BLACK HOLES IN AND FROM HIGHER DIMENSIONS

- Introduction (``*In*" and ``*From*")
- From higher dimensions
  1. Stationary Spacetime with Intersecting Branes
  2. Stationary Black Holes
  3. Time Dependent Intersecting Branes
  4. Black Holes in the Universe
  5. Summary & Future Work

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With
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1. Introduction (``*In*" and ``*From*")

Unification of Fundamental Interactions

most promising candidate

Superstring / M-theory ⇒ Higher Dimensions

Strong gravitational field: very interesting

Early Universe

Black Holes

Black holes

- **The origin of BH entropy** A. Strominger and C. Vafa ('96)

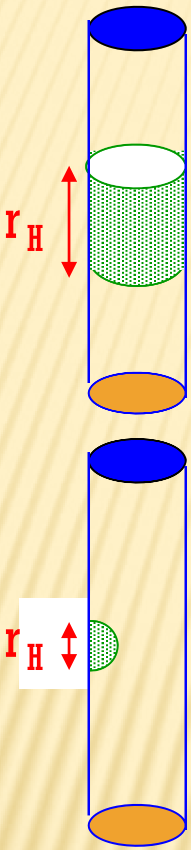
From

From black holes in 4D from higher dimensions

In black holes in higher dimensions

higher dimensional objects? or 4D objects ?

◆ compactified extra-dimensions (Kaluza-Klein type)



large BH in 4D

~ a black string in higher-dimensions

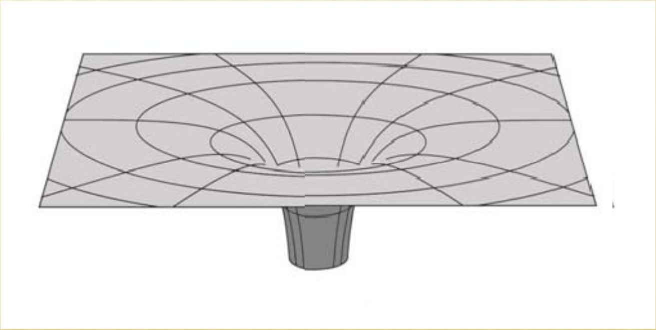
phase transition

small BH in 4D

~ a black hole in higher-dimensions

◆ brane world

~ a black hole in higher-dimensions



In higher dimensional GR

Black Holes in Higher-dimensions

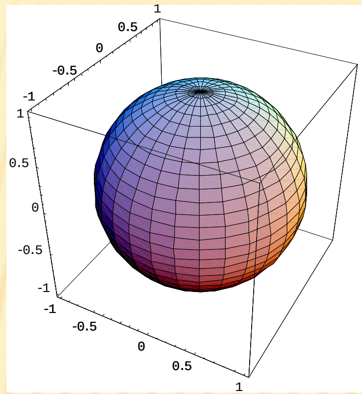
- variety of black objects

R.C. Myers and M.J. Perry (1986)

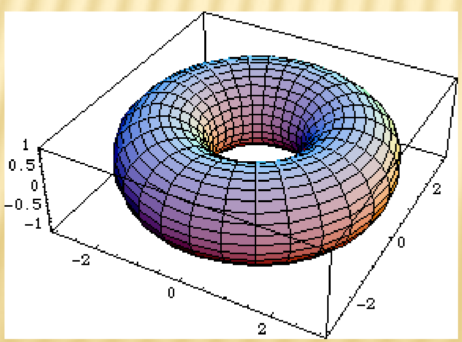
R. Emparan and H.R. Reall (2000)

H. Elvang and P. Figueras (2007)

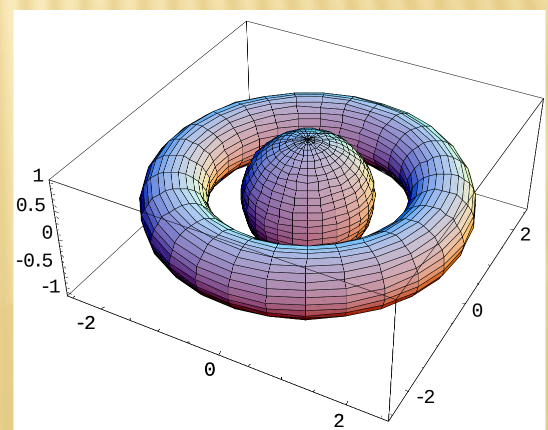
a black hole



a black ring



a black saturn



- exact solutions — inverse scattering method (5D) /other topology
- uniqueness/classification — topology/symmetry/conserved charges
- stability/other properties
- thermodynamics
- formation/evolution — numerical relativity
- evaporation

⋮

In & ***From***

dilaton

other fields in string

modification of gravity higher curvature term

ambiguity by field re-definition

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other related approaches

AdS/CFT (or *gauge/gravity*)

matrix models, fuzzball based on string theory

From higher dimensions

KM & M. Tanabe NPB738 (2006) 184

1. Stationary spacetime with branes

D-dimensional effective action

$$S = \frac{1}{16\pi G_D}\int d^D X\sqrt{-g}\left[\mathcal{R} - \frac{1}{2}(\nabla\varphi)^2 - \sum_A \frac{1}{2\cdot n_A!}e^{a_A\varphi}F_{\mathbf{n_A}}^2\right]$$

φ dilaton $F_{\mathbf{n_A}}$ n_A form fields

A: type of branes (2-brane, 5-brane etc)

Basic equations

$$\mathcal{R}_{\mu\nu} = \frac{1}{2}\partial_\mu\varphi\partial_\nu\varphi + \sum_A \Theta_{\mathbf{n_A}\mu\nu}$$

$\Theta_{\mathbf{n_A}\mu\nu}$: em tensor of $F_{\mathbf{n_A}}$

$$\nabla^2\varphi = \sum_A \frac{a_A}{2\cdot n_A!}e^{a_A\varphi}F_{\mathbf{n_A}}^2$$

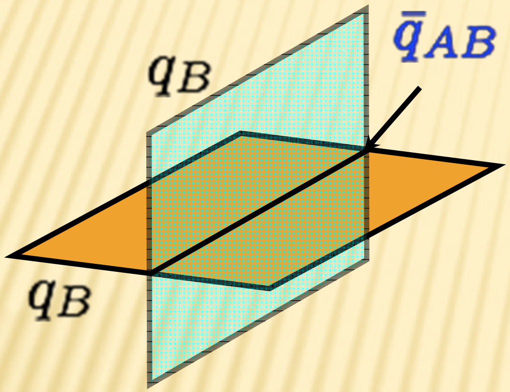
$$\partial_{\mu_1}(\sqrt{-g}\ e^{a_A\varphi}F_{\mathbf{n_A}}^{\mu_1\cdots\mu_{n_A}}) = 0$$

$$\partial_{[\mu}F_{\mathbf{n_A}\mu_1\cdots\mu_{n_A}]} = 0$$

- microscopic description by branes (10D or 11D)

S. R. Das ('96),
M. Cvetic and C. M. Hull ('88)

Branes in some dimensions → gravitational sources



BHs (Black objects) in 4 or 5 dim

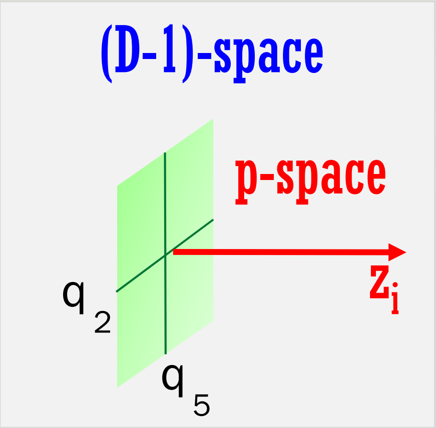
branes ~ charges

⇕

area of horizon (BH entropy)

Ansatz:

Source: Several types of branes in p-dim space



$D = d + p$

- d: t, (d−1)-space (z_i)
- p: branes (x_α)

x_1 · · · wave

uniform: smearing

metric ansatz:

$$u = \frac{1}{\sqrt{2}}(t - x_1) \qquad v = \frac{1}{\sqrt{2}}(t + x_1)$$

(1) null branes

$$ds^2 = 2e^{2\xi}\left(du + \frac{\tilde{\mathcal{A}}}{\sqrt{2}}\right)\left(dv + f du + \frac{\mathcal{A}}{\sqrt{2}}\right) + e^{2\eta}\sum_{i=1}^{d-1} ds_{d-1}^2 + \sum_{\alpha=2}^{p} e^{2\zeta_\alpha}(dx^\alpha)^2$$

ds_{d-1}^2 : Ricci flat

(2) timelike branes

$$ds^2 = -e^{2\xi}\left(dt + f dx_1 + \mathcal{A}_i dz^i\right)^2 + e^{2\eta}\sum_{i=1}^{d-1} ds_{d-1}^2 + \sum_{\alpha=2}^{p} e^{2\zeta_\alpha}(dx^\alpha)^2$$

stationary spacetime ⟹ $\xi, f, \mathcal{A}, \eta, \zeta_\alpha$ depend on z_i

- **null branes** $\tilde{A} = 0$ ds^2_{d-1} : Ricci flat

source term ・・・ charged branes (ellectric type)

$$F_{\mathbf{n_A}} = \partial_j E_A dz^j \wedge du \wedge dv \wedge dx_2 \wedge \cdots \wedge dx_{q_A}$$
$$+\frac{1}{\sqrt{2}} \partial_i B^A_j dz^i \wedge dz^j \wedge du \wedge dx_2 \wedge \cdots \wedge dx_{q_A}$$

$^*F_{\mathbf{n_A}}$ dual expression (magnetic type)

- **Assumption (BPS type relation)**

$$\partial_j \xi = \frac{1}{2(D-2)} \sum_A (D - q_A - 3) H_A^2 \tilde{E}_A \partial_j \tilde{E}_A$$
$$\partial_j \zeta_\alpha = \frac{1}{2(D-2)} \sum_A \delta_{\alpha A} H_A^2 \tilde{E}_A \partial_j \tilde{E}_A$$
$$\partial_j \varphi = -\frac{1}{2} \sum_A \epsilon_A a_A H_A^2 \tilde{E}_A \partial_j \tilde{E}_A$$

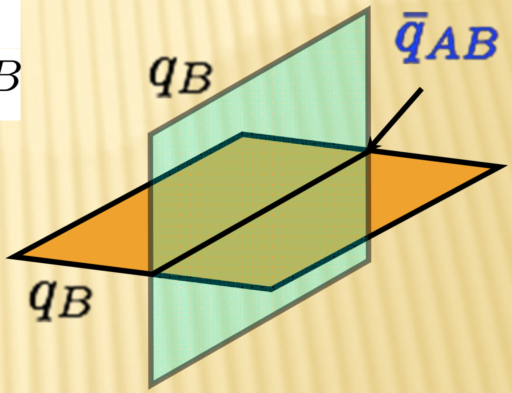
$\tilde{E}_A = E_A + 1$

Equation for η

$A \neq B$: intersection rule

$$\bar{q}_{AB} = \frac{(q_A+1)(q_B+1)}{D-2} - 1 - \frac{1}{2}\epsilon_A a_A \epsilon_B a_B$$

Interesecting dimensions



$A = B$: solution for $\tilde{E}_A$

$$\tilde{E}_A = \sqrt{\frac{2(D-2)}{\Delta_A}\frac{1}{H_A}} \quad (\text{or} \quad \tilde{E}_A = \text{const})$$

$$H_A = \exp\left[-\left(2\xi + \sum_{\alpha=\alpha_2}^{\alpha_{q_A}} \zeta_\alpha - \frac{1}{2}\epsilon_A a_A \varphi\right)\right]$$

gauge condition (V=1)

$$V = \exp\left[2\xi + (d-3)\eta + \sum_{\alpha=2}^{p} \zeta_\alpha\right]$$

$$\xi = -\sum_A \frac{D - q_A - 3}{\Delta_A} \ln H_A$$

$$\eta = \sum_A \frac{q_A + 1}{\Delta_A} \ln H_A$$

$$\zeta_\alpha = -\sum_A \frac{\delta_{\alpha A}}{\Delta_A} \ln H_A$$

$$\varphi = (D-2) \sum_A \frac{\epsilon_A a_A}{\Delta_A} \ln H_A$$

$$\Delta_A = \frac{(q_A+1)(D-q_A-3)}{D-2} + \frac{a_A^2}{2}$$

$$\partial^2 H_A = 0$$

harmonic function on R[4]

$$\partial^j \mathcal{F}_{ij} = 0$$

$\mathcal{A}_i$: vector harmonic function on R[4]

$$B_i^A = -\tilde{E}_A \mathcal{A}_i = -\sqrt{\frac{2(D-2)}{\Delta_A}} \frac{\mathcal{A}_i}{H_A}$$

$$\partial^2 f = \frac{\beta}{2} \prod_A H_A^{-\frac{2(D-2)}{\Delta_A}} \mathcal{F}_{ij}^2$$

:Poisson equation (Laplace eq. for $\beta = 0$)

$$\beta = 1 - \sum_{A'} \frac{(D-2)}{\Delta_{A'}}$$

(constant)

$H_A, \mathcal{A}_j$: (vector) harmonic functions

f : Poisson eq. (or Laplace eq.)

The general solutions are obtained by superposing the above (vector) harmonic functions

- timelike branes

We find the similar solutions e.g. for M2M2M2 brane

2. Stationary Black Holes from higher dimensions

D=11, supergravity

4-form $q_2=2$ M2 brane

dual: 7-form $q_5=5$ M5 brane

intersection rule

$M2 \cap M2 \quad \to \bar{q}_{22} = 0$ $\quad M2 \cap M5 \quad \to \bar{q}_{25} = 1$ $\quad M5 \cap M5 \quad \to \bar{q}_{55} = 3$

d=5

	t	x^1	x^2	x^3	x^4	x^5	x^6	z^1	z^2	z^3	z^4
M2	○	○	○								
M2	○			○	○						
M2	○					○	○				

	t	x^1	x^2	x^3	x^4	x^5	x^6	z^1	z^2	z^3	z^4
M2	○	○	○								
M5	○	○		○	○	○	○				
W	○	○									

black holes in "our" world

Solution in D-dim spacetime

$$ds_D^2 = \left[(1+f)\prod_A H_A^{\frac{2}{\Delta_A}[D-d-q_A(d-2)]}\right]^{-\frac{1}{d-2}} ds_d^2$$

$$+\prod_A H_A^{-2\frac{D-q_A-3}{\Delta_A}}(1+f)\left[dx_1 - \frac{1}{1+f}\left(f\,dt - \frac{\mathcal{A}}{2}\right)\right]^2 + \sum_{\alpha=2}^{p}\prod_A H_A^{-2\frac{\delta_{\alpha A}}{\Delta_A}}\, dx_\alpha^2$$

$$\delta_A^\alpha = \begin{cases} D-q_A-3 \\ -(q_A+1) \end{cases} \text{for} \begin{cases} x^\alpha \text{ belonging to } q_A\text{-brane} \\ \text{otherwise} \end{cases} .$$

compactification

Einstein frame in *d*-dimensions

$$d\bar{s}_d^2 = -\Xi^{d-3}\left(dt + \frac{\mathcal{A}}{2}\right)^2 + \Xi^{-1}\sum_{i=1}^{d-1} dz_i^2$$

$$\Xi \equiv (1+f)^{-1/(d-2)}\prod_A H_A^{-\frac{2(D-2)}{(d-2)\Delta_A}}$$

null branes

- 5 dimensional black hole

$M2 \perp M5$

	t	x^1	x^2	x^3	x^4	x^5	x^6	z^1	z^2	z^3	z^4
M2	○	○	○								
M5	○	○		○	○	○	○				
W	○	○									

$$d\bar{s}_5^2 = -\Xi^2 \left(dt + \frac{\mathcal{A}}{2} \right)^2 + \Xi^{-1} ds_{E^4}^2$$

$$\Xi = [H_2 H_5 (1+f)]^{-1/3}$$

$$\partial^2 H_2 = 0 \qquad \partial^2 H_5 = 0 \qquad \partial^2 f = 0$$

$$\partial^j \mathcal{F}_{ij} = 0$$

Hypersperical coordinates

$$z_1 + iz_2 = r\cos\theta e^{i\phi} \quad z_3 + iz_4 = r\sin\theta e^{i\psi}$$

$$ds^2_{\mathbf{E}^4} = dr^2 + r^2 d\theta^2 + r^2\cos^2\theta d\phi^2 + r^2\sin^2\theta d\psi^2$$

general solution

$$f = \sum_{\ell=0}^{\infty} \frac{a_\ell}{r^{2(\ell+1)}} P_\ell(\cos 2\theta)$$

$$H_A = 1 + \sum_{\ell=0}^{\infty} \frac{b_\ell^{(A)}}{r^{2(\ell+1)}} P_\ell(\cos 2\theta)$$

$$\mathcal{A}_\psi = \sum_{n=1}^{\infty} \frac{b_n^{(\psi)}}{r^{2n}} F(-n, n, 1, \cos^2\theta)$$

$$\mathcal{A}_\phi = \sum_{m=1}^{\infty} \frac{b_m^{(\phi)}}{r^{2m}} F(-m, m, 1, \sin^2\theta)$$

$F(\alpha, \beta, \gamma, z)$: hypergeometric function

The lowest order: 5D supersymmetric rotating BH

BMPV BH J. C. Breckenridge, R. C. Myers, A. W. Peet and C. Vafa, Phys. Lett. B391 (1993) 93.

$$d\bar{s}_5^2 = -\Xi^2 \left[dt + \frac{1}{2}\left(\mathcal{A}_\phi d\phi + \mathcal{A}_\psi d\psi\right)\right]^2$$

$$+\Xi^{-1}\left(dr^2 + r^2 d\theta^2 + r^2 \cos^2\theta d\phi^2 + r^2 \sin^2\theta d\psi^2\right)$$

$$\Xi = [H_2 H_5 (1+f)]^{-1/3}$$

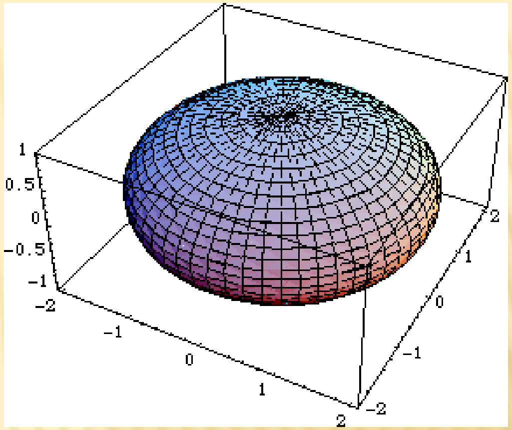
$$H_A = 1 + \frac{\mathcal{Q}_H^{(A)}}{r^2} \qquad (A = 2, 5),$$

$$f = \frac{\mathcal{Q}_0}{r^2},$$

$$\mathcal{A}_\phi = \frac{J_\phi \cos^2\theta}{r^2} \qquad \mathcal{A}_\psi = \frac{J_\psi \sin^2\theta}{r^2}$$

$$M_{\mathrm{ADM}} = \frac{\pi}{4G_5}\left(\mathcal{Q}_0 + \mathcal{Q}_H^{(2)} + \mathcal{Q}_H^{(5)}\right) \qquad S = \frac{\pi^2}{2G_5}\sqrt{\mathcal{Q}_0 \mathcal{Q}_H^{(2)} \mathcal{Q}_H^{(5)} - \frac{J^2}{8}}$$

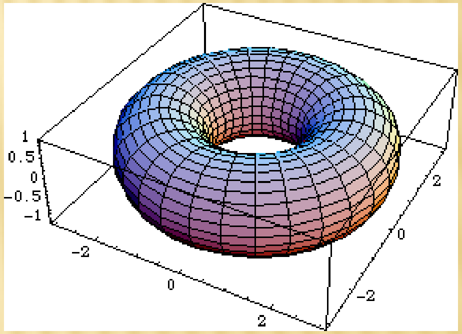
Hyperelliptical coordinates



$$ds^2_{\mathbf{E}^4} = R^2(\sinh^2\xi + \sin^2\eta)(d\xi^2 + d\eta^2)$$
$$+R^2\cosh^2\xi\cos^2\eta d\phi^2 + R^2\sinh^2\xi\sin^2\eta d\psi^2$$

the lowest order :

$$H_A = 1 + Q_A \ln\tanh\xi, \quad f = Q_w \ln\tanh\xi$$
$$\mathcal{A}_\phi = -2\sqrt{2}J_1^{(\phi)}(1 + 2\cosh^2\xi\ln\tanh\xi)\cos^2\eta$$
$$\mathcal{A}_\psi = 2\sqrt{2}J_1^{(\psi)}(1 + 2\sinh^2\xi\ln\tanh\xi)\sin^2\eta$$



Hyperpolorical coordinates

$$ds^2_{\mathbf{E}^4} = \frac{R^2}{(\cosh\xi - \cos\eta)^2}(d\xi^2 + \sinh^2\xi d\psi^2 + d\eta^2 + \sin^2 d\phi^2)$$

the lowest order :

$$H_A = Q_A(\cosh\xi - \cos\eta), \quad f = -1 + Q(\cosh\xi - \cos\eta)$$
$$\mathcal{A}_\phi = \sqrt{2}J_1^{(\phi)}\cosh\xi\sin^2\eta, \quad \mathcal{A}_\psi = \sqrt{2}J_1^{(\psi)}\sinh^2\xi\cos\eta$$

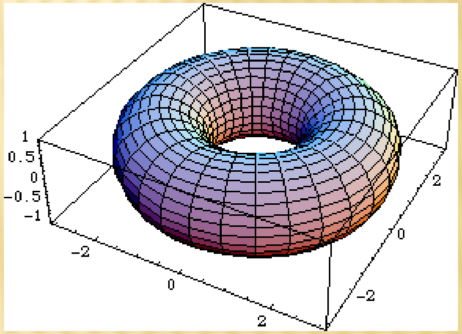
Both solutions have naked singularities.

timelike M2M2M2 branes

We find a supersymmetric black ring and a black saturn.

$$ds_4^2 = \frac{R^2}{(x-y)^2}\left[\frac{dy^2}{y^2-1} + (y^2-1)d\phi^2 + \frac{dx^2}{1-x^2} + (1-x^2)d\psi^2\right]$$

a supersymmetric black ring I. Elvang et al (2004)



$$H_A = 1 + \frac{Q_A - q_B q_C}{2R^2}(x-y) - \frac{q_B q_C}{4R^2}(x^2-y^2)$$

$$\mathcal{A}_i dx^i = \mathcal{A}_\phi d\phi + \mathcal{A}_\psi d\psi$$

$$\mathcal{A}_\phi = -\frac{1-x^2}{8R^2}\left[\sum_{I=1}^{3} q_I Q_I - q_1 q_2 q_3 (3+x+y)\right]$$

$$\mathcal{A}_\psi = \frac{1}{2}(q_1+q_2+q_3)(1+y) - \frac{y^2-1}{8R^2}\left[\sum_{I=1}^{3} q_I Q_I - q_1 q_2 q_3 (3+x+y)\right]$$

$$A^I = \frac{1}{H_I}(dt + \mathcal{A}_i dx^i) - \frac{1}{2} q_i^{(I)} dx^i$$

$$A^I = \frac{1}{H_I}(dt + \mathcal{A}_i dx^i) - \frac{q_I}{2}\left[(1+y)d\phi + (1+x)d\psi\right]$$

a supersymmetric black saturn

I. Benna and N.P. Warner (2004)

$$H_A = 1 - \frac{Q_A^{(B)}}{R^2}\frac{x-y}{x+y} + \frac{Q_A^{(R)} - q_B q_C}{2R^2}(x-y) - \frac{q_B q_C}{4R^2}(x^2-y^2)$$

$$A_\phi = (x^2-1)\left[\frac{B}{2} + \frac{C}{3}(x+y) + \frac{D}{x+y} - \frac{J_\phi^{(B)}}{R^2(x+y)^2}\right]$$

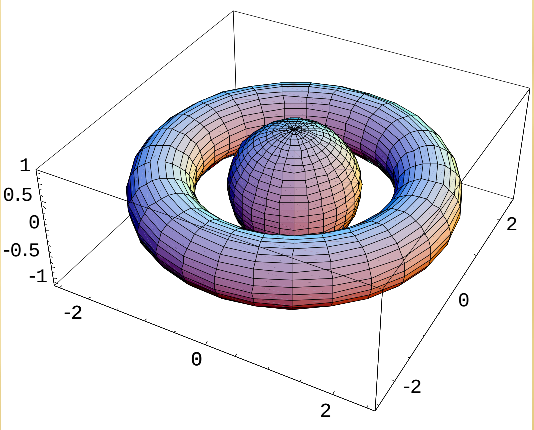
$$A_\psi = A(y+1) - (y^2-1)\left[\frac{B}{2} + \frac{C}{3}(x+y) + \frac{D}{x+y} - \frac{J_\psi^{(B)}}{R^2(x+y)^2}\right]$$

$$A = \frac{1}{2}\sum_I q_I$$

$$B = \frac{1}{4R^2}\left(\sum_I q_I Q_I^{(R)} - 3q_1q_2q_3\right)$$

$$C = -\frac{3q_1q_2q_3}{8R^2}$$

$$D = \frac{1}{2R^2}\sum_I q_I Q_I^{(B)}$$

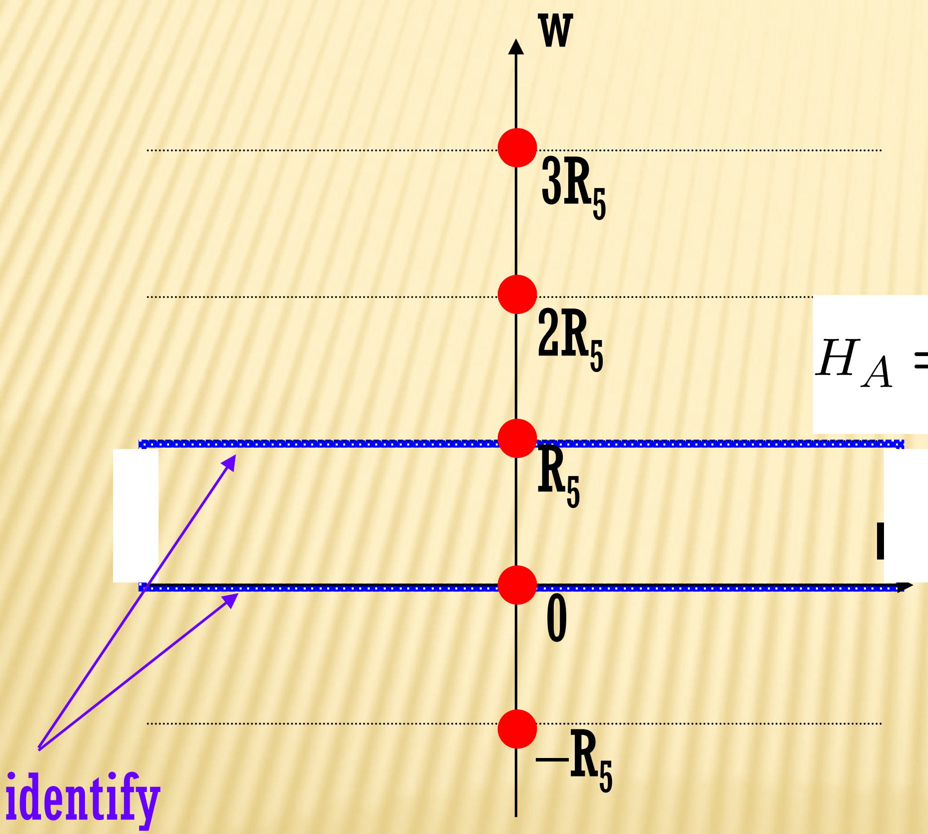


Rotating BH in a Compactified Spacetime

KM, N. Ohta, M. Tanabe PRD74 (2006) 104002

a periodic solution by superposing 5D BMPV BH $\longrightarrow$

a rotating BH in a compactified space



$$ds^2_{E^4} = dx^2 + dy^2 + dz^2 + dw^2$$

$$r^2 = x^2 + y^2 + z^2$$

$$H_A = 1 + Q_H^{(A)} \sum_{n=-\infty}^{\infty} \frac{1}{r^2 + (w + nR_5)^2}$$

$$f = Q_0 \sum_{n=-\infty}^{\infty} \frac{1}{r^2 + (w + nR_5)^2}$$

$$\mathcal{A}_\phi = J_\phi \sum_{n=-\infty}^{\infty} \frac{x^2 + y^2}{r^2 + (w + nR_5)^2}$$

$$\mathcal{A}_\psi = J_\psi \sum_{n=-\infty}^{\infty} \frac{z^2 + (w + nR_5)^2}{r^2 + (w + nR_5)^2}$$

$$F(\xi,\eta) \equiv \sum_{n=-\infty}^{\infty} \frac{1}{\xi^2 + (\eta + 2\pi n)^2} = \frac{\sinh \xi}{2\xi\left(\cosh \xi - \cos \eta\right)}$$

$$\bar{r} = r/R_5, \bar{x} = x/R_5, \bar{y} = y/R_5, \bar{z} = z/R_5, \bar{w} = w/R_5$$

$$H_A = 1 + \frac{Q_H^{(A)}}{2R_5^2} \frac{\sinh \bar{r}}{\bar{r}\left(\cosh \bar{r} - \cos \bar{w}\right)}, \qquad f = \frac{Q_0}{2R_5^2} \frac{\sinh \bar{r}}{\bar{r}\left(\cosh \bar{r} - \cos \bar{w}\right)},$$

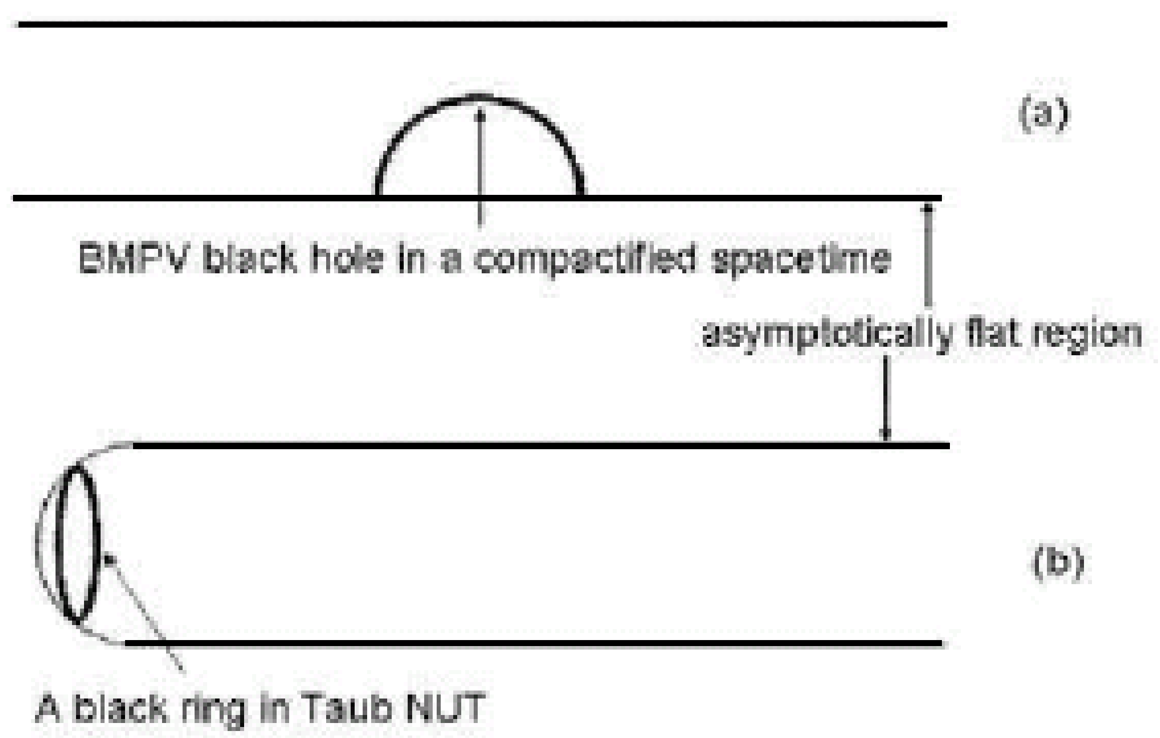
$$\mathcal{A}_\phi = \frac{J_\phi}{8R_5^2} \frac{\left(\bar{x}^2 + \bar{y}^2\right)}{\bar{r}^2} \left[\frac{\left(\cosh \bar{r} \cos \bar{w} - 1\right)}{\left(\cosh \bar{r} - \cos \bar{w}\right)^2} + \frac{\sinh \bar{r}}{\bar{r}\left(\cosh \bar{r} - \cos \bar{w}\right)}\right],$$

$$\mathcal{A}_\psi = \frac{J_\psi}{8R_5^2} \left[-\frac{\left(\bar{x}^2 + \bar{y}^2\right)}{\bar{r}^2} \frac{\left(\cosh \bar{r} \cos \bar{w} - 1\right)}{\left(\cosh \bar{r} - \cos \bar{w}\right)^2} + \frac{\left(\bar{r}^2 + \bar{z}^2\right)}{\bar{r}^2} \frac{\sinh \bar{r}}{\bar{r}\left(\cosh \bar{r} - \cos \bar{w}\right)}\right]$$

$$d\bar{s}_5^2 = -\Xi^2 \left[dt + \frac{1}{2}\left(\mathcal{A}_\phi d\phi + \mathcal{A}_\psi d\psi\right)\right]^2 + \Xi^{-1} ds_{E^4}^2$$

$$\Xi = \left[H_2 H_5 (1 + f)\right]^{-1/3}$$

ours

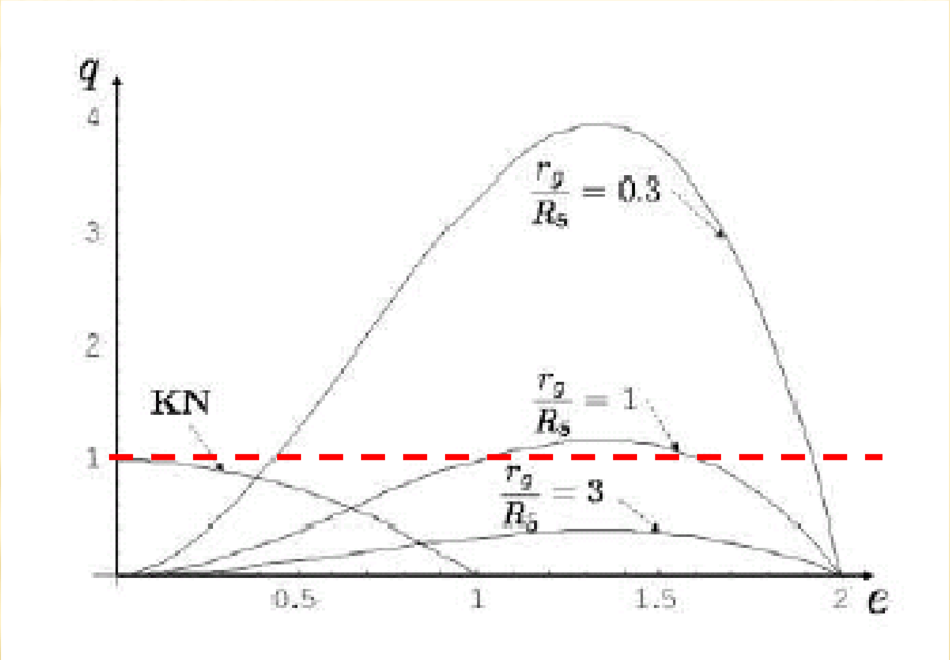


cf

a black ring in Taub NUT space ➡ effectively 4D rotating BH

H. Elvang, R. Emparan, D. Mateos and H.S. Reall (2005)
D. Gaiotto, A. Strominger and X. Yin (2005).
I. Bena, P. Kraus and N.P. Warner (2005)

q >1 is possible for small black holes $q = \frac{a}{G_4 M}$



cf Kerr-Newman BH $q < 1$

3. TIME DEPENDENT INTERSECTING BRANES

Time dependent intersecting in 10D or 11D

⇨ **time-dependence in 4D or 5D spacetime ?**

Cosmology

Time dependent Black Holes

Hawking evaporation ??

D-dimensional effective action

P. Binetruy, M. Sasaki , K. Uzawa (07)
KM, N. Ohta, K. Uzawa (09).
KM, N. Ohta, M. Tanabe, and R. Wakebe (09)

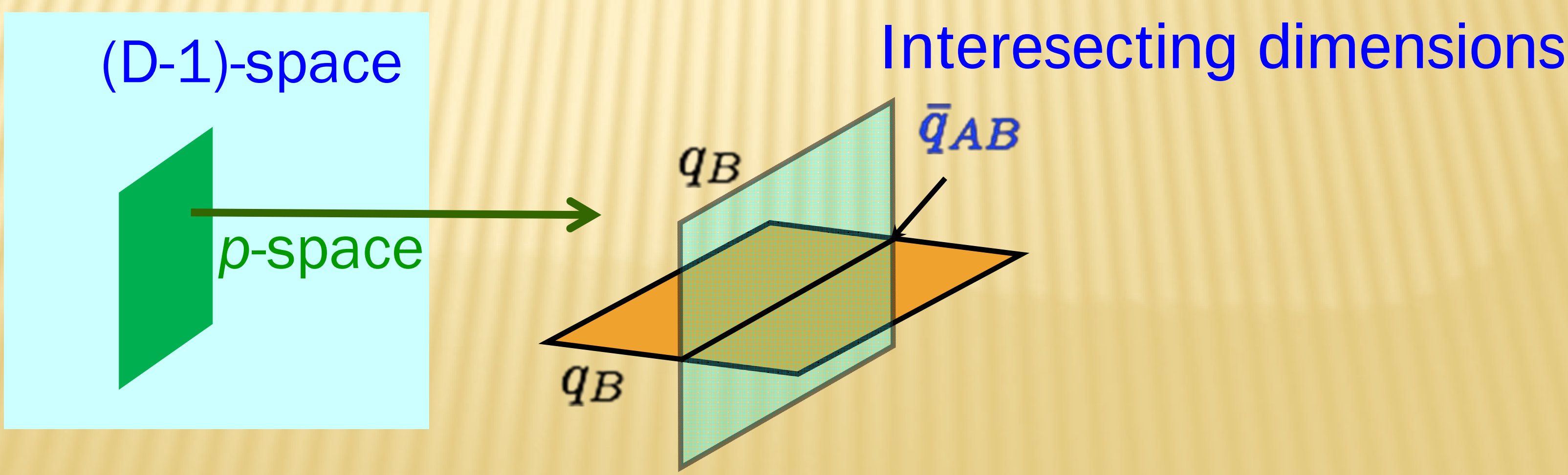
$$S = \frac{1}{2\kappa^2}\int d^D X\sqrt{-g}\left[R - \frac{1}{2}(\nabla\phi)^2 - \sum_A \frac{1}{2\cdot n_A!} e^{a_A\phi} F_{n_A}^2\right]$$

ϕ : **dilaton** F_{n_A} ***: n_A* form fields**

A: type of branes (2-brane, 5-brane etc)

Ansatz:

Source: Several types of branes in *p*-dim space



time dependence

branes

$$ds^2 = -\prod_A h_A^{-\frac{D-q_A-3}{D-2}}(t,z)dt^2 + \sum_{\alpha=1}^{p}\prod_A h_A^{\frac{\delta_A^\alpha}{D-2}}(t,z)(dx^\alpha)^2(X)$$

$$+\prod_A h_A^{\frac{q_A+1}{D-2}}(t,z)u_{ij}(Z)dz^i dz^j$$

$$\delta_A^\alpha = \begin{cases} D-q_A-3 \\ -(q_A+1) \end{cases} \text{for} \begin{cases} x^\alpha \text{ belonging to } q_A\text{-brane} \\ \text{otherwise} \end{cases}.$$

Forms

$$F_{(q_A+2)} = d(h_A^{-1}) \wedge \Omega(X_A)$$

One brane ($\tilde{A}$) can be time dependent

$$h_{\tilde{A}} = at + H_{\tilde{A}} \qquad \Delta_Z H_{\tilde{A}} = 0$$

$$h_A = H_A \quad (A \neq \tilde{A}) \qquad \Delta_Z H_A = 0$$

$$R_{ij}(Z) = 0$$ Ricci flat

4. Black Holes in an Expanding Universe

Time-dependent black hole ?

M2M2M5M5 4 charges (4 branes)

	0	1	2	3	4	5	6	7	8	9	10
	t	x^1	x^2	x^3	x^4	x^5	x^6	x^7	z^1	z^2	z^3
M2	○	○	○								
M2	○			○	○						
M5	○	○		○		○	○	○			
M5	○		○		○	○	○	○			

compactification

our 3-space

Intersecting brane solution

spherical symmetry in our space

$$ds^2 = h_2^{1/3} h_5^{2/3} h_{2'}^{1/3} h_{5'}^{2/3} \left\{ -h_2^{-1} h_5^{-1} h_{2'}^{-1} h_{5'}^{-1} dt^2 \right.$$

$$+ h_5^{-1} h_{5'}^{-1} \left[(dx^1)^2 + (dx^2)^2 + (dx^3)^2 \right]$$

$$+ h_2^{-1} h_5^{-1} (dx^4)^2 + h_{2'}^{-1} h_5^{-1} (dx^5)^2$$

$$+ h_2^{-1} h_{5'}^{-1} (dx^6)^2 + h_{2'}^{-1} h_{5'}^{-1} (dx^7)^2$$

$$\left. + \left(dr^2 + r^2 d\Omega^2 \right) \right\}$$

1 time-dependent brane

$$h_T = \frac{t}{t_0} + \frac{Q_T}{r}$$

harmonics

3 static branes

$$h_S = 1 + \frac{Q_S}{r} \qquad h_{S'} = 1 + \frac{Q_{S'}}{r} \qquad h_{S''} = 1 + \frac{Q_{S''}}{r}$$

compactification

$$ds^2 = -\Xi dt^2 + \Xi^{-1}\left(dr^2 + r^2 d\Omega^2\right)$$

isotropic coordinates

$$\Xi = (H_T H_S H_{S'} H_{S''})^{-1/2}$$

$$H_T = \frac{t}{t_0} + \frac{Q_T}{r} \qquad H_A = 1 + \frac{Q_A}{r}, \quad A = S, S', S''$$

$$\frac{t}{t_0} = \left(\frac{\bar{t}}{\bar{t}_0}\right)^{4/3}$$

$$ds^2 = -\bar{\Xi} d\bar{t}^2 + \frac{a^2(\bar{t})}{\bar{\Xi}}\left(dr^2 + r^2 d\Omega^2\right)$$

$$a = \left(\frac{\bar{t}}{\bar{t}_0}\right)^{1/3}$$

$$\bar{\Xi} = (\bar{H}_T H_S H_{S'} H_{S''})^{-1/2} \qquad \bar{H}_T = 1 + \frac{\bar{Q}_T(\bar{t})}{r} \qquad \bar{Q}_T = \frac{Q_T}{a^4(\bar{t})}$$

same charges $Q_T = Q_S = Q_{S'} = Q_{S''} = Q$

- **static** $a = 1$

Extreme RN BH

$$ds^2 = -H^{-2}dt^2 + H^2\left(dr^2 + r^2 d\Omega^2\right) \qquad H = 1 + \frac{Q}{r}$$

$$\bar{r} = r + Q$$

$$ds^2 = -\left(1 - \frac{Q}{\bar{r}}\right)^2 dt^2 + \frac{1}{\left(1 - \frac{Q}{\bar{r}}\right)^2} d\bar{r}^2 + \bar{r}^2 d\Omega^2$$

$r = 0 \ (\bar{r} = Q)$: horizon

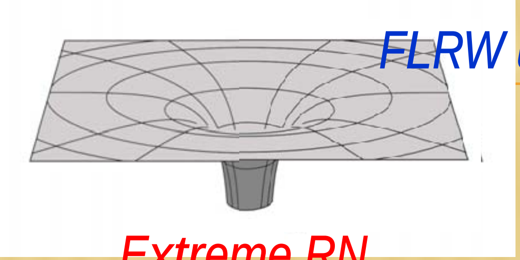
- $r \to \infty$

$$ds^2 = -d\bar{t}^2 + a^2(\bar{t})\left(dr^2 + r^2 d\Omega^2\right)$$

$$a = \left(\frac{\bar{t}}{\bar{t}_0}\right)^{1/3}$$

The FLRW universe with stiff matter

⇨ **The BH in the Universe ?**



$$ds_4^2 = -\Xi\, dt^2 + \frac{1}{\Xi}\left(dr^2 + r^2 d\Omega_2^2\right)$$

$$\Xi = \left(\frac{t}{t_0} + \frac{Q}{r}\right)^{-1/2}\left(1 + \frac{Q}{r}\right)^{-3/2}$$

$$ds_4^2 = -\bar{\Xi}\, d\bar{t}^2 + \frac{a^2}{\bar{\Xi}}\left(dr^2 + r^2 d\Omega_2^2\right)$$

$$\bar{\Xi} = \left(1 + \frac{Q}{a^4 r}\right)^{-1/2}\left(1 + \frac{Q}{r}\right)^{-3/2} \qquad a = \left(\frac{\bar{t}}{\bar{t}_0}\right)^{1/3}$$

This is an exact solution of the Einstein-*Maxwell*-scalar system

$$\mathcal{S} = \int d^4x \sqrt{-g}\left[\frac{1}{2\kappa^2}\mathcal{R} - \frac{1}{2}(\nabla\Phi)^2 - \frac{1}{16\pi}\sum_A e^{\lambda_A \kappa \Phi}(F_{\mu\nu}^{(A)})^2\right]$$

$$\lambda_T = \sqrt{6} \qquad \lambda_S = \lambda_{S'} = \lambda_{S''} = -\frac{\sqrt{6}}{3}$$

$$\kappa\Phi = \frac{\sqrt{6}}{4} \ln \left(\frac{H_T}{H_S} \right)$$

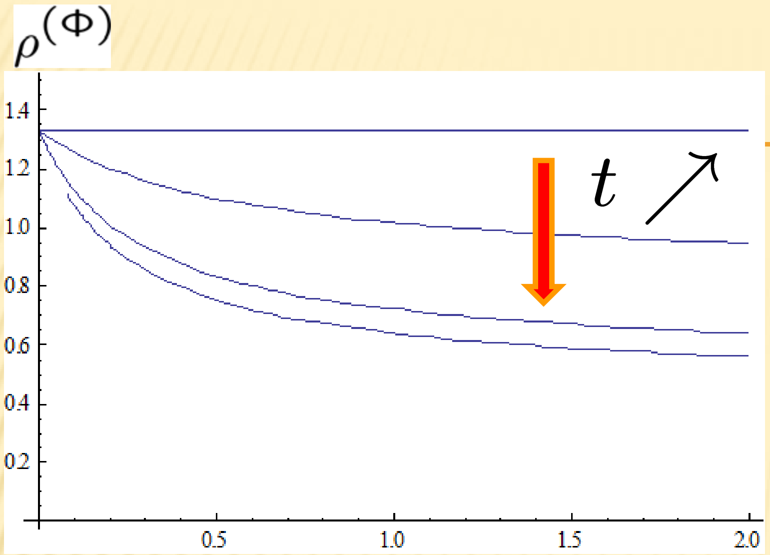
$$\kappa A_0^{(T)} = \frac{\sqrt{2\pi}}{H_T}, \quad \kappa A_0^{(S)} = \frac{\sqrt{2\pi}}{H_S}$$

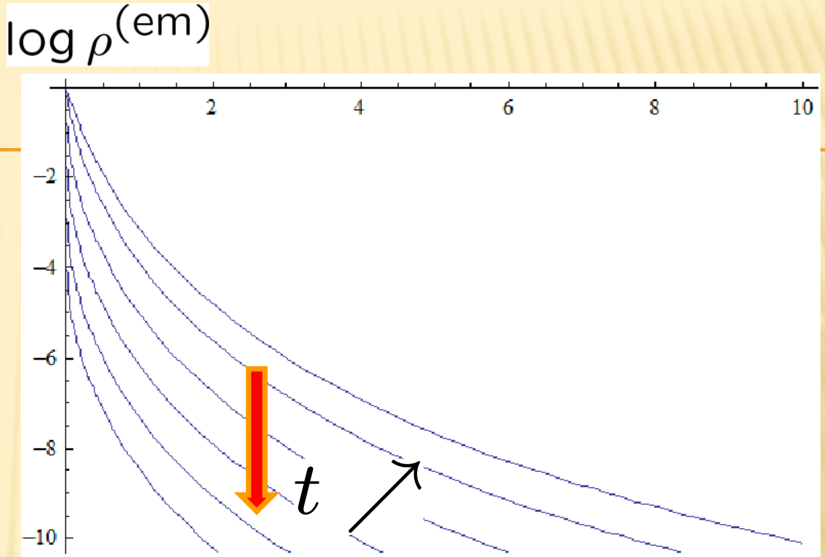
$$\kappa F_{01}^{(T)} = -\sqrt{2\pi}\frac{Q}{r^2 H_T^2}, \quad \kappa F_{01}^{(S)} = -\sqrt{2\pi}\frac{Q}{r^2 H_S^2}$$

$$\rho^{(\Phi)} = P_r^{(\Phi)} = \frac{3}{16\kappa^2}\left[\frac{1}{t_0^2}\left(\frac{H_S}{H_T}\right)^{3/2} + \frac{Q^2}{r^4}\frac{(H_T - H_S)^2}{H_T^{5/2} H_S^{7/2}}\right]$$

$$P_\theta^{(\Phi)} = P_\phi^{(\Phi)} = \frac{3}{16\kappa^2}\left[\frac{1}{t_0^2}\left(\frac{H_S}{H_T}\right)^{3/2} - \frac{Q^2}{r^4}\frac{(H_T - H_S)^2}{H_T^{5/2} H_S^{7/2}}\right]$$

$$\rho^{(\mathrm{em})} = -P_r^{(\mathrm{em})} = P_\theta^{(\mathrm{em})} = P_\phi^{(\mathrm{em})} = \frac{Q^2}{4\kappa^2 r^4}\left[\frac{1}{H_T^4}\left(\frac{H_T}{H_S}\right)^{3/2} + \frac{3}{H_S^4}\left(\frac{H_T}{H_S}\right)^{-1/2}\right]$$





Energy flux: ingoing $\rightarrow$
scalar field energy is falling toward the black hole

$$\kappa^2 T^{\hat{0}}{}_{\hat{1}} = \kappa^2 T^{(\Phi)\hat{0}}{}_{\hat{1}} = -\frac{3Q}{2t_0}\frac{(H_T - H_S)}{r^2 H_T^2 H_S} < 0$$

However, it never gets into the black hole.

global structure

KM & M. Nozawa arXiv:0912.2811[hep-th]

$$d\tilde{s}_4^2 = -\tau^2 \tilde{\Xi} d\tilde{t}^2 + \tilde{\Xi}^{-1} d\tilde{r}^2 + \tilde{R}^2 d\Omega_2^2$$

$$d\tilde{s}^2 = ds^2 / Q^2$$

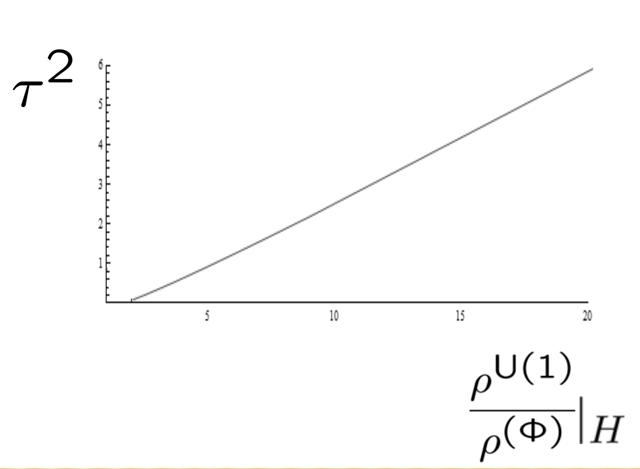
$$\tilde{\Xi} = \frac{\tilde{r}^2}{\tilde{R}^2} \qquad \tau = \frac{t_0}{Q} \qquad \tilde{r} = \frac{r}{Q} \qquad \tilde{t} = \frac{t}{t_0}$$

circumference radius

$$R^2 = (1 + \tilde{t}\tilde{r})^{1/2} (1 + \tilde{r})^{3/2}$$

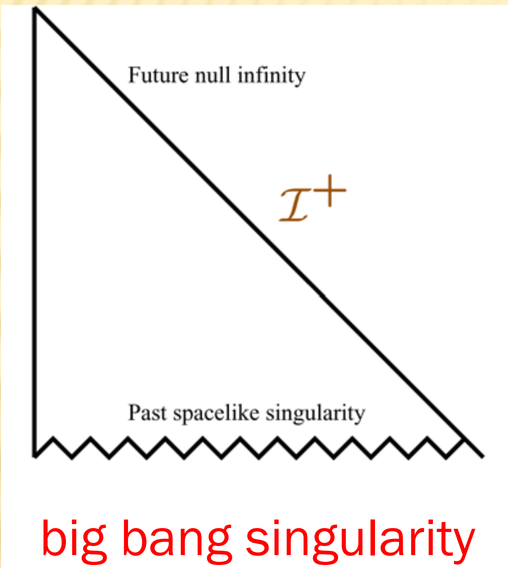
horizon radius

$$R_\pm^2 = \frac{\pm 1 + \sqrt{1 + 4\tau^2}}{2\tau}$$



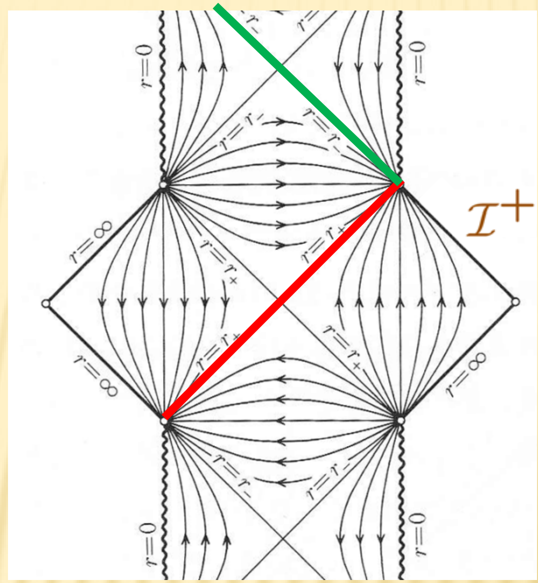
Penrose diagram

FLRW spacetime

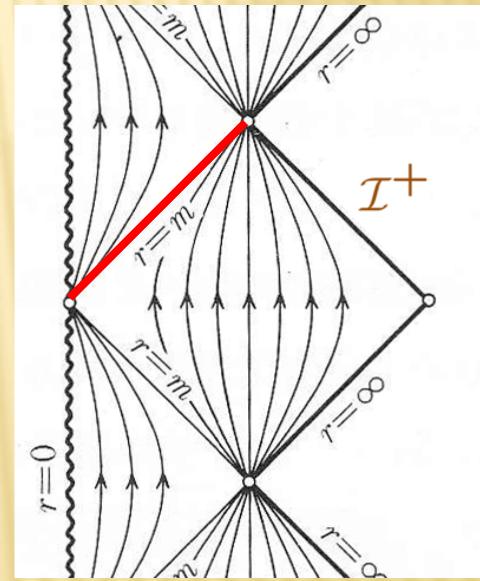


$(P = \rho)$

RN spacetime

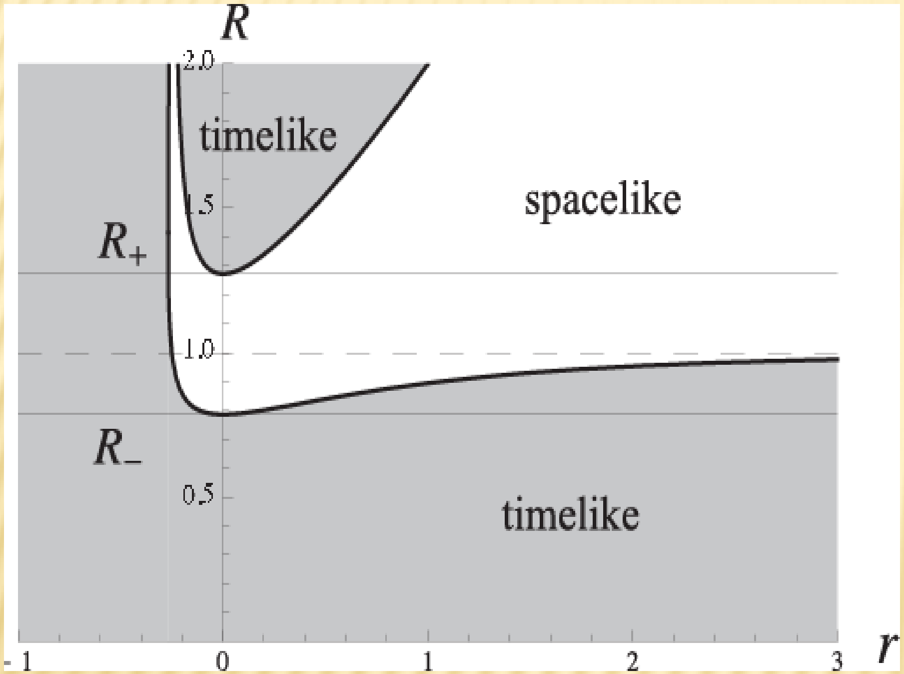


non-extreme

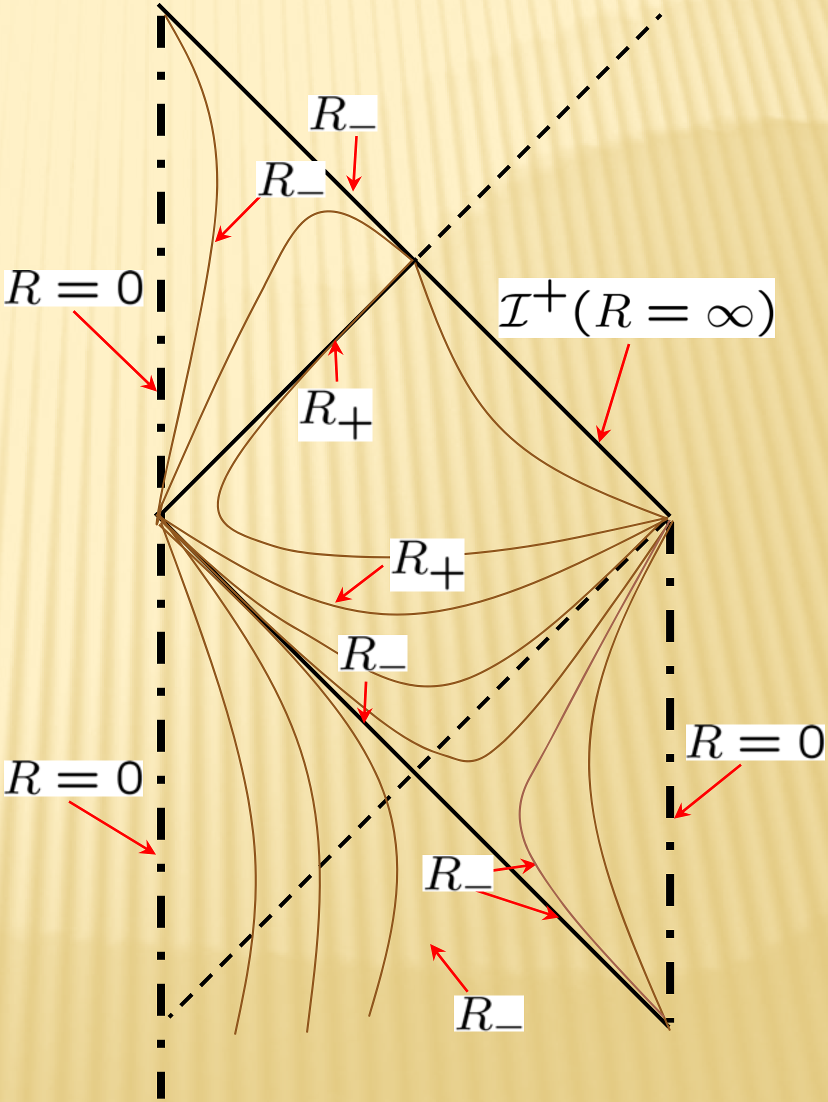


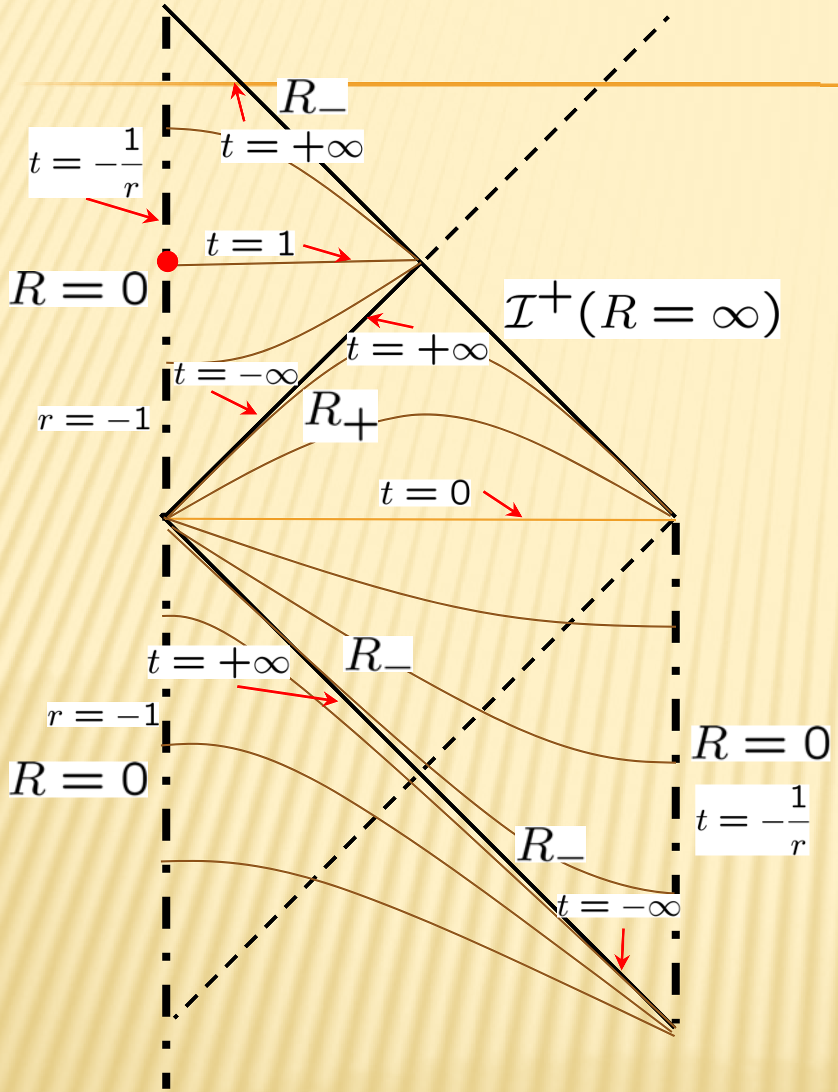
extreme

R=constant curve

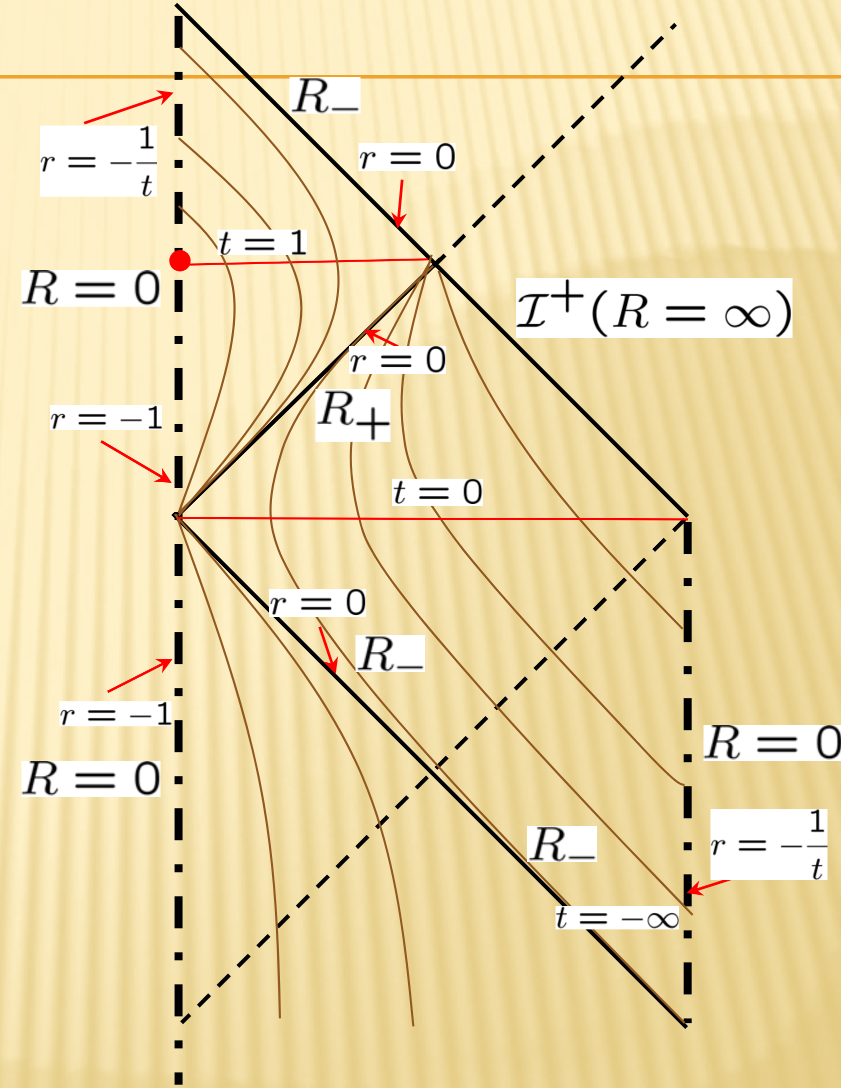


R = constant





t = constant



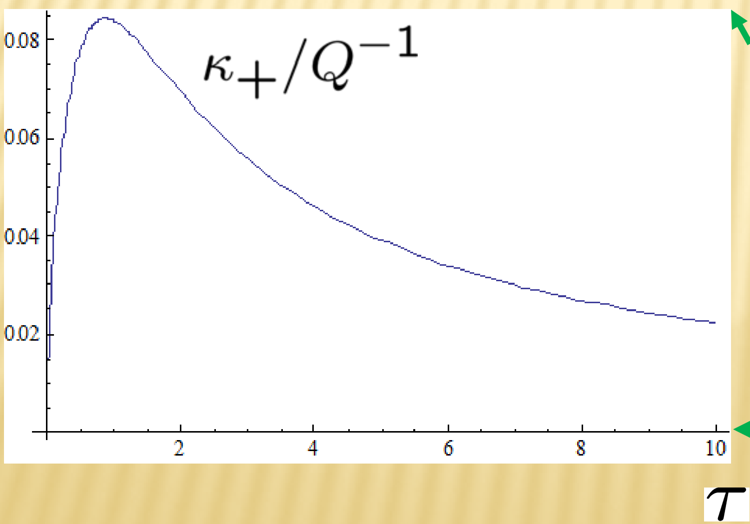
r = constant

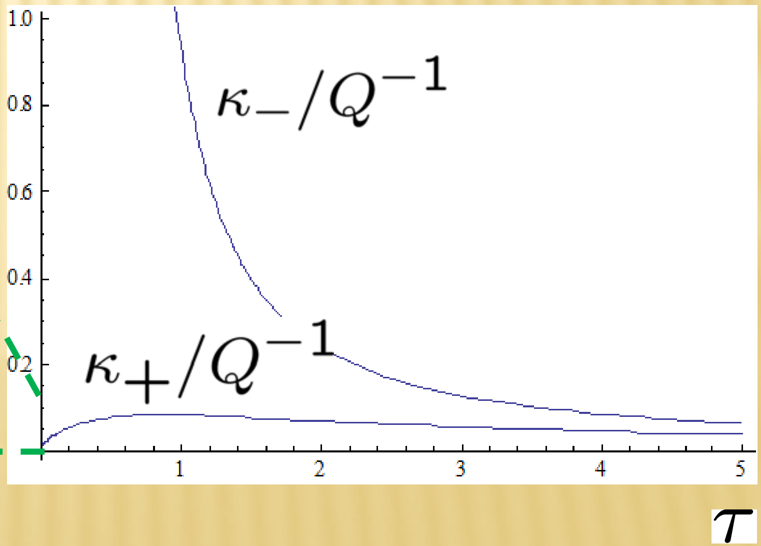
surface gravity (~temperature)

$$T = \frac{\kappa}{2\pi}$$

$$K^A = -\epsilon^{AB}\nabla_B R \quad \Rightarrow \quad K_{[A;B]}K^B = \kappa K_A$$

$$\kappa_\pm = \frac{\tau^{1/2}(1+4\tau^2)^{1/2}}{\sqrt{2}\left(\pm 1 + \sqrt{1+4\tau^2}\right)^{5/2}} \times \frac{1}{Q}$$





BH in the expanding universe with arbitrary expansion law

G.W. Gibbons, KM arXiv:0912.2809

$$ds^2 = -U^{-2}d\bar{t}^2 + U^2a^2d\vec{x}^2$$

Kastor-Traschen

$$U = 1 + \sum_i \frac{Q_i}{ar_i} \qquad a = e^{H\bar{t}} \qquad r_i = |\mathbf{r} - \mathbf{x}_i|$$

Intersecting branes (M2M2M5M5)

$$U = \left(1 + \sum_i \frac{Q_i}{a^4 r_i}\right)^{1/4} \left(1 + \sum_i \frac{Q_i}{r_i}\right)^{3/4} \qquad a = \bar{t}^{1/3}$$

extension to arbitrary power

$$U = \left(1 + \sum_i \frac{Q_i}{a^{n+1} r_i}\right)^{\frac{1}{n+1}} \left(1 + \sum_i \frac{Q_i}{r_i}\right)^{\frac{n}{n+1}}$$

$a = \bar{t}^p$ $\quad p = \frac{1}{n}$

Intersecting branes $n = 3$ Kastor-Traschen $n = 0$

---

background $r \to \infty$

$$ds^2 = -d\bar{t}^2 + a^2(\bar{t})\left(dr^2 + r^2 d\Omega^2\right)$$

$$a = \left(\frac{\bar{t}}{\bar{t}_0}\right)^p$$

The FLRW universe with EOS $P = w\rho$

$$p = \frac{1}{n} = \frac{2}{3(w+1)} \qquad w = \frac{2n-3}{3}$$

scalar field → power law expansion

exponential potential $V = V_0 \exp^{-\alpha\kappa\Phi}$

$$p = \frac{2}{\alpha^2}\left(= \frac{1}{n}\right)$$

brane type

$$\frac{t}{t_0} = \left(\frac{\bar{t}}{\bar{t}_0}\right)^{\frac{n+1}{n}}$$

$$ds^2 = -\Xi dt^2 + \Xi^{-1}\left(dr^2 + r^2 d\Omega^2\right)$$

$$\Xi = H_T^{-\frac{n_T}{2}} H_S^{-\frac{n_S}{2}}$$

$$H_T = \frac{t}{t_0} + \sum_i \frac{Q_i^{(T)}}{r_i}, \quad H_S = 1 + \sum_i \frac{Q_i^{(S)}}{r_i}$$

$$n_T = \frac{4}{n+1}, n_S = \frac{4n}{n+1}$$

$$n_T + n_S = 4$$

This metric with

$$\kappa\Phi = \frac{1}{2}\sqrt{\frac{n_T n_S}{2}}\ln\left(\frac{H_T}{H_S}\right) \qquad \kappa^2 V_0 t_0^2 = \frac{n_T(n_T-1)}{4}$$

$$\kappa A_0^{(T)} = \frac{\sqrt{2\pi}}{H_T}, \quad \kappa A_0^{(S)} = \frac{\sqrt{2\pi}}{H_S}$$

is an exact solution of the following system

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}R - \frac{1}{2}(\nabla\Phi)^2 - V(\Phi) - \frac{1}{16\pi}\sum_{A=T,S} g_A e^{\lambda_A\kappa\Phi}(F_{\mu\nu}^{(A)})^2\right]$$

$$g_T = n_T \quad , \quad g_S = n_S \qquad \lambda_T = \sqrt{\frac{2n_S}{n_T}} \quad , \quad \lambda_S = -\sqrt{\frac{2n_T}{n_S}}$$

$$V = V_0 \exp^{-\alpha\kappa\Phi} \qquad \alpha = \sqrt{\frac{2n_S}{n_T}}$$

Type	n_T	n_S	The coefficients in the action					Models of the Universe			$\kappa^2 V_0 t_0^2$
			α	g_T	g_S	λ_T	λ_S	p	(expansion law)	w	
I	4	0	0	4	0	0	$-\infty$	∞	(**de Sitter**)	-1	3
	3	1	$\sqrt{6}/3$	3	1	$\sqrt{6}/3$	$-\sqrt{6}$	3	(**quintessence**)	$-7/9$	$3/2$
II	2	2	$\sqrt{2}$	2	2	$\sqrt{2}$	$-\sqrt{2}$	1	(**Milne**)	$-1/3$	$1/2$
III	1	3	1	3	$\sqrt{6}$	$\sqrt{6}$	$-\sqrt{6}/3$	$1/3$	(**stiff matter**)	1	0
	0	4	∞	0	4	∞	0	0	(**static**)	0	0
	$8/5$	$12/5$	$\sqrt{3}$	$8/5$	$12/5$	$\sqrt{3}$	$-2/\sqrt{3}$	$2/3$	(dust)	0	$6/25$
	$4/3$	$8/3$	2	$4/3$	$8/3$	2	-1	$1/2$	(radiation)	$1/3$	$1/9$

expansion law $a = \left(\frac{\bar{t}}{\bar{t}_0}\right)^p$

equation of state $P = w\rho$ $\quad -1 \le w \le 1 \quad 4 \ge n_T \ge 1$

Single BH

circumference radius

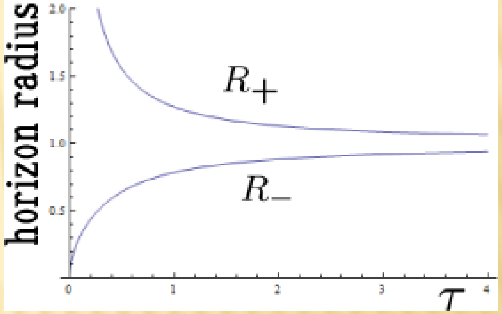
horizon radius

$$\tilde{R}^2 = (1+\tilde{t}\tilde{r})^{\frac{n_T}{2}} (1+\tilde{r})^{\frac{n_S}{2}}$$

$$\tau(\tilde{R}^{4/n_T} - 1) = \pm \tilde{R}^2$$

- Type I — decelerating expansion

$1 \le n_T < 2$ — 2 roots $R_\pm$

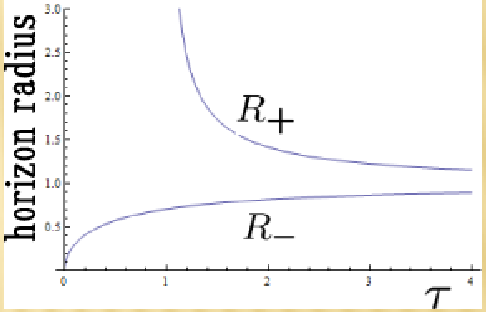


- Type II — Milne

$n_T = 2$

$\tau > 1$ — 2 roots $R_\pm$

$\tau \le 1$ — 1 root R_H



- Type III — accelerating expansion

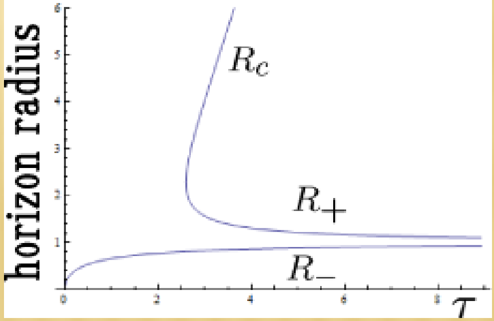
$2 < n_T \le 4$

$$\tau_{\rm cr} = \frac{1}{2} n_T^{\frac{n_T}{2}} (n_T - 1)^{-\frac{(n_T-2)}{2}}$$

$\tau > \tau_{\rm cr}$ — 3 roots $R_\pm, \; R_c$

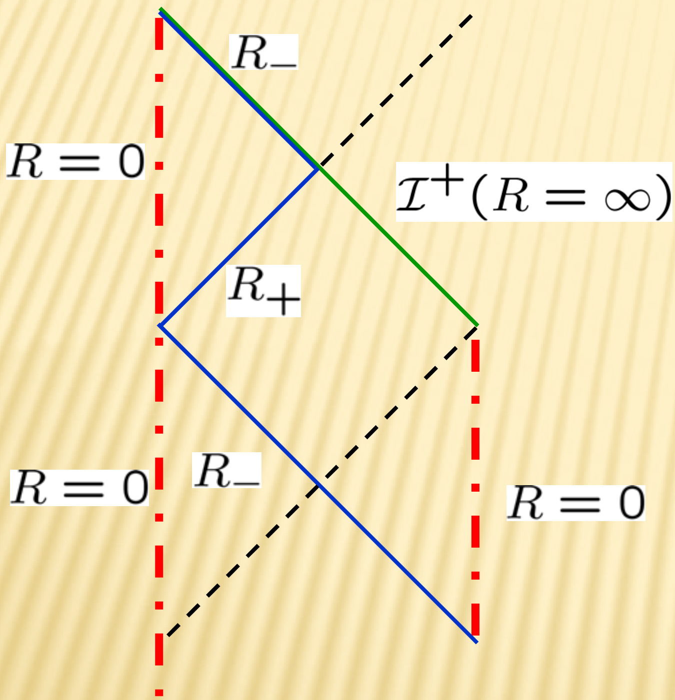
$\tau = \tau_{\rm cr}$ — 2 roots $R_H, \; R_c$

$\tau < \tau_{\rm cr}$ — *1 root* R_c

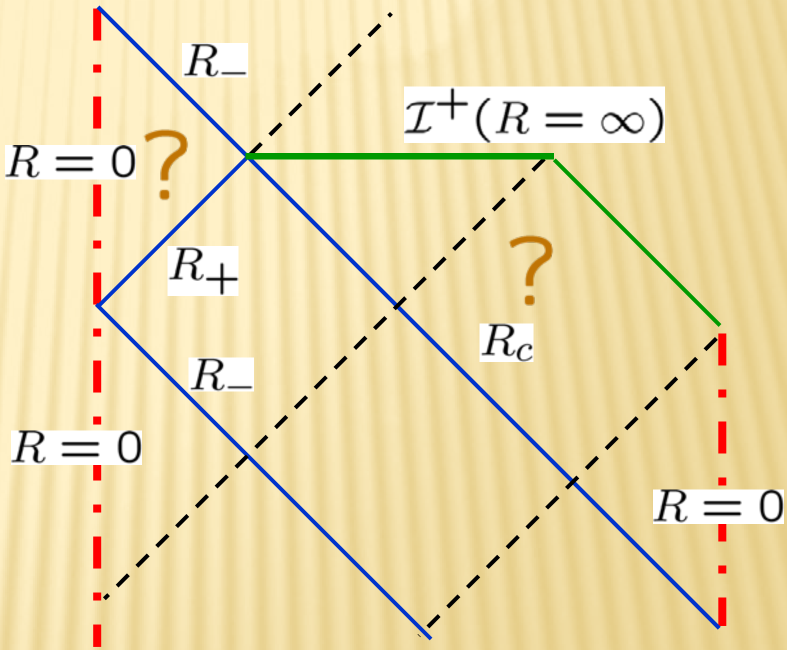


Penrose diagram (*under investigation*)

decelerating (–1/3<w<1)



Accelerating (–1<w<–1/3)

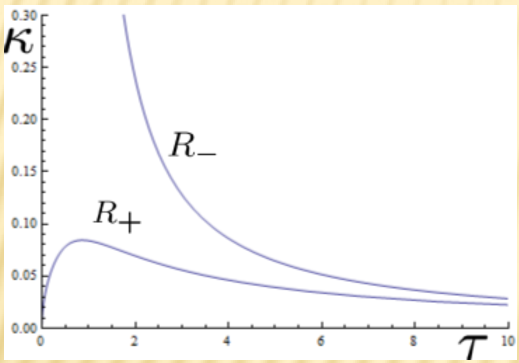


surface gravity (~ temperature)

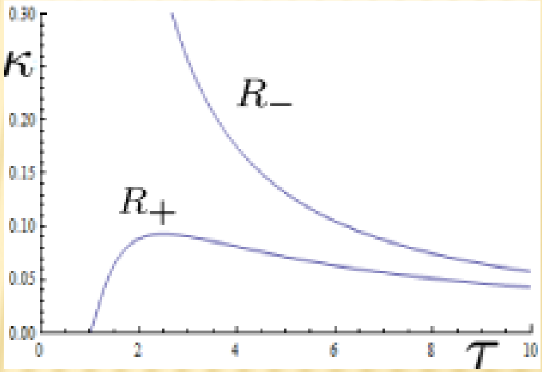
$$T = \frac{\kappa_H}{2\pi}$$

$$\kappa_\pm = \frac{n_T \left| 2\tau \tilde{R}_\pm^{\frac{n_S}{n_T}-1} \mp n_T \right|}{8\tau^2 \tilde{R}_\pm^{\frac{2n_S}{n_T}-1}}$$

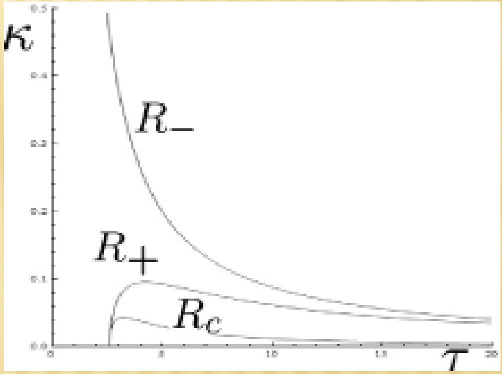
$$\tau(R_\pm^{4/n_T} - 1) = \pm R_\pm^2$$



$n_T = 1$



$n_T = 2$



$n_T = 3$

The origin of the exponential potential ?

From higher dimensions

n_T : integer

M2M2M5M5 intersecting branes

n_T timdependent branes

n_S static branes

$$n_T + n_S = 4$$

+

The VEV of 4 form field in 4D spacetime (Freund-Rubin)

$$\overset{(4)}{F}_{\mu\nu\rho\sigma} = F_0 \gamma^{-3/2} \overset{(4)}{\epsilon}_{\mu\nu\rho\sigma}$$

γ det. of the internal space metric

previous action

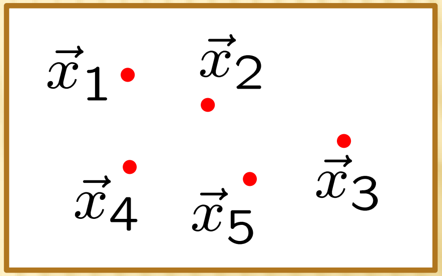
$$\kappa^2 V_0 = F_0^2/2$$

Collision of multi black holes ?

Kastor-Traschen (1993)

Multi-Extreme RN BHs

BHs in de Sitter spacetime



$$ds^2 = -\Xi dt^2 + \Xi^{-1} d\vec{x}^2$$

$$\Xi = \left(1 + \sum_i \frac{m_i}{r_i}\right)^{-2}$$

$$r_i = \sqrt{(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2}$$

Time-dependent

Contracting universe

$$ds^2 = -\bar{\Xi} d\bar{t}^2 + \frac{a^2(\bar{t})}{\bar{\Xi}} d\vec{x}^2$$

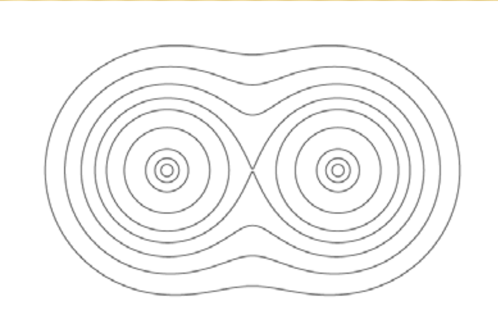
$$\bar{\Xi} = \bar{H}_T^{n_T/4} H_S^{n_S/4}$$

$$\bar{H}_T = 1 + \frac{1}{a^{4/n_T}} \sum_i \frac{Q_T^{(i)}}{r_i}$$

$$H_S = 1 + \sum_i \frac{Q_S^{(i)}}{r_i}$$

$$a \sim \bar{t}^{n_T/n_S}$$

G.W. Gibbons, H. Lu, C.N. Pope (PRL 2005)
Brane Worlds in Collision



5. Summary & Future work

- We find a black hole system in the Universe with arbitrary expansion law
- The effective action can be derived from supergravity in higher dimensions
- It may be worth to work in higher dimensions
  - new time-dependent solutions
    from intersecting branes in supergravity model ?
  - rotating black holes in the Universe ? *T. Shiromizu (1999)*
  - neutral black holes in the Universe ? *non-BPS*
  - thermodynamics *time-dependent spacetime*
  - black hole collision/brane collision ?
  - black hole evaporation ?

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