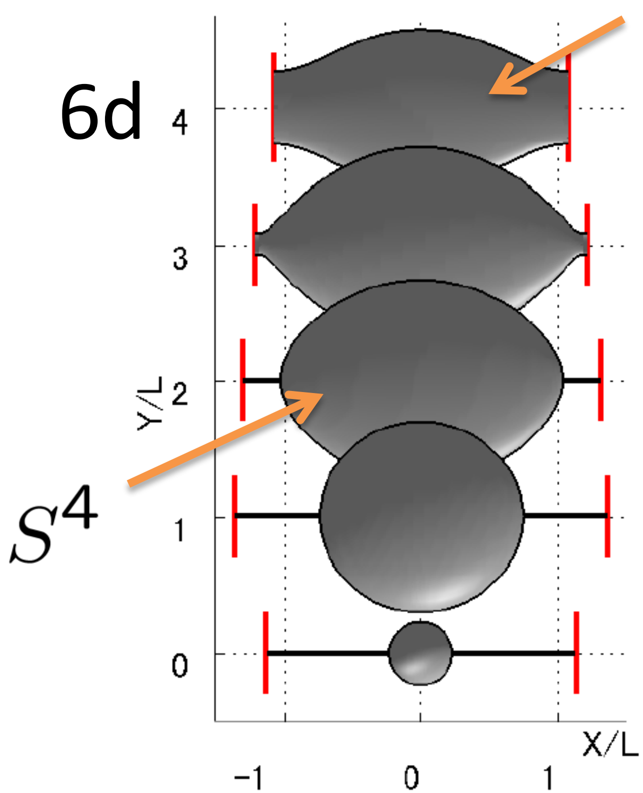




$$R_{\mu\nu} = 0, \ (5\mathrm{d})$$

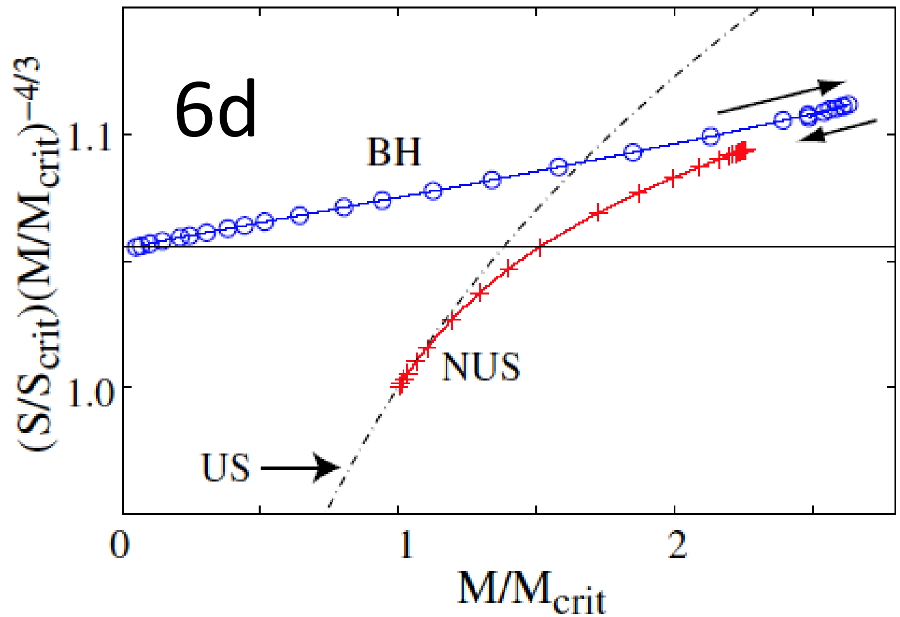
1. Universal instability of black objects (string, brane, ring, warped string…)
2. Get strong in high-d
3. Nobody knows endpoint

Caged black holes



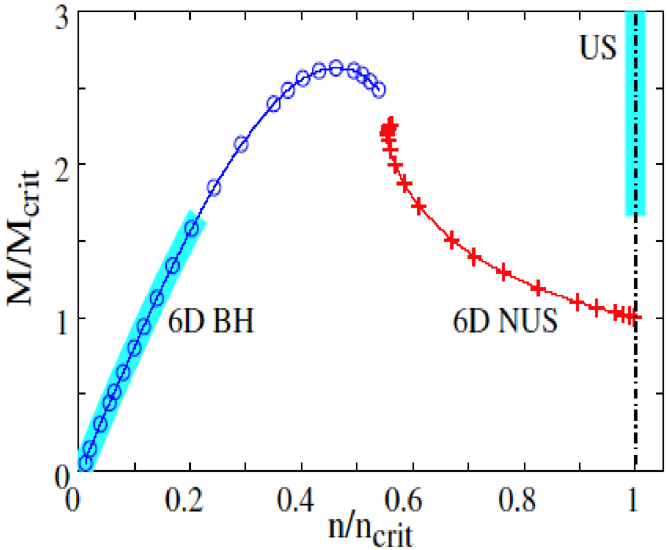
$R_{\mu\nu} = 0$

Gubser , Kol, Kol-Sorkin-Piran
Wiseman, Kudoh-Wiseman ‘04 &
many

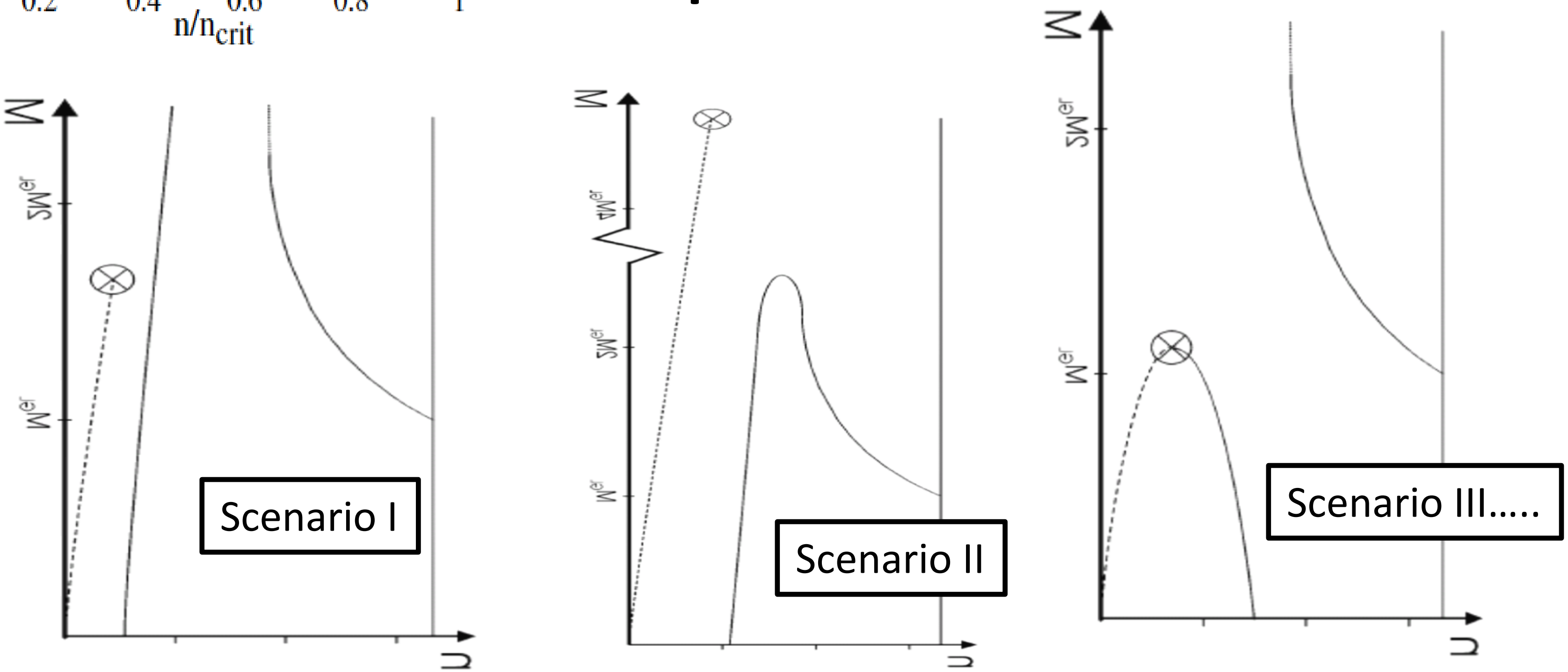


1. 1st order trans. from **Uniform String** to **BH**
2. **Non-Uniform String** never favored
3. Phase diagrams available in 5 & 6d only

Mass-Tension diagram (Kudoh-Wiseman 2004)



In past.... e.g. Harmark & Obers 2004

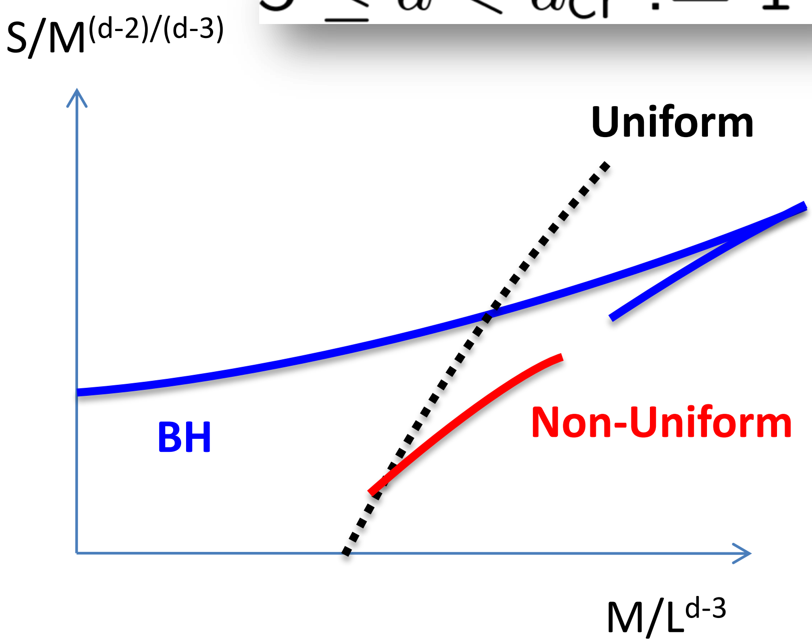


Critical dimension(s)

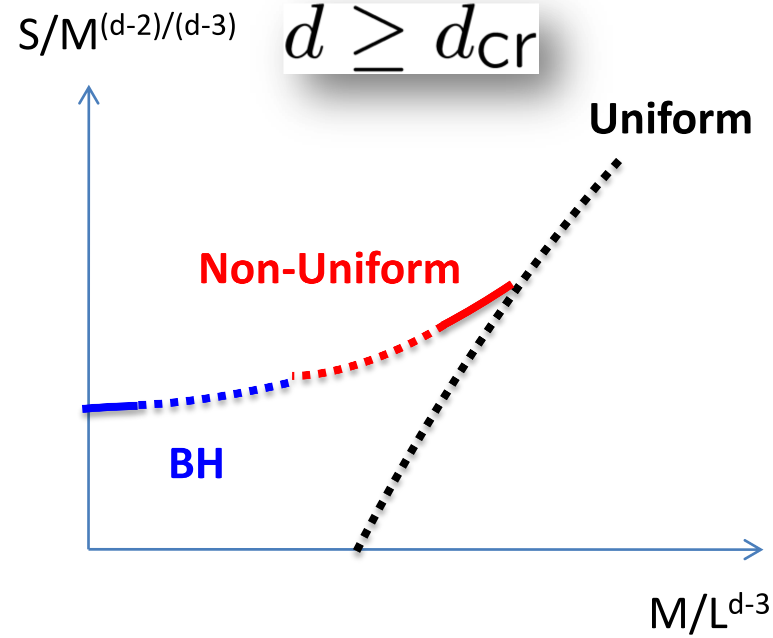
(predicted by higher-order perturbations)

Sorkin '04, Kol-Sorkin
Kudoh-Miyamoto

$5 \leq d < d_{cr} := 14$



$d \geq d_{cr}$

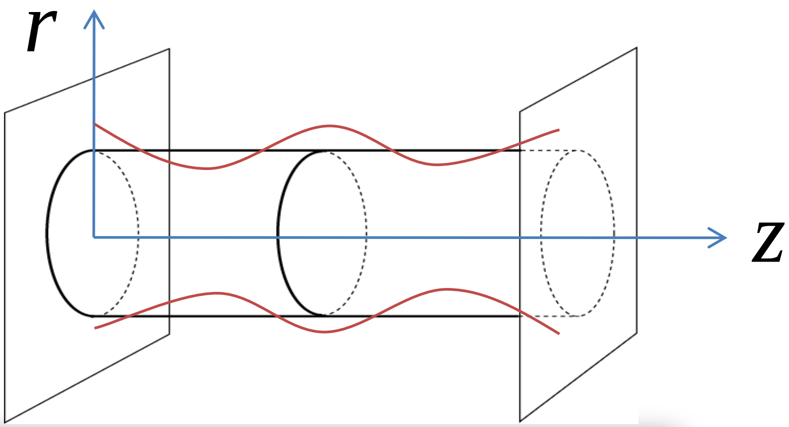


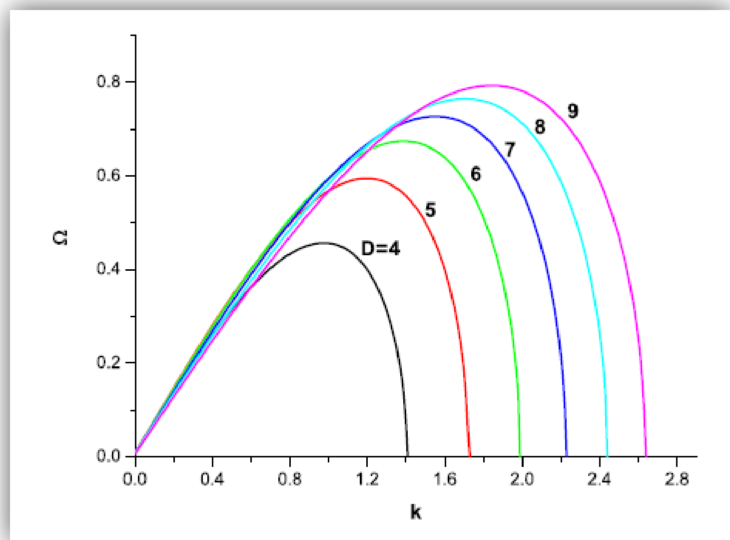
1. 2nd order trans. from **Uniform String** to **Non-Uniform String** above d_{cr}
2. Stability of **NUS** changes at around d_{cr}
3. d_{cr} depends on ensemble, charge, and momentum

Rayleigh-Plateau instability

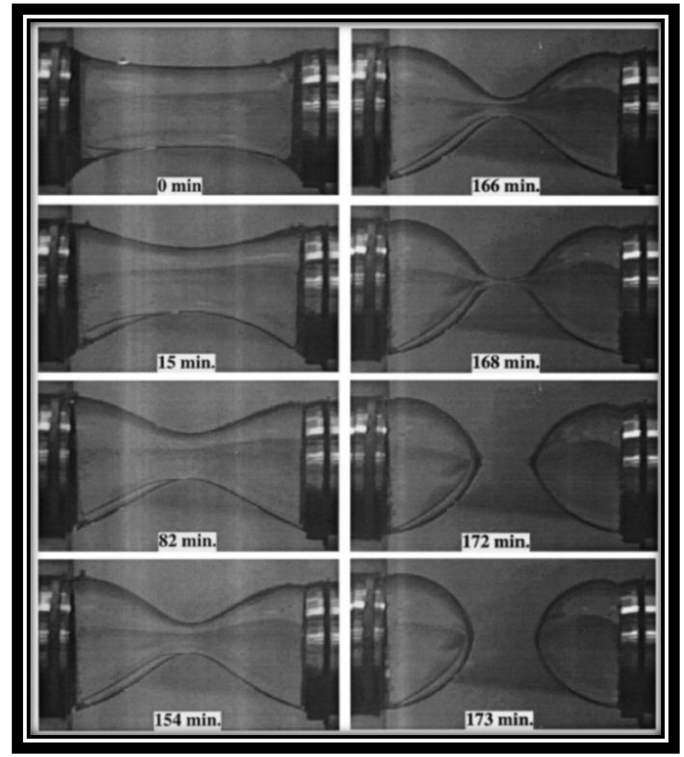
Plateau 1873, Rayleigh 1878
Cardoso-Dias-Gualtieri 06

D-dim. cylindrical fluid
held by ***surface tension/Newtonian gravity.***





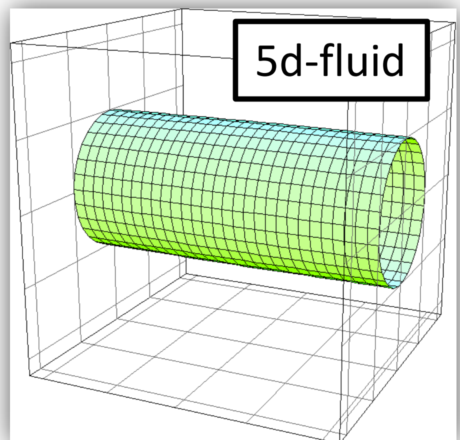
Time evolution (**experiment**)



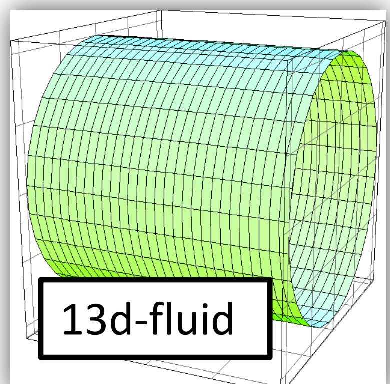
1. Many similarities with GL
2. Get strong in high-d
3. Everybody knows endpoint (in 4d)

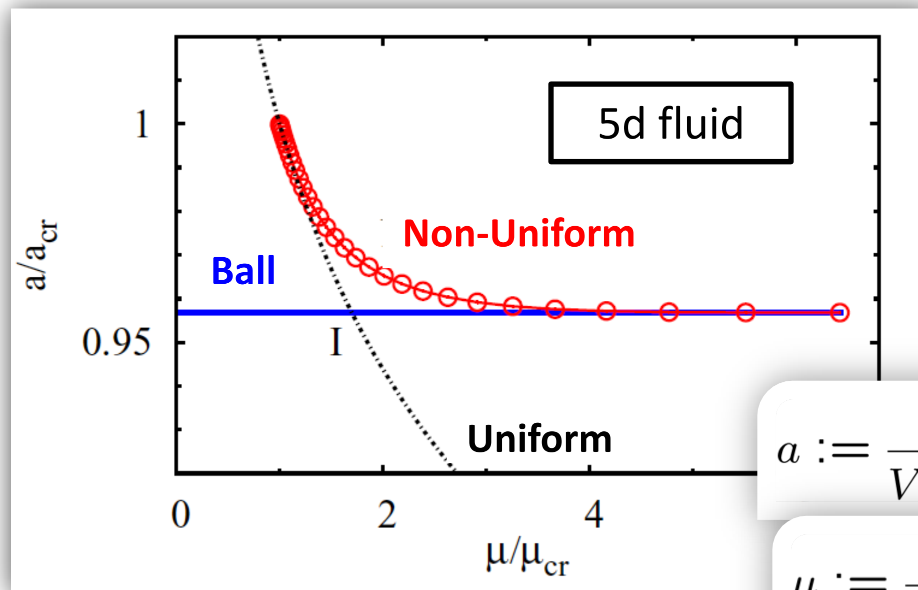
Geometry of fluid lumps

U.M. and K.-i. Maeda
Phys. Lett. B 664(2008)103



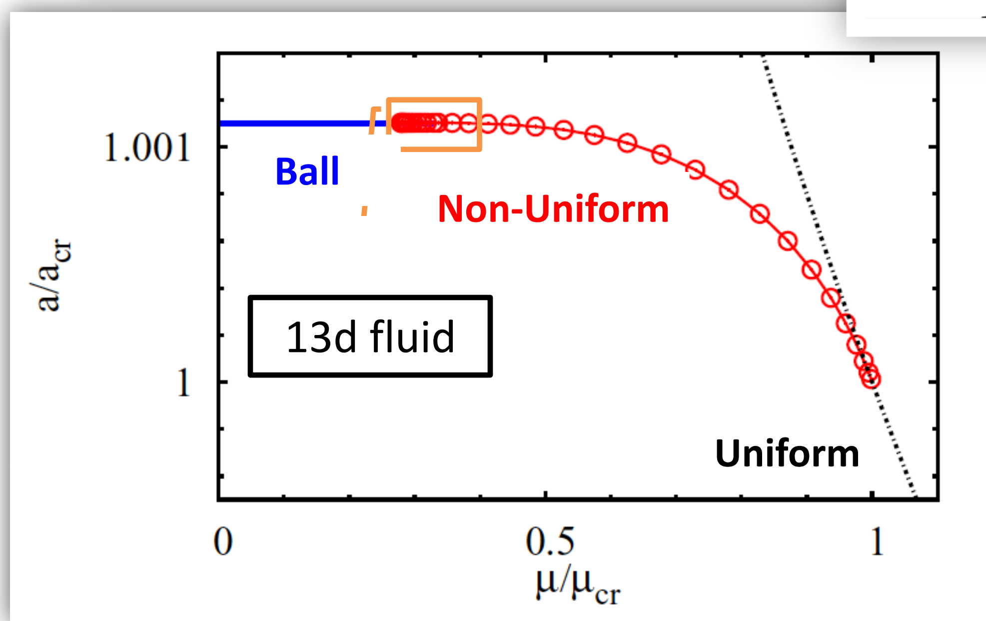
Equilibrium configurations of a fluid lump held by surface tension





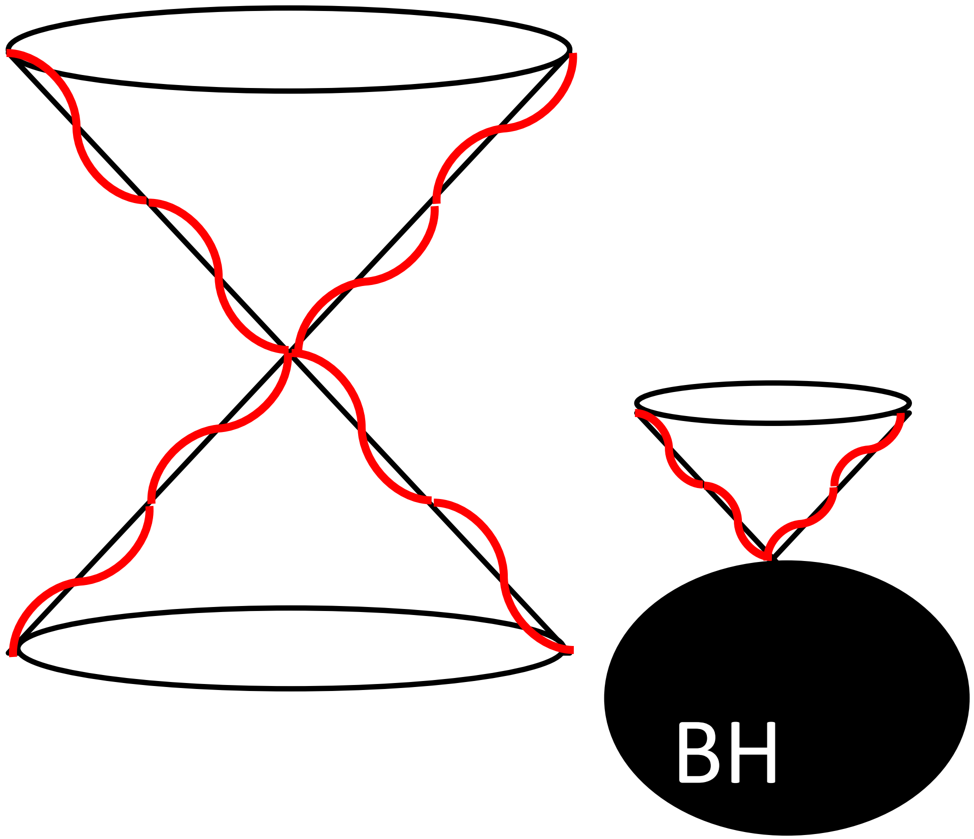
$$a := \frac{A}{V^{(n+1)/(n+2)}}$$

$$\mu := \frac{V}{L^{n+2}}$$



PR: Critical dimension of minimal cone

CQG 26 (2009) 185008
w/ G.W. Gibbons & K.-i. Maeda



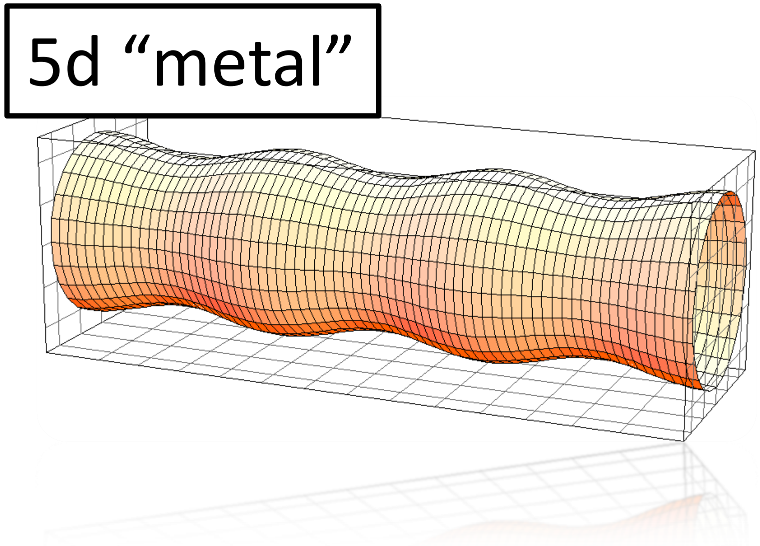
Cone is unstable d<9
Cone is stable d>=9=d*

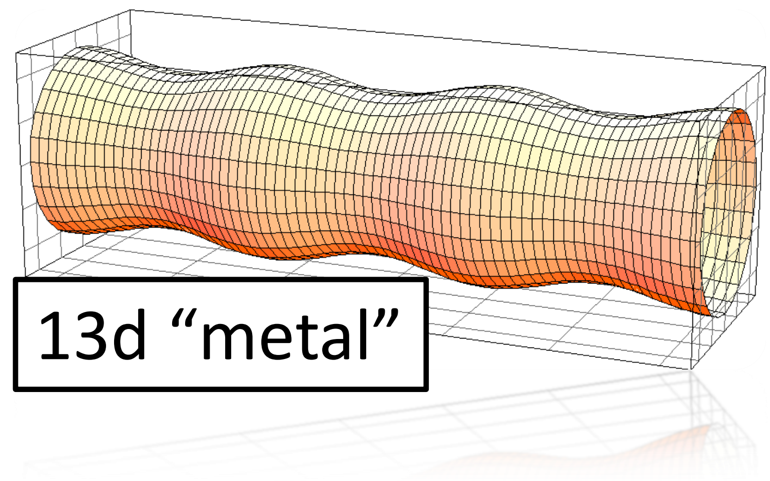
Related to Hashimoto-san's talk…..?

Final fate of RP instability
in surface diffusion model

U.M.
Phys. Rev. D 78, 026001 (2008)

- Surface diffusion = Microscopic model of surface tension
  W.W. Mullins, *"Theory of Thermal Grooving,"*
  J. Appl. Phys. 28, 333 (1957)





“Black-hole black-string phase transitions from hydrodymamics”

JHEP 0903(2009)066 [arXiv:0811.2305]

w/ K.-I. Maeda

“Black Holes as Lumps of Fluid”
0811.2381
Caldarelli, Dias, Emparan, Klemm

Fluid/Gravity correspondence

- Fluid/Gravity correspondence
  = AdS/CFT (Gravity/Gauge) correspondence
  + Effective fluid description of QFT

  – Hawking-Page transition ⇔ Confine-Deconfine transition
    (Witten '98)

  – Einstein Eq. w/ $\Lambda<0$ ⇔ Relativistic Navier-Stokes Eq.
    (Bhattacharyya-Minwalla-Hubney-Rangamani '07 and many)

  – Finite energy BHs ⇔ Lumps of deconfined plasma
    (Aharony-Minwalla-Wiseman '05)

Hawking-Page tr. ⇔ Deconfining phase tr.

$$R_{AB} = -\frac{d+1}{\ell^2} g_{AB}$$

Hawking-Page '83, Witten '98

AdS soliton (≒Scherk-Schwarz compactified AdS)

$$ds^2_{d+2} = \ell^2 \left(e^{2u}[-dt^2 + T_{2\pi}(u)d\theta^2 + dw_i^2] + \frac{1}{T_{2\pi}(u)} du^2 \right)$$

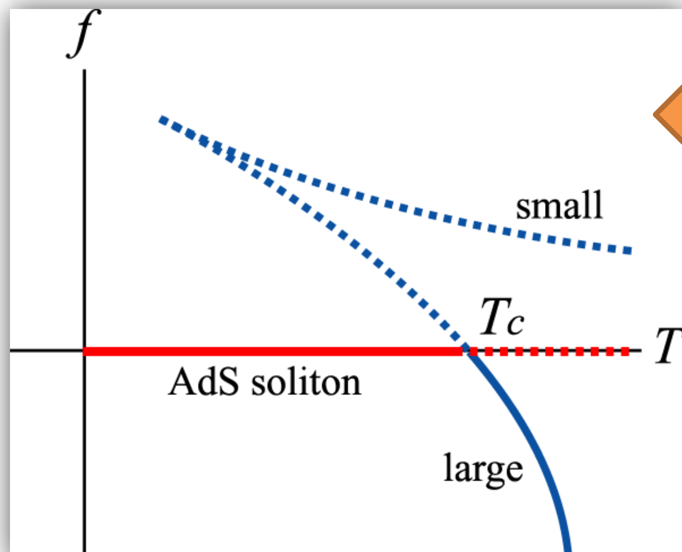
$$\theta \equiv \theta + 2\pi$$

$$i = 1, 2, \ldots, d-1$$

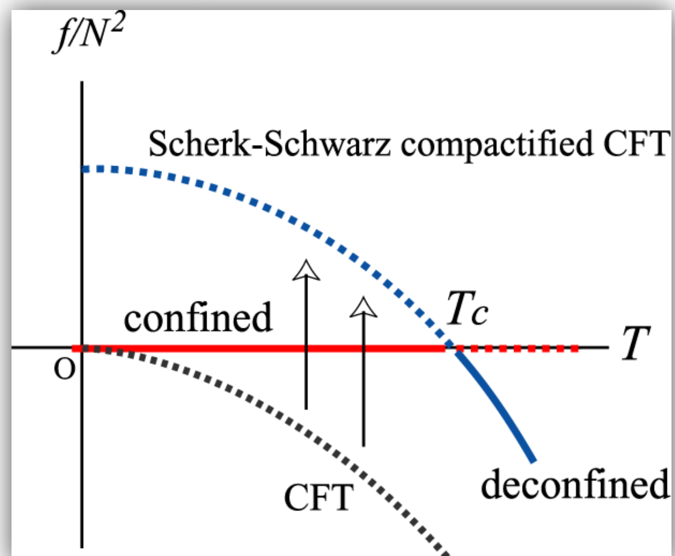
Black brane of T=1/β

$$ds^2_{d+2} = \ell^2 \left(e^{2u}[-T_\beta(u)dt^2 + d\theta^2 + dw_i^2] + \frac{1}{T_\beta(u)} du^2 \right)$$

$$T_x(u) = 1 - \left[\frac{x}{4\pi}(d+1)e^u\right]^{-(d+1)}$$



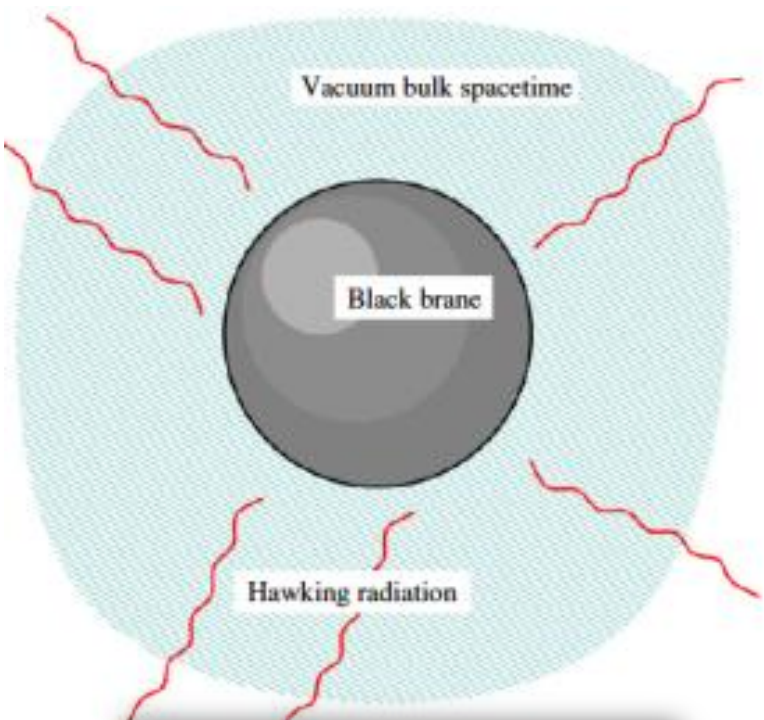
Hawking-Page transition

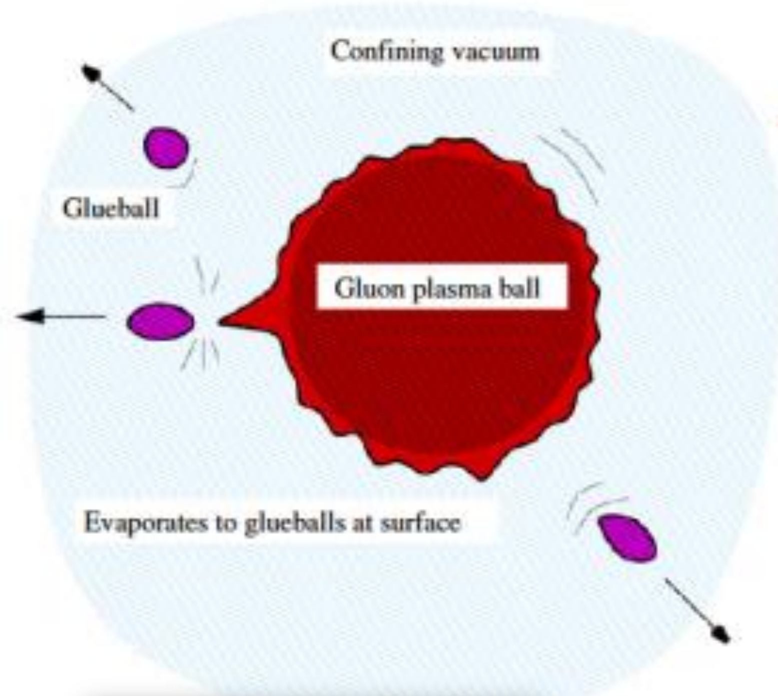


Confine-Deconfine transition

Finite energy BHs ⇔ Lumps of deconfined plasma

(Aharony-Minwalla-Wiseman '05)





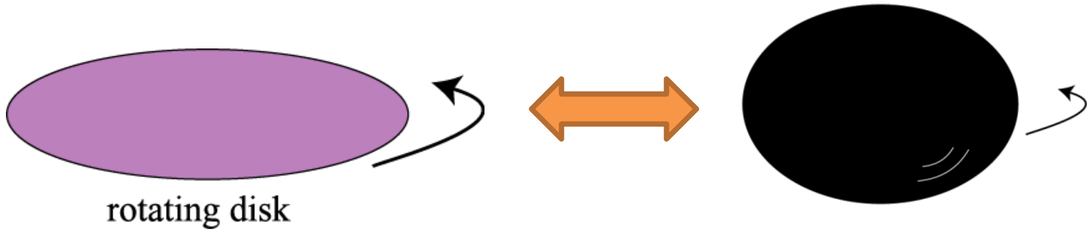
$$R_{AB} = -\frac{d+1}{\ell^2} g_{AB}$$

$$\partial_\mu T^{\mu\nu} = 0$$

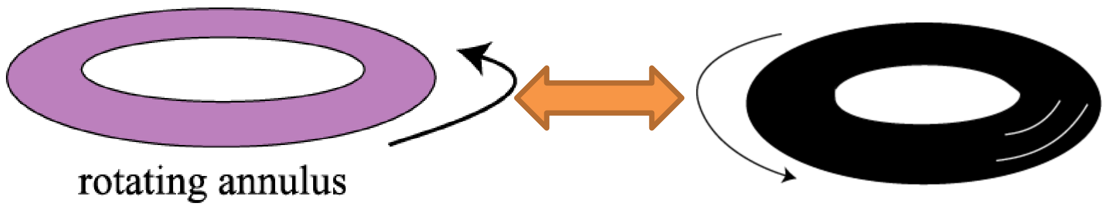
Bhattacharyya-Minwalla-Hubney-Rangamani '07
And many

Plasma Rings as dual black rings

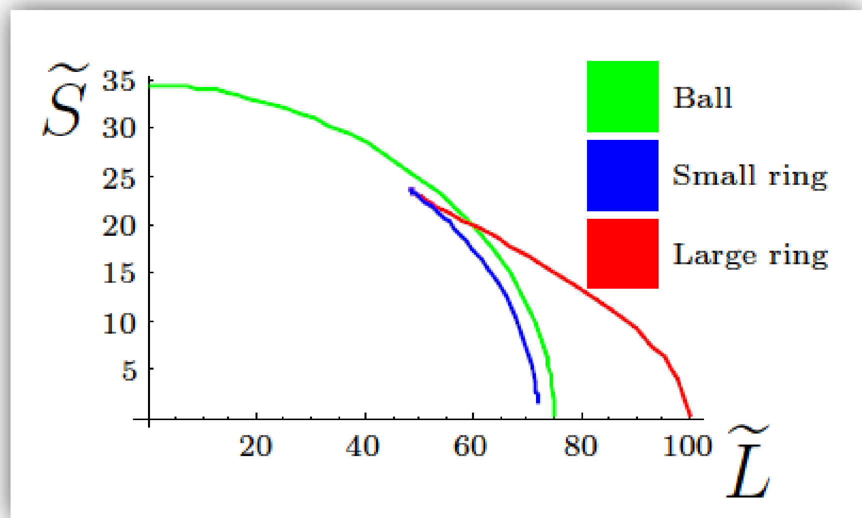
Lahiri-Minwalla '07

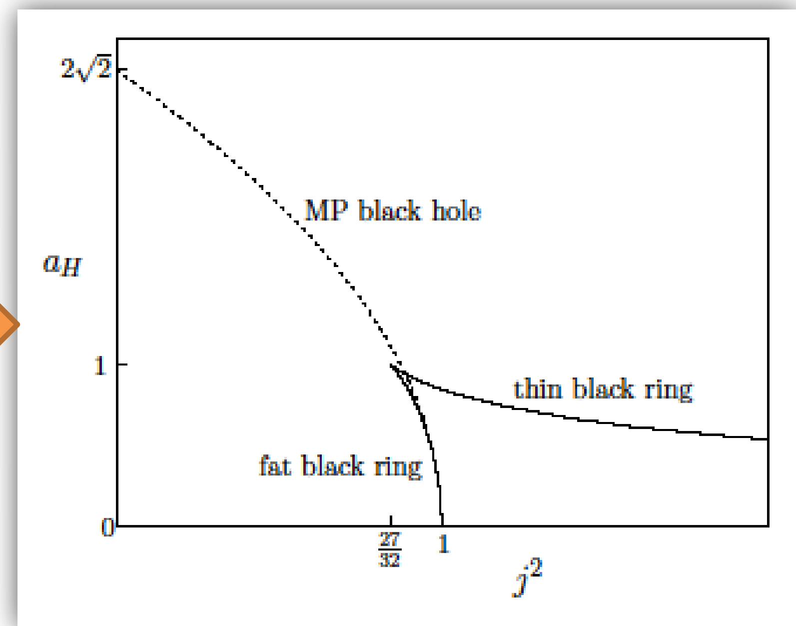


Black Hole in Scherk-Schwarz compactif. AdS_5

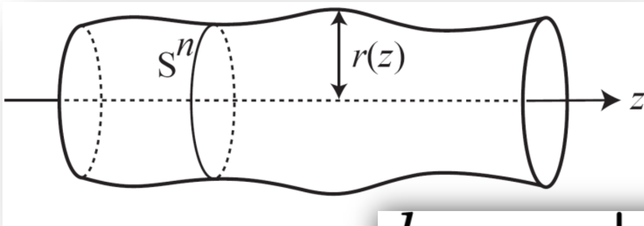


Black Ring in SS compactif. AdS_5

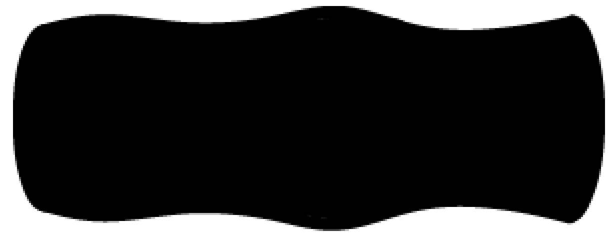




Plasma tubes as dual black strings



$$d = n + 3$$



Black String in AdS_(d+2)

EOM: Rela. Navier-Stokes eq.

$$\partial_\mu T^{\mu\nu} = \partial_\mu \left(T^{\mu\nu}_{\text{pefect}} + T^{\mu\nu}_{\text{surface}} \right) = 0$$

$$T^{\mu\nu}_{\text{perfect}} = (\rho + P) u^{\mu\nu} + P g^{\mu\nu}$$

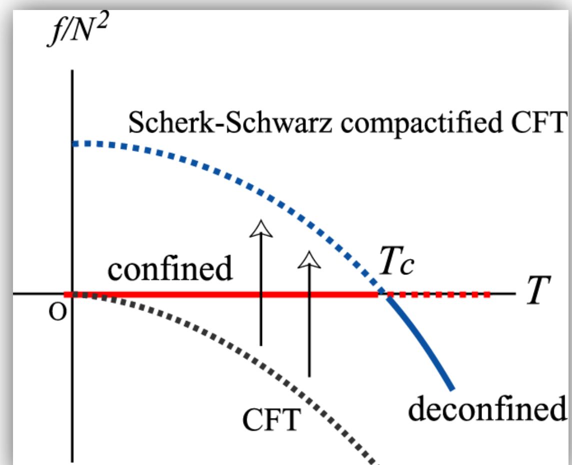
$$T^{\mu\nu}_{\text{surface}} = \sigma (n^\mu n^\nu - g^{\mu\nu}) \sqrt{\partial\Phi \cdot \partial\Phi}\, \delta(\Phi)$$

$$P = \text{const.} \qquad \kappa(z) = \frac{P}{\sigma}$$

EOS: SS compactified (d+1)-dim CFT

$$\mathcal{F} = (\rho_0 - \alpha T^{d+1}) \mathcal{V}$$

$$P = -\rho_0 + \alpha T^{d+1}$$

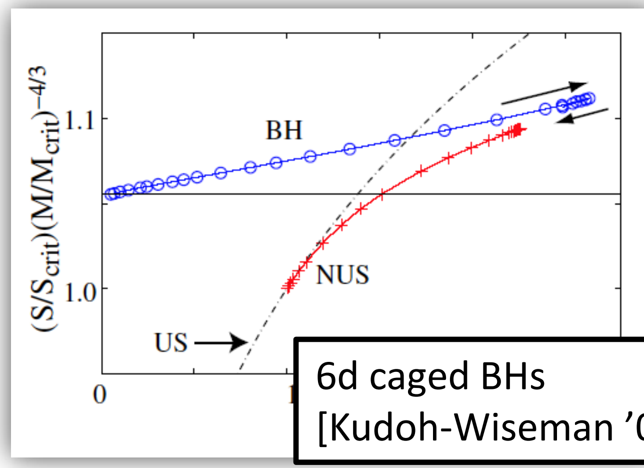


Phase diagrams in micro-canonical ensemble

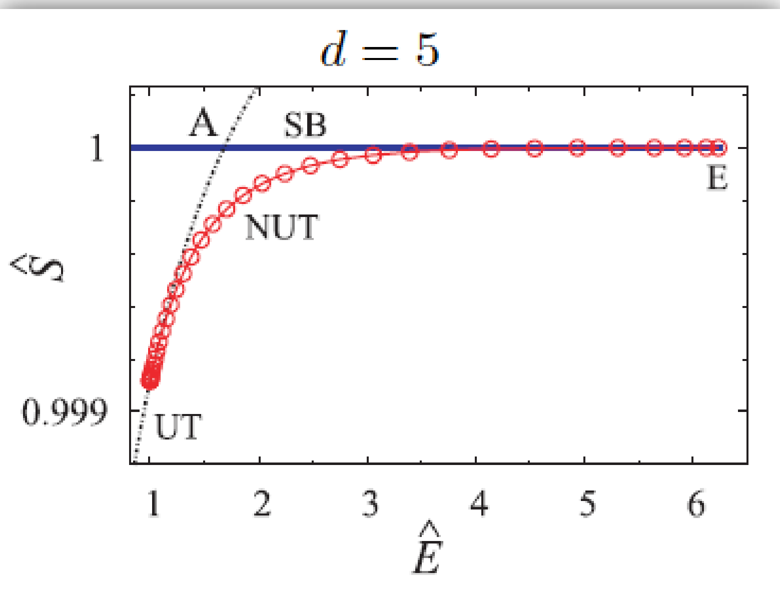
$$S_{\text{lump}} = sV$$

$$E_{\text{lump}} = \rho V + \sigma A$$

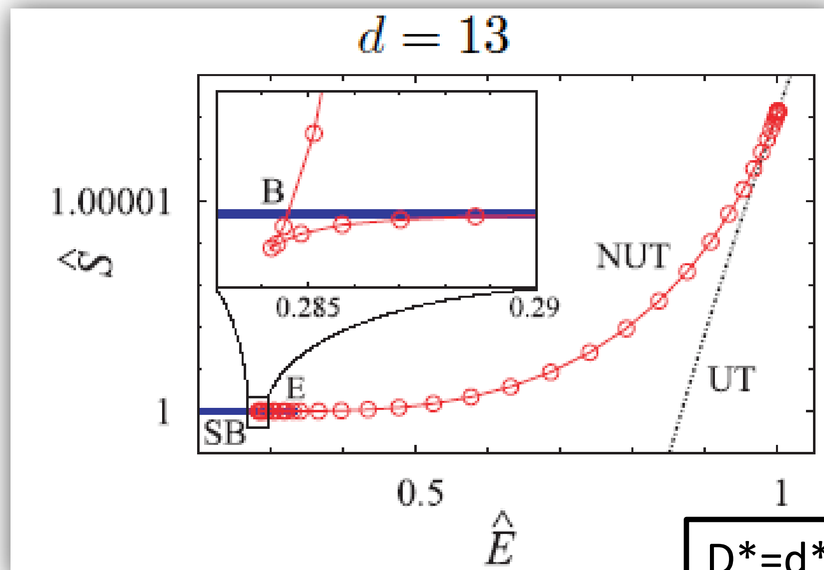
Similarity
(not duality)



6d caged BHs
[Kudoh-Wiseman '04]



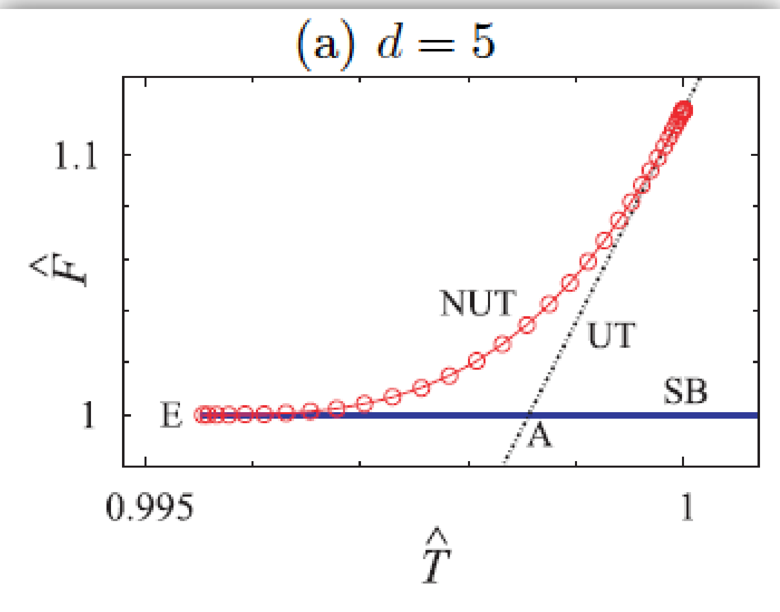
BH-BS phase diagram
In D=d+2=7 dim AdS

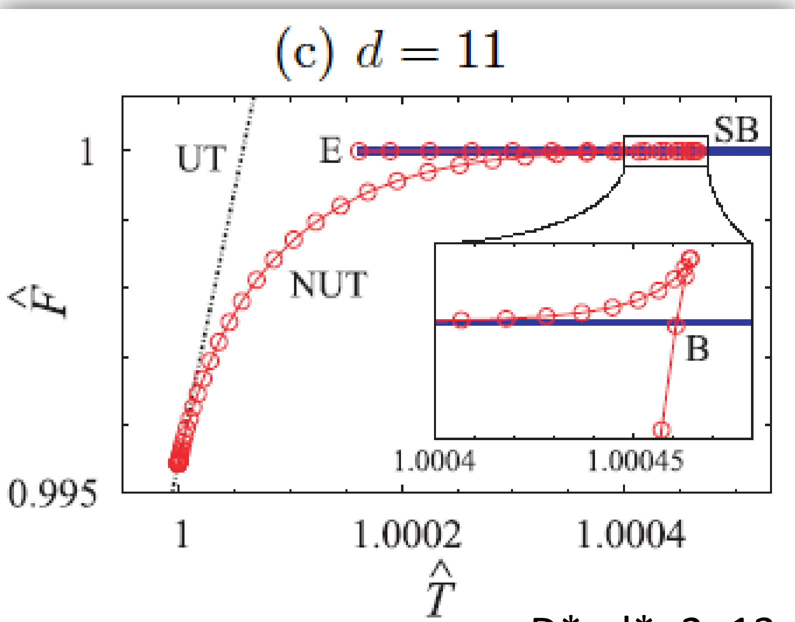


D*=d*+2=15

Phase diagrams in Canonical ensemble

$$T_{\text{lump}} = T_c \left[1 + (n+1) \frac{\sigma}{\rho_0} \kappa \right]^{1/(n+4)} \qquad F_{\text{lump}} = -PV + \sigma A$$





D*=d*+2=13

Conclusion

- Similarity b/w fluid & BH goes beyond analogy w/ Λ<0 (& SS compactif.)
  - Fluid eq. + appropriate EoS ➔ finite energy BH in AdS

Future prospects

- (In)stabilitity of Non-Uniform Tubes (w/ R.Emparan)
- Final fate(s) of RP instability (共同研究者募集中)
- Final fate(s) of GL instability (SACRA N-D…?, Crease set…?)
- Other topologies….
- …..