高次元Black Hole研究最前線研究会@基礎物理学研究所

Toroidal Spiral Strings around Five-dimensional Black Holes

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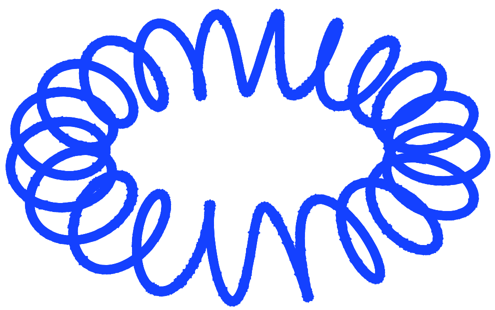
[contents]

Abstract

We examine the dynamics of the Nambu-Goto test strings in a closed shape of toroidal spiral in a 5D spacetime. We discuss

- stationary configuration,
- separability in E.O.M.,
- dynamical solution,

of the *Toroidal Spiral String*.



Ⅰ. INTRODUCTION

important tasks

Identification of the spacetime dim.

(could be done by obs. of phenomena concerning to BHs)

what to do

– revealing
- properties of higher−dim. BH
- differences between 4D and higher dim.

powerful tools

test objects
- test particles [stars, galaxies, electrons…]
- test strings [cosmic string, …]

Here, as a probe of BH spacetime, we concentrate on

test string

Ⅰ－Ⅰ. Test Objects

a typical HD BH exact sol.

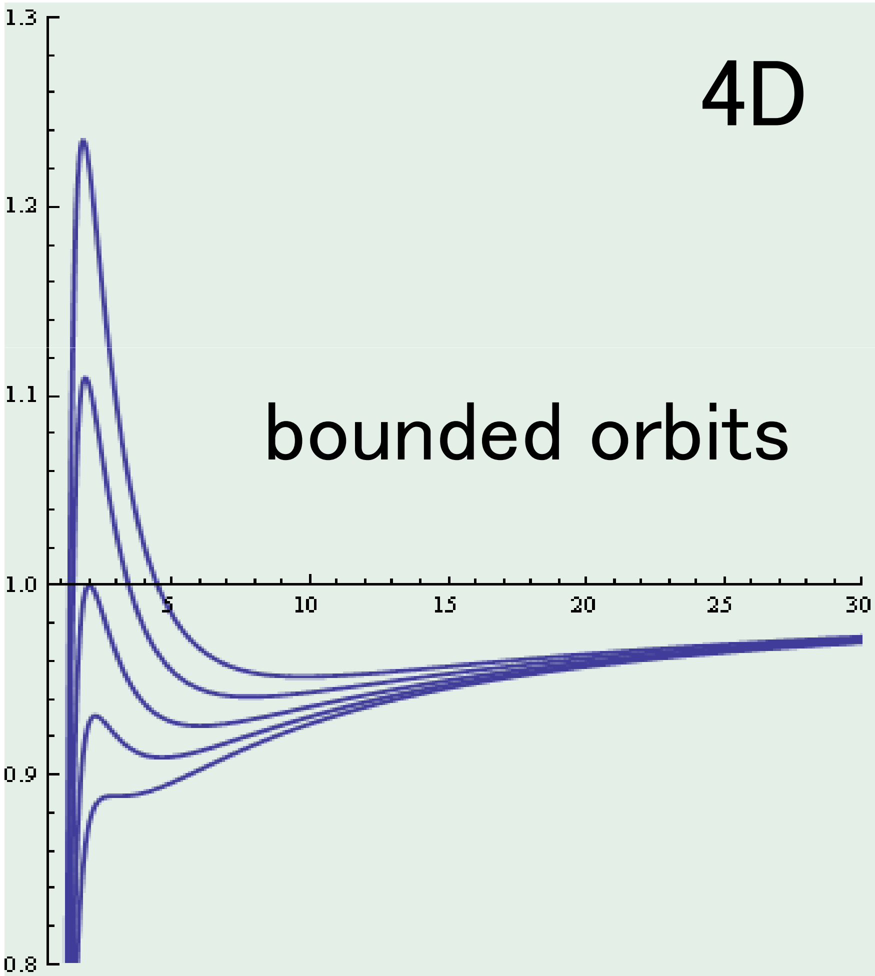
Kerr black hole

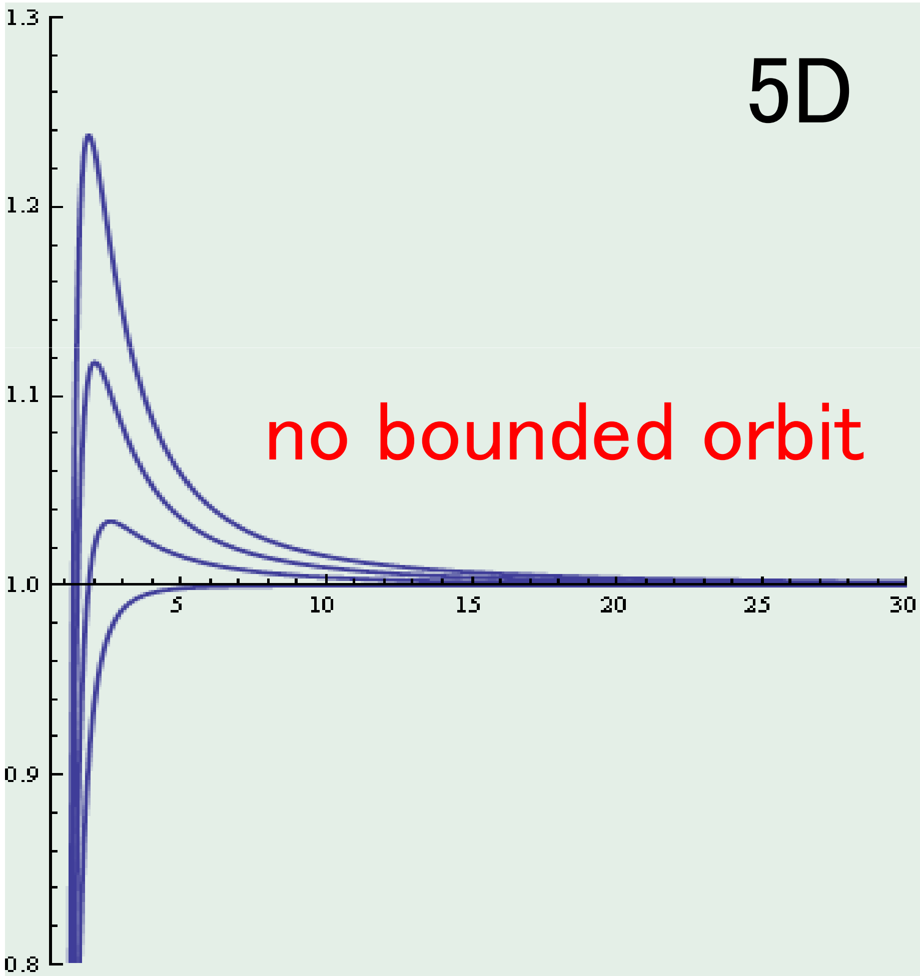
－ free test particle

	bounded orbits	separability	
4D Kerr BH	○	○	B.Carter (1968)
5D Kerr BH	×	○	V.P.Frolov and D.Stojkovic (2003)

Ⅰ－Ⅱ. Radial Motion of a Test Particle

<u>effective potential</u> (in Sch. BH)





Ⅰ−Ⅲ. Test Objects

− free test particle

	bounded orbits	separability	
4D Kerr BH	○	○	B.Carter (1968)
5D Kerr BH	×	○	V.P.Frolov and D.Stojkovic (2003)

The separability is related to KV fields and KT fields.

Today's topics is ***dynamical string***

− test *stationary string*

separable in E.O.M. in 5D Kerr BH [V.P.Frolov and K.A.Stevens (2004)]

Ⅱ. COHOMOGENEITY-ONE STRING

cohomogeneity-one string : a KV field ξ is tangent to Σ

Nambu-Goto action

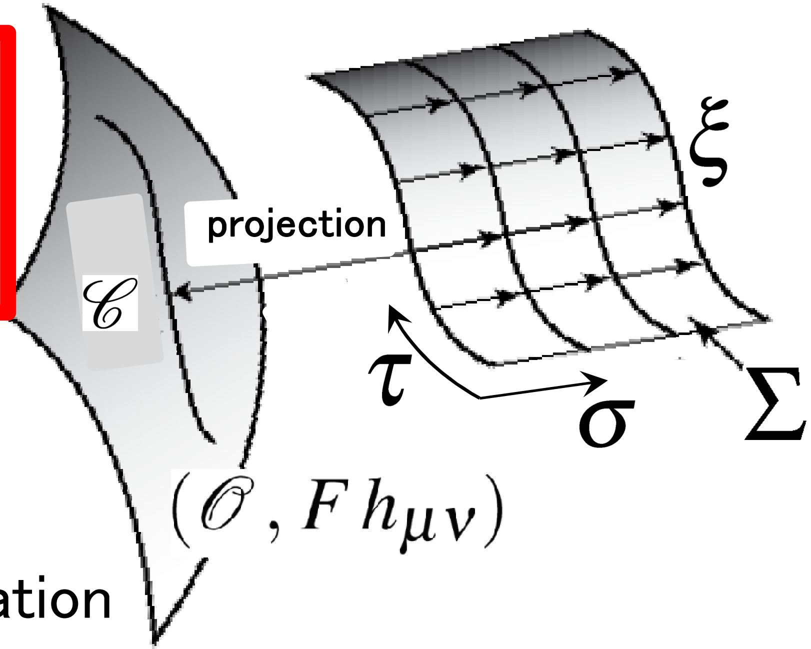
$$S_{NG} = -\mu \int_{\Sigma} d\tau d\sigma \sqrt{-\gamma}$$ Σ : worldsheet

$\Downarrow$ reduction ($\xi = \partial_\sigma$ dir.)

<u>action for a free particle</u>

$$S_{NG} = -\mu \Delta\sigma \int_{\mathscr{C}} \sqrt{-F h_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau}} d\tau$$

F : norm of ξ



ξ
- timelike ⋯ stationary
- spacelike ⋯ sym. of configuration

Ⅲ. Toroidal Spiral String in 5D Flat Background

$$
\begin{aligned}
ds^2 &= -dt^2 + \underbrace{dx^2 + dy^2}_{x-y\,\text{plane}} + \underbrace{dz^2 + dw^2}_{z-w\,\text{plane}} \\
&= -dt^2 + d\rho^2 + \rho^2 d\Phi^2 + d\zeta^2 + \zeta^2 d\Psi^2
\end{aligned}
$$

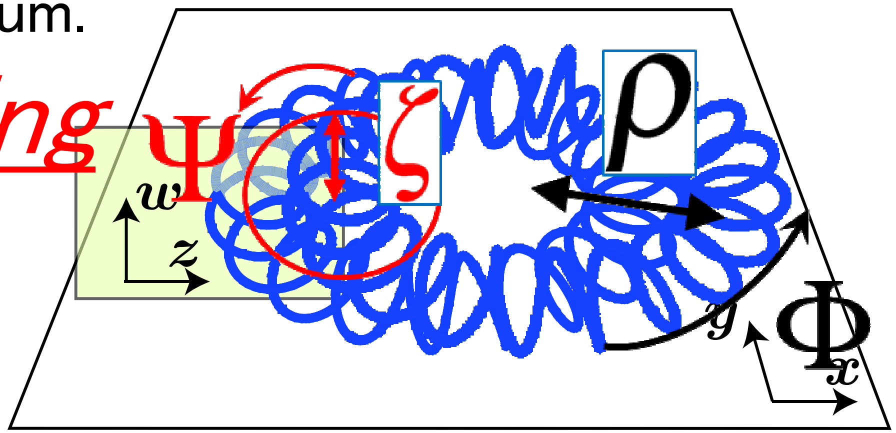
independent planes

Suppose Σ is tangent to the KV field

$$\xi = \partial_\Phi + \alpha \partial_\Psi$$

α : const. ratio of winding num.

Toroidal Spiral String



note

- ang. momentum
- closed string $(\alpha \in \mathbf{Q})$
- exist commutable closed KVs in higher-dim. spacetime

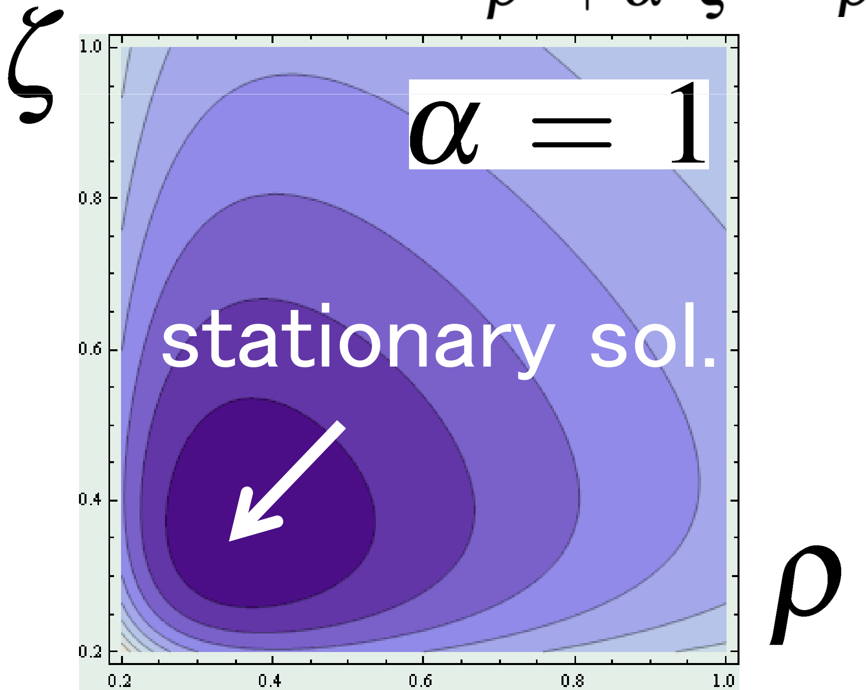
Ⅲ－Ⅱ. Stationary Configuration

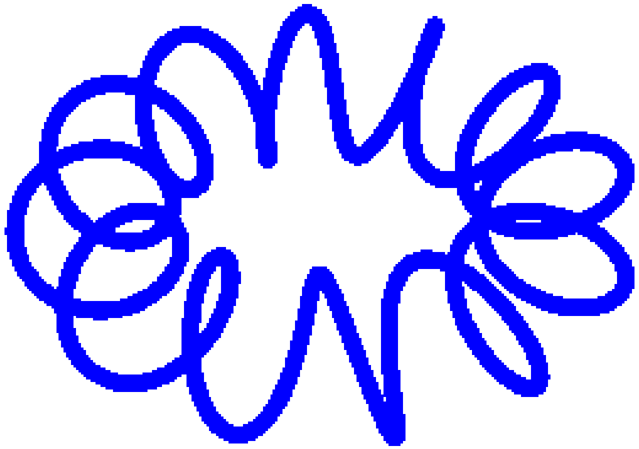
<u>effective potential</u>

$$H = \frac{1}{2}\left[\frac{p_\rho^2 + p_\zeta^2}{\rho^2 + \alpha^2\zeta^2} + V_{\rm eff}(\rho,\zeta)\right]$$

Hamiltonian for TSS

$$V_{\rm eff}(\rho,\zeta) = -\frac{E^2}{\rho^2+\alpha^2\zeta^2} + \frac{L^2}{\rho^2\zeta^2} + 1$$





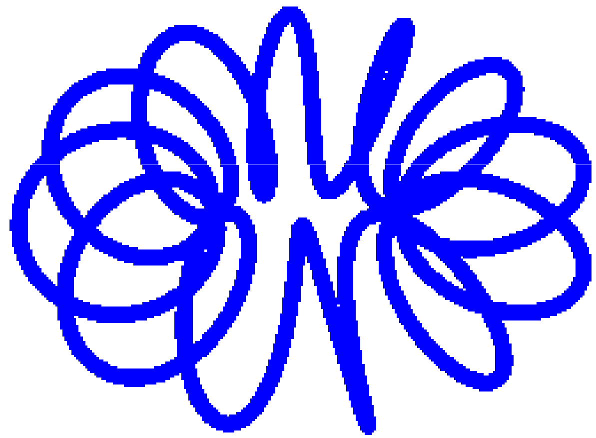
realized by a balance of tension and centrifugal force

Ⅲ−Ⅲ. Dynamical solution

The E.O.M. is separable for general TSS in the flat background.

Dynamical Solution

$$\begin{aligned} \rho^2 &= \frac{\rho_{\rm max}^2-\rho_{\rm min}^2}{2}\cos(2\tau+\delta_\rho)+\frac{\rho_{\rm max}^2+\rho_{\rm min}^2}{2}, \\ \zeta^2 &= \frac{\zeta_{\rm max}^2-\zeta_{\rm min}^2}{2}\cos(2\alpha\tau+\delta_\zeta)+\frac{\zeta_{\rm max}^2+\zeta_{\rm min}^2}{2}, \\ t &= E\tau \end{aligned}$$

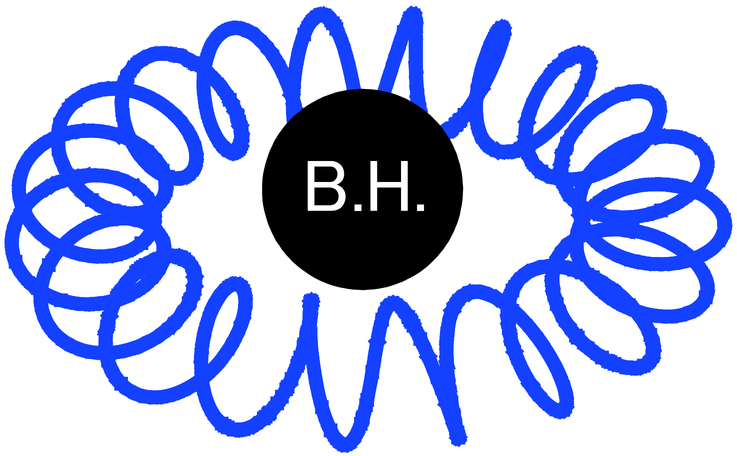


note

- periodic motion $(\alpha \in \mathbf{Q})$
- no cusp
- J.J.Blanco−Pillado, R.Emparan and A.Iglesias (2008)

Ⅳ. Toroidal Spiral String around a 5D Black Hole

Ⅳ- Ⅰ. TSS around the 5D Kerr–AdS BH



<u>note</u>

balance
- tension,
- centrifugal force,
- gravitational force

<u>5D Kerr–AdS BH metric</u>

$$ds^2 = -\frac{\Delta_\theta \, \Xi_r \, dt^2}{\Xi_a \Xi_b} + \frac{2M}{\Sigma}\left(\frac{\Delta_\theta \, dt}{\Xi_a \Xi_b} - \nu\right)^2 + \frac{\Sigma \, dr^2}{\Delta_r} + \frac{\Sigma \, d\theta^2}{\Delta_\theta} + \frac{r^2 + a^2}{\Xi_a}\sin^2\theta \, d\Phi^2 + \frac{r^2 + b^2}{\Xi_b}\cos^2\theta \, d\Psi^2$$

$$\Xi_a = 1 - a^2\lambda^2, \qquad \Xi_b = 1 - b^2\lambda^2, \qquad \Xi_r = 1 + \lambda^2 r^2 ,$$

$$\Delta_r = \frac{(r^2+a^2)(r^2+b^2)(1+\lambda^2 r^2)}{r^2} - 2M, \qquad \Delta_\theta = 1 - a^2\lambda^2\cos^2\theta - b^2\lambda^2\sin^2\theta ,$$

$$\nu = a\sin^2\theta \frac{d\Phi}{\Xi_a} + b\cos^2\theta \frac{d\Psi}{\Xi_b}, \qquad \Sigma = r^2 + a^2\cos^2\theta + b^2\sin^2\theta,$$

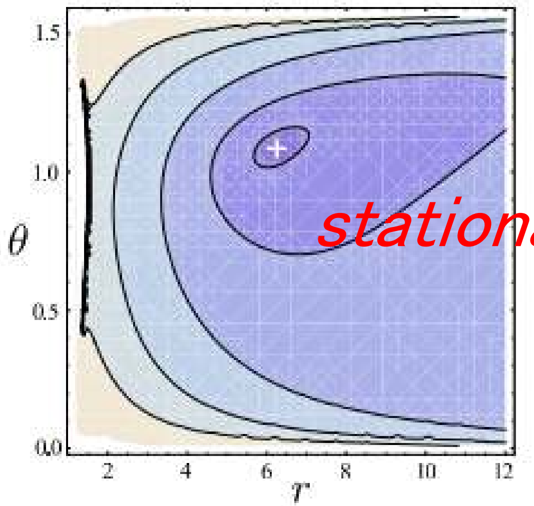
Ⅳ−Ⅱ. Effective Potential

$$H = \frac{1}{2}\left[F^{-1}\left(h^{rr}p_r^2 + h^{\theta\theta}p_\theta^2\right) + V_{\rm eff}(r,\theta)\right]$$

Hamiltonian for TSS

$$\begin{aligned} V_{\rm eff}(r,\theta) = & -\frac{1}{\Sigma}\left[(r^2+2M) + \frac{4M^2}{\Delta} + (a^2\cos^2\theta + b^2\sin^2\theta)\right]\frac{E^2}{F} \\ & + \frac{1}{\Sigma}\left[\frac{(a^2-b^2)\left[r^2(1-\alpha^2)+(a^2-\alpha^2 b^2)\right] - 2M(a-\alpha b)^2}{r^2\Delta} + \frac{1}{\cos^2\theta} + \frac{\alpha^2}{\sin^2\theta}\right]\frac{L^2}{F} \\ & + \frac{4M(-b(r^2+a^2)+\alpha a(r^2+b^2))}{r^2\Delta\Sigma}\frac{EL}{F} + 1 \end{aligned}$$

effective potential



stationary configuration

realized by a balance of tension, centrifugal force, and grav. force

Ⅳ−Ⅲ. Separability

✕ Separation of variables for a general TSS

[outline of analysis]

By the Hamilton–Jacobi eqn,

Hamilton's principal fct.

$$S = \frac{1}{2}\mu^2\tau - Et + L\psi + S_r(r) + S_\theta(\theta)$$

$$2F\Sigma \times \left[\frac{1}{2}\frac{h^{\mu\nu}}{F}\frac{\partial S}{\partial x^\mu}\frac{\partial S}{\partial x^\nu} + \frac{\partial S}{\partial \tau}\right] = R(r) + \Theta(\theta) + \boxed{\mu^2 F\Sigma} = 0$$

The separability depends on this term.

where

$$\mu^2 F\Sigma = \mu^2(r^2 + a^2\cos^2\theta + b^2\sin^2\theta)\left[\frac{(r^2+b^2)\alpha^2\cos^2\theta}{\Xi_b} + \frac{(r^2+a^2)\sin^2\theta}{\Xi_a}\right] + 2M\mu^2\left(\frac{\alpha b\cos^2\theta}{\Xi_b} + \frac{a\sin^2\theta}{\Xi_a}\right)^2$$

Ⅳ−Ⅳ. Separability

For TSS with $\alpha^2 = 1$, complete separation of variables in the H−J eqn. occurs in two cases:

(A) Kerr background $a \neq b \, , \lambda = 0$

(B) Kerr−AdS background with two equal angular momenta

$$a = b \, , \lambda \neq 0$$

Ⅳ−Ⅴ. Hopf loop string

r constant surface on a timeslice $\cdots$ S^3 (Hopf bundle)

metric on the Hopf bundle embedded in 5D Kerr−AdS BH

S^2 base space twisted S^1 fiber

$$ds^2_{S^3} = \frac{1}{4}\left[g_{\theta\theta}d\theta_E^2 + \frac{4}{F}(g_{\Phi\Phi}g_{\Psi\Psi} - g^2_{\Phi\Psi})d\phi_E^2\right] + \frac{F}{4}\left[d\psi_E + \frac{1}{F}(g_{\Psi\Psi} - g_{\Phi\Phi})d\phi_E\right]^2$$

where θ_E, ϕ_E, ψ_E : Euler angles

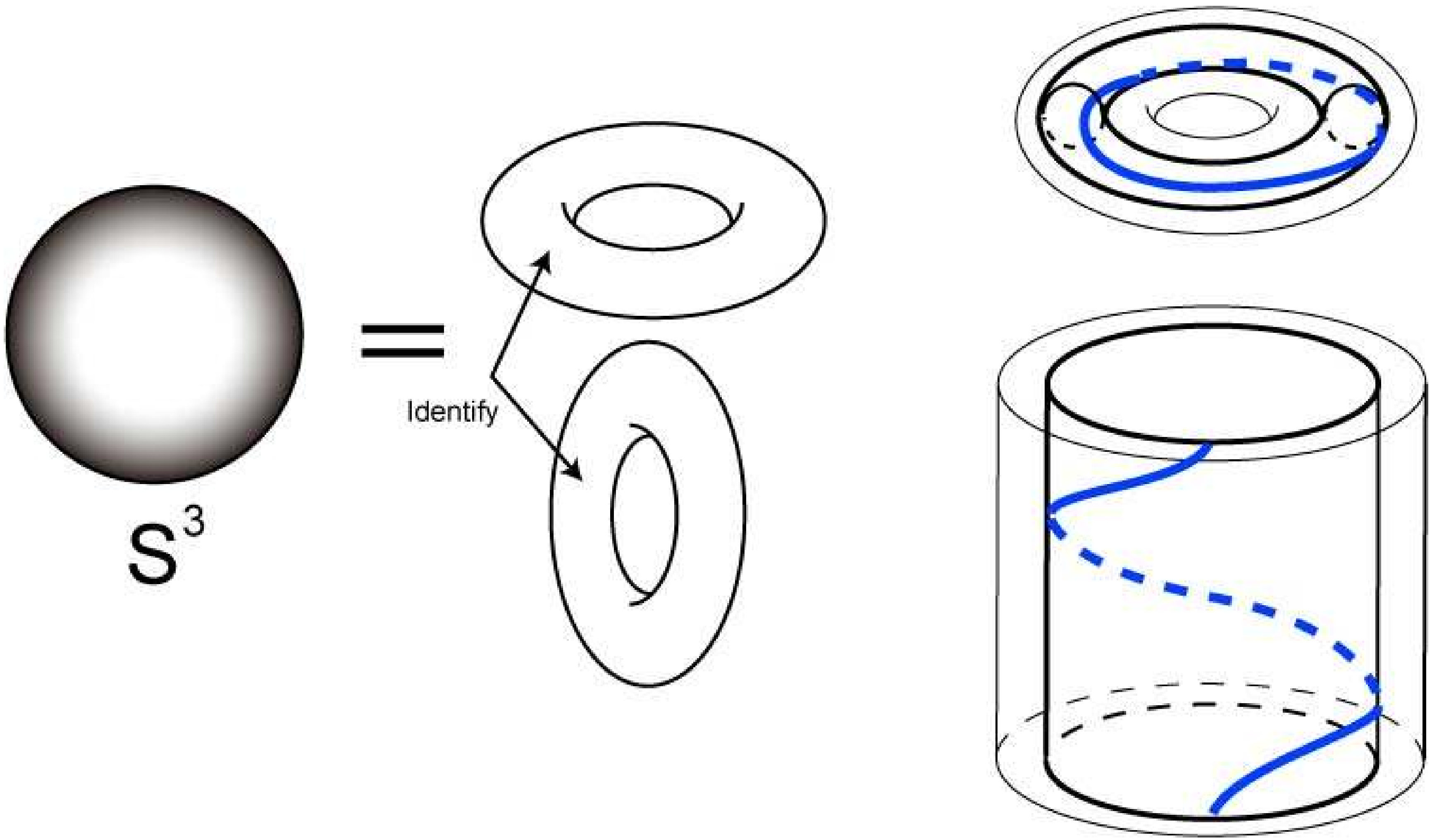
coord. along the fiber

$$\xi = \partial_\Phi + \partial_\Psi = 2\partial_{\psi_E}$$

TSS with $\alpha^2 = 1$ lies along a fiber of Hopf fibration

Hopf loop

Ⅳ−Ⅵ. Hopf loop



Ⅳ-Ⅶ. Dynamics

For a *Hopf loop*, complete separation of variables in the H-J eqn. occurs in two cases:

(A) Kerr background $a \neq b\,,\,\lambda = 0$

(B) Kerr-AdS background with two equal angular momenta

$$a = b\,,\,\lambda \neq 0$$

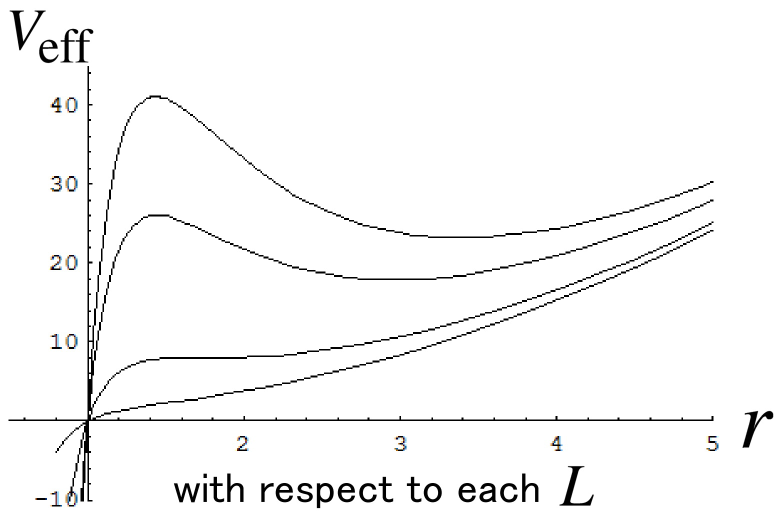
Ⅳ−Ⅷ. Innermost Stable Orbit

radial motion of Hopf loop in the 5D Sch. BH

$$\dot{r}^2 + V_{\rm eff} = E^2,$$

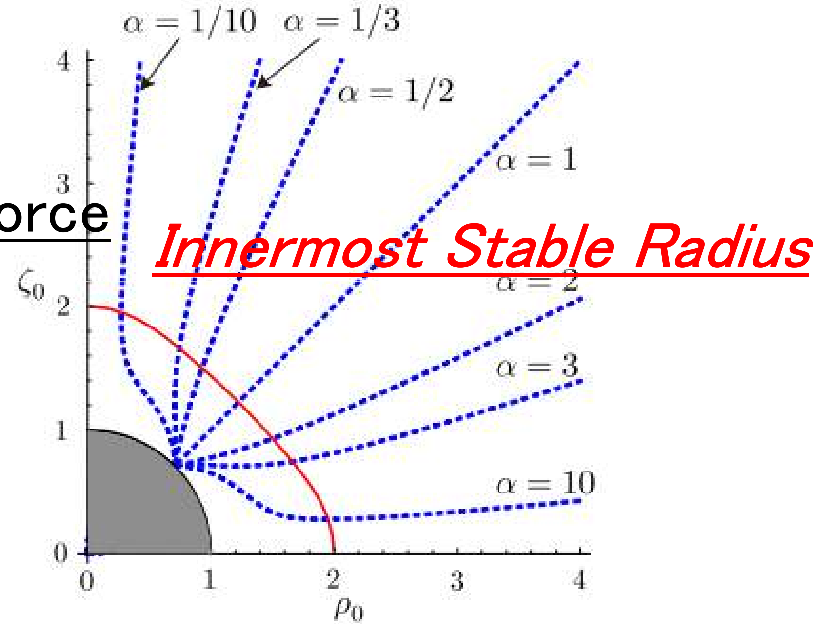
$$V_{\rm eff} = -2M\mu^2 \boxed{-\frac{8ML^2}{r^4}} + \boxed{\frac{4L^2}{r^2}} + \boxed{\mu^2 r^2}$$

grav. force — centrifugal force — tension



with respect to each L

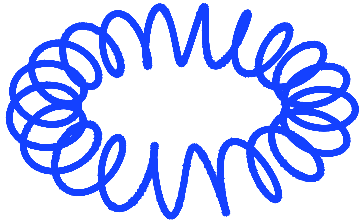
the competition of three forces



$r_g = 2M = 1, \ \Lambda/6 = -\lambda^2 = 0, \ a = 0$

Ⅳ. SUMMARY

– *Toroidal Spiral String*



- sym. of configuration (C–1 string)
- closed $(\alpha \in \mathbf{Q})$
- angular momentum
- commutable closed KVs

– TSS in 5D spacetime

- stationary configuration

  by a balance of centrifugal force, tension, and grav. force

- separability

  for general TSS in M^5

  for *Hopf loop* (TSS with $\alpha^2 = 1$) in

  (A) Kerr $(a \neq b, \lambda = 0)$

  (B) Kerr–AdS $(a = b, \lambda \neq 0)$

- dynamics

  existence of bounded orbits, innermost stable orbits

Ⅳ. DISCUSSION and FUTURE WORK

- no cusp (while closed strings in 4D must have cusp)

gravitational wave
- burst (4D)
- periodic (HD)

difference between 4D and HD

- existence of bounded orbits

useful as a probe of HD spacetime

test particle in 4D BH …… Hopf loop in 5D BH

- TSS in more higher-dimensional spacetime
- TSS around black ring

straightforward

- stability of TSS