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@ YITP workshop on BH in higher dimensions 09

# Application of String Theory and Sugra Solution


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arXiv: 0909.12196



w/ Heng-Yu Chen (Madison),
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# Relax…

| | |
|---|---|
| KT | [Klebanov, Tseytlin, 0002159] |
| MN | [Maldacena, Nunez, 0008001] |
| KS | [Klebanov, Strassler, 0007191] |
| BMN | [Berenstein, Maldacena, Nastase, 0202021] |
| LLM | [Lin, Lunin, Maldacena, 0409174] |

These are important geometries for string theorists

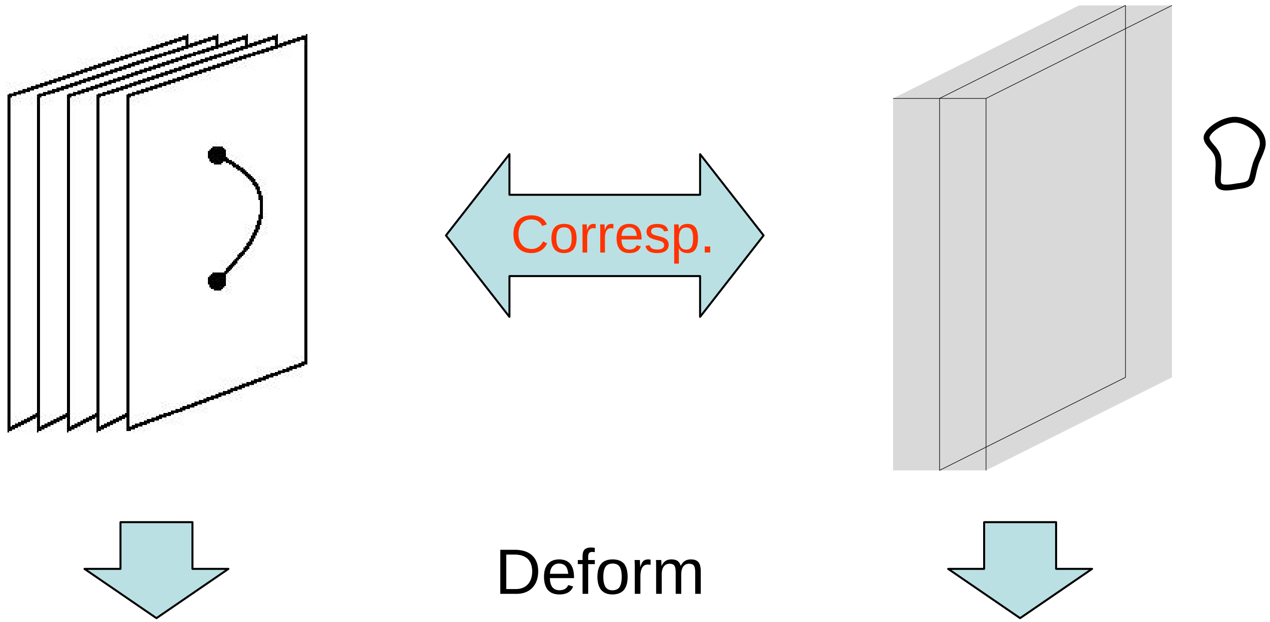
…….. Why important?

## Plan

# 1. Need of gravity solutions

# Motivation for a superstring theorist

N=4 Super YM in 4d     $AdS_5 \times S^5$ geometry of IIB sugra




Gauge theory
in different dimensions,
with lower supersymmetries,
with various matter fields

?

# Possible ways of the deformations

Requirement : Clear deformations on the both sides !

Ex.1) Introduction of matter fields by “flavor D-branes”

“Probe approximation” : Nc >> Nf

Throwing away important gauge dynamics…

“Nc ~ Nf” effects : beyond quenching, Seiberg dualities, Color-flavor locking, …

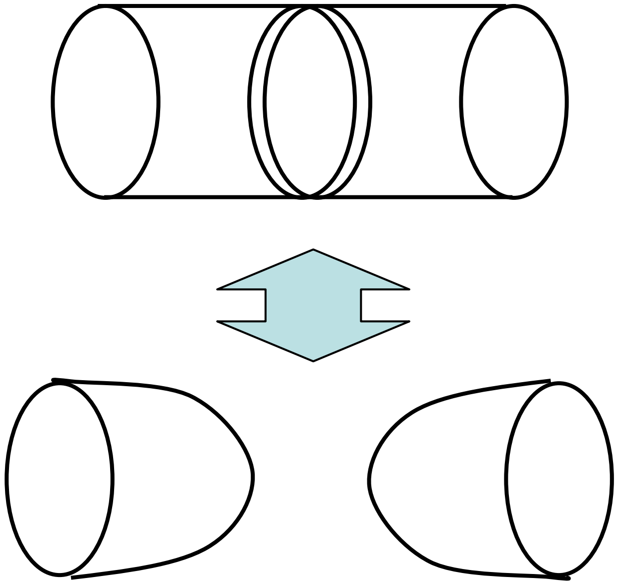
Ex.2) SUSY + boundary informations

Higher supersymmetries constrains geometries, and AdS/CFT dictionary at boundary fixes them.

Non-commutative YM, bubbling geometry

Ex.3) Geometrically clear deformations

# Gravity solution for pure YM without SUSY



*Nc* D4-branes wrapping $S^1$

Gaugino : antiperiodic

→ 4d bosonic YM at low energy

Gravity solution [Gibbons, Maeda (88)] [Witten (98)]

Double-Wick rotated $AdS_7 \times S^4$ blackhole

$$ds^2 = \frac{r^2}{L^2}\left(\eta_{\mu\nu}dx^\mu dx^\nu + f(r)d\tau^2\right) + \frac{L^2}{r^2}f^{-1}(r)dr^2$$

(written with 11 dim. supergravity notation) $f \equiv 1 - R^6/r^6$

What is cleaver about this geometry :

How to break susy is specified → field theory dual is clear

# Techniques demanded

D-brane consruction of gauge theories : well established

Supergravity solutions ???

Gravity solutions for even simple intersecting D-branes have not been constructed explicitly [Lunin (0706.3396)]

What have been done :

Back-reaction solved order by order in Nf / Nc ….

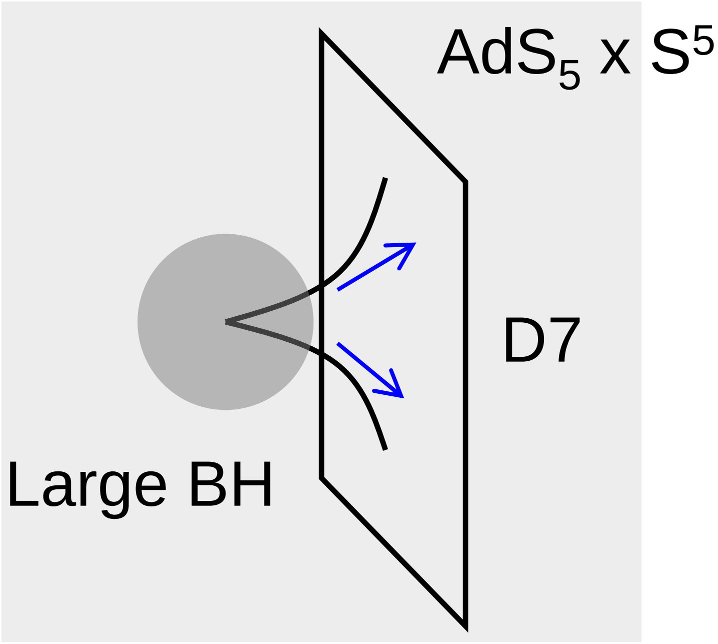
Smearing of D-branes….

# 2. My Sugra analysis

# My SUGRA analysis

I would like to compute …

| A back reaction of spiky D7-brane probe in $AdS_5$ BH |
|---|

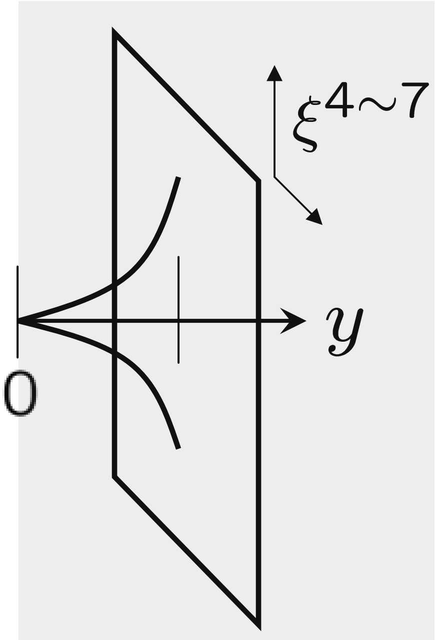



Spike (scalar charge) is translation invariant under $x^1$, $x^2$, $x^3$

On the D7, from the tip, electric flux is generated

# D7 spike solution

See [Karch,O'Bannon(07)]




Background: $AdS_5 \times S^5$

$$ds^2 = \frac{r_6^2}{R^2} dx^\mu dx^\nu \eta_{\mu\nu} + \frac{R^2}{r_6^2} dr_6^2 + R^2 ds_5^2.$$

$$r_6^2 = r^2 + y^2 + z^2 \qquad r^2 = (\xi^4)^2 + (\xi^5)^2 + (\xi^6)^2 + (\xi^7)^2$$

$$g_s C_4 = \frac{r_6^4}{R^4} dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 \,,$$

$$g_s F_5 = (1 + *_{10}) d(g_s C_4) = 4R^4 (d\Omega_5 + *_{10} d\Omega_5) \,.$$

D7-action : $S = -T_{\mathrm{D7}} \int d^4x \int d^4\xi \; \mathrm{tr} \sqrt{-\det(G_{\mu\nu} + 2\pi\alpha' F_{\mu\nu})}$

We turn on only $y(r), A_t(r)$ → Spike solution

$$2\pi\alpha' A_t'(r) = \frac{\mathbf{d}}{\mathcal{N}\sqrt{r^6 + r_0^6}}, \quad y'(r) = \frac{\mathbf{c}}{\mathcal{N}\sqrt{r^6 + r_0^6}} \qquad r_0^6 = \frac{\mathbf{d}^2 - \mathbf{c}^2}{\mathcal{N}^2}$$

# A SUGRA back reaction : step 1/3

We show D7 electric field back-reacts to generate $F^{(3)}_{123}$

Electric flux on D7 = source for NSNS B-field in the bulk

$$S_{\rm D7} = -N_f \mathcal{T}_{\rm D7} \int d^4x d^4\xi \sqrt{-\det(g_{\mu\nu} + 2\pi\alpha' F_{\mu\nu} + \tilde{B}_{\mu\nu})}$$

Expansion of this in terms of B gives a source term

$$S^{\rm D7}_{\rm DBI}\bigg|_{\mathcal{O}(\hat{B})} = -\int d^4x dr\ \hat{B}_{0r} \left[\frac{\delta L}{\delta(2\pi\alpha' A_t')}\right]_{B=0}$$

Using $z = r_6 \cos\theta_1\,, \quad y = r_6 \sin\theta_1 \cos\theta_2\,, \quad r = r_6 \sin\theta_1 \sin\theta_2$

The tip ~ rigid cone, $\theta_2 \sim \theta_2^{(0)} \equiv \dfrac{\sqrt{\mathbf{d}^2 - \mathbf{c}^2}}{\mathbf{c}} \qquad \theta_1 = \pi/2$

$$S^{\rm D7}_{\rm DBI}\bigg|_{\mathcal{O}(B)} = -\frac{\mathbf{d}}{2\pi^2} \int d^4x \int dr_6\, d\Omega_5\, \delta(\theta_1 - \pi/2)\delta(\theta_2 - \theta_2^{(0)}) \frac{B_{0r_6}}{\sin^3\theta_2^{(0)}}$$

# A SUGRA back reaction : step 2/3

Relevant part of the IIB SUGRA is

$$S_{\rm B} = -\frac{1}{4\kappa_{10}^2}\int d^{10}x\sqrt{-g_{10}}\, e^{-2\Phi}|H_3|^2 + \frac{1}{4\kappa_{10}^2}\int F_5\wedge B_2\wedge F_3$$

Substituting the background 5-form flux, we obtain

$$S_{\rm B} = -\frac{1}{2(2\pi)^7\alpha'^4 g_s^2}\int d^4x dr_6 d\Omega_5\; r_6^3\left[H_{0r_6\theta_1}^2 + \frac{1}{\sin^2\theta_2}H_{0r_6\theta_2}^2\right] + \frac{1}{(2\pi)^7\alpha'^4}\int d^4x dr_6 d\Omega_5\; B_{0r_6}F_{123}^{(3)} 2^4\pi N_c(\alpha')^2\,.$$

With the D7 source term, the EOM is

$$0 = \frac{r_6^3}{(2\pi)^7\alpha'^4 g_s^2}\left[\partial_{\theta_1}\left(\sin^4\theta_1\sin^3\theta_2 H_{0r_6\theta_1}\right) + \partial_{\theta_2}\left(\sin^2\theta_1\sin^3\theta_2 H_{0r_6\theta_2}\right)\right] + \frac{1}{(2\pi)^7\alpha'^4}F_{123}^{(3)}\sin^4\theta_1\sin^3\theta_2\, 2^4\pi N_c\alpha'^2 - \delta(\theta_1-\pi/2)\delta(\theta_2-\theta_2^{(0)})\frac{\mathbf{d}}{2\pi^2}\,.$$

# A SUGRA back reaction : step 3/3

Integrating this equation over the $\theta_1, \theta_2$ space, we get

$$\frac{1}{(2\pi)^7 \alpha'^4} F^{(3)}_{123} \int_0^\pi d\theta_1 \ \sin^4\theta_1 \int_0^\pi d\theta_2 \ \sin^3\theta_2 \ (2^4 \pi N_c \alpha'^2) = \frac{\mathbf{d}}{2\pi^2}$$

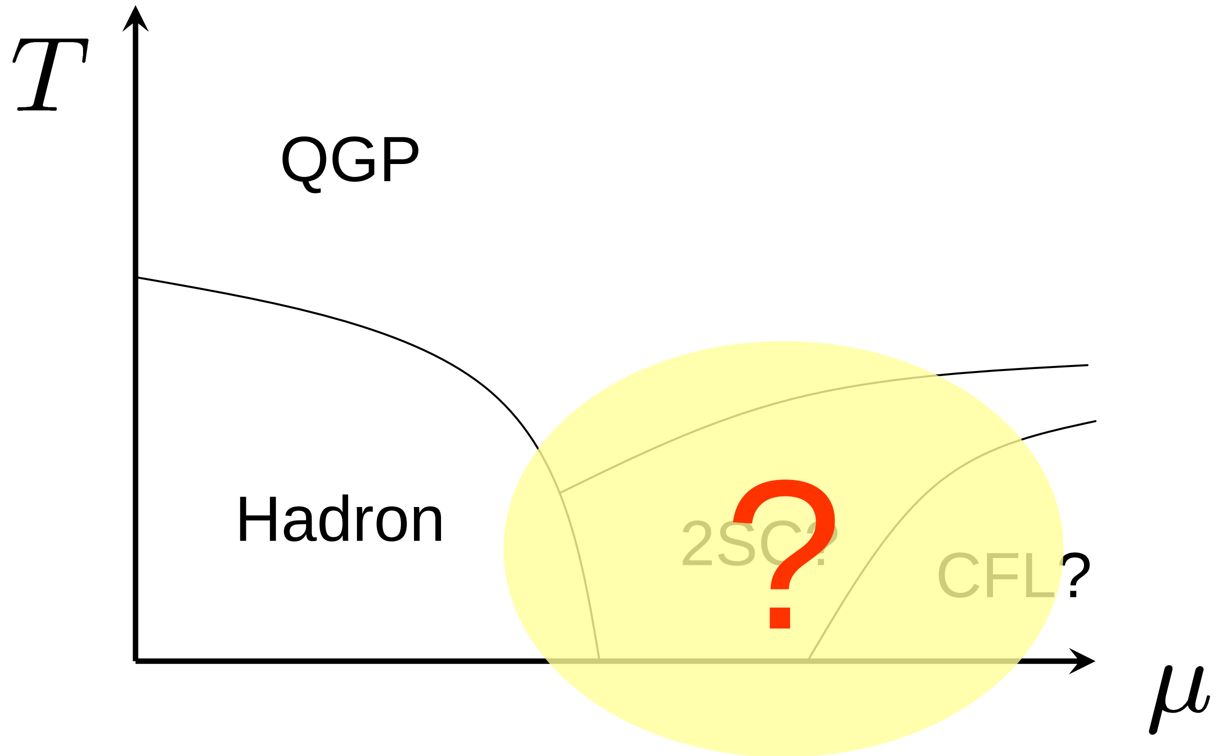
This provides a back-reaction for the constant 3-form flux,

$$F^{(3)}_{123} = \frac{8\pi^3 \alpha'^2 \mathbf{d}}{N_c}$$

Note: (1) Consistent with the $F^{(3)}$ EOM. “Baryon vertex”!
(2) It is a back reaction, $\mathcal{O}(1/N_c)$
(3) This computation holds also for BH case.
(4) Dilaton/gravity back-reactions ignored.

# 3. Motivation for the computations

# Exploring QCD phase diagram




2SC : $\langle \epsilon_{\alpha\beta 3}\epsilon^{ij}\psi_i^\alpha \psi_j^\beta \rangle \neq 0$

CFL : $\langle \epsilon_{\alpha\beta A}\epsilon^{ijA}\psi_i^\alpha \psi_j^\beta \rangle \neq 0$

# CFL in strongly-coupled QCD?

For large chemical potential, perturbative QCD helps.

For very small chemical potential, Lattice QCD helps.

Phase transition cannot be studied by these

↓

Holographic QCD may help?

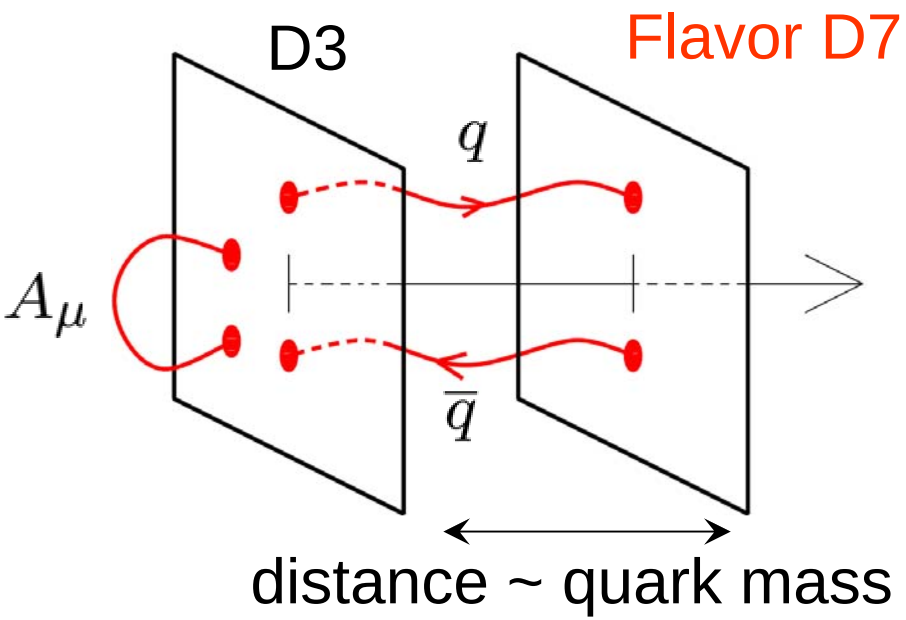
# Toy model of QCD : *N*=2 SQCD

D3-D7 model

[Karch, Katz (0205236)]

- Nc D3: 0123
- Nf D7: 0123 4567

*N*=2 SQCD

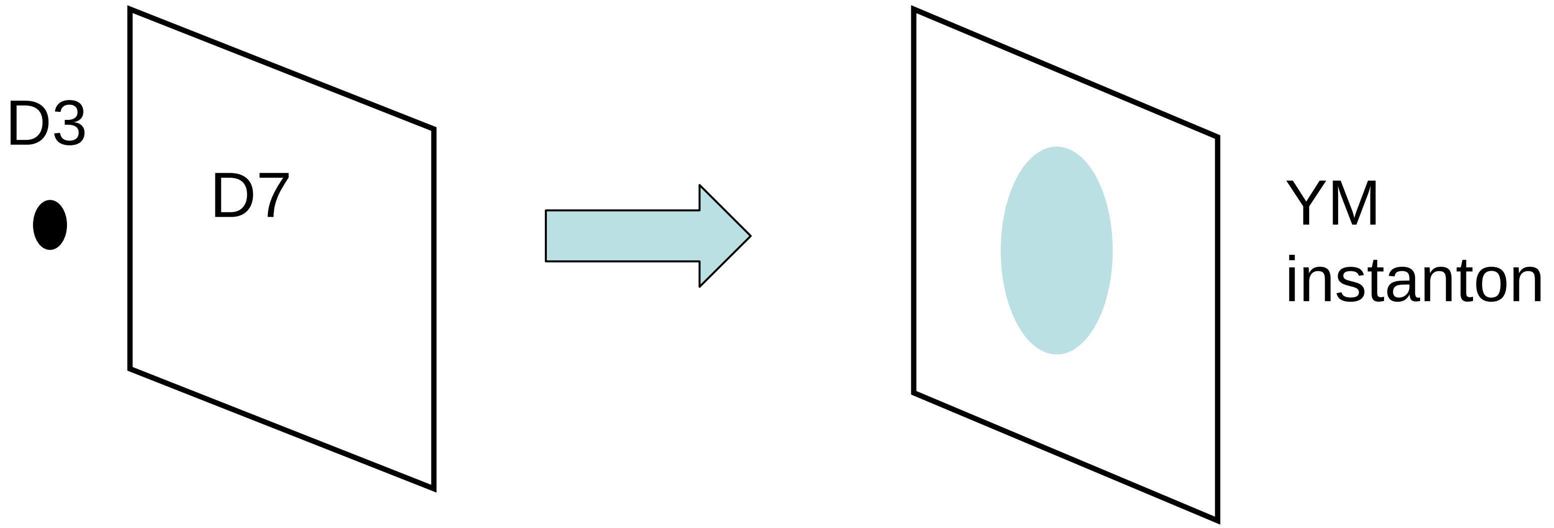



Quarks are accompanied by Squarks

➔ When baryon chemical potential $\mu$ is turned on, the squarks get condensed first!

➔ **Color-Flavor Locking = Higgs phase**

# Realization of Higgs phase




Dissolution of D3 into D7 = YM instanton

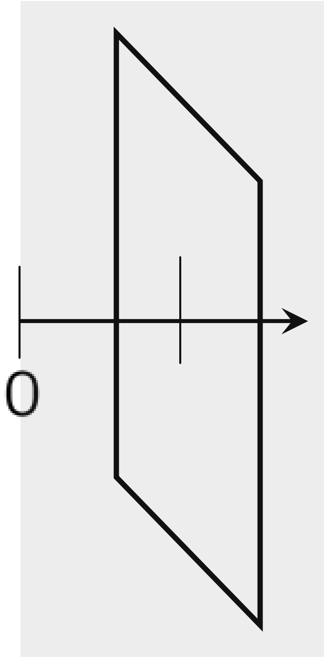
= Squark condensation

(Higgs phase)

We will show : In D3-D7 set up of holographic N=2 SQCD, baryon chemical potential induces a size moduli potential for YM instanton on D7

# 4. Sugra back-reaction and "physics"

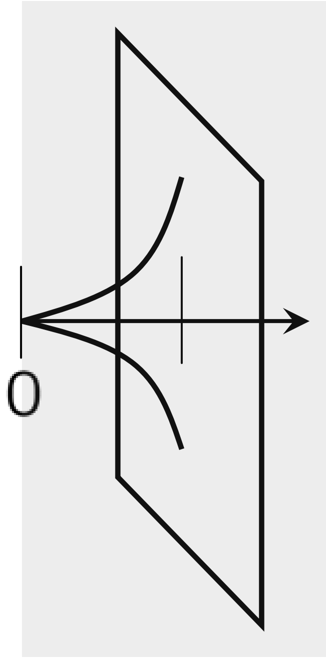
# Chemical potential and phase transition at T=0




$\mu < m$

"Minkowski" embedding

$A_t, y$ :constant




$\mu > m$

"Black Hole" embedding

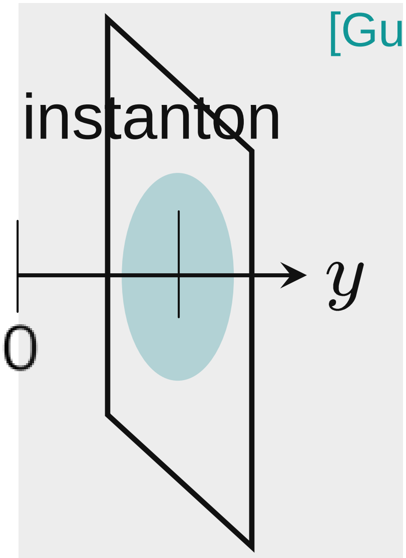
Spike solution

Chemical potential $\mu$ is introduced as asymptotic value of $A_t$ on the D7.

BH embedding is favored for $\mu > m$

[Kobayashi,Mateos,Matsuura, Myers,Thomson (06)]
[Nakamura,Seo,Sin,Yogendran (07)]
[Gohroku,Ishihara,Nakamura(07)]
[Karch,O'Bannon(07)]

# Instanton on the flat D7




[Guralnik,Kovacs,Kulik(04)] [Apreda,Erdmenger,Evans,Guralnik(05)]

$$S = -T_{\rm D7} \int d^4x \int d^4\xi \ {\rm tr} \sqrt{\det \left( \tilde{\tilde{G}}_{ij} + 2\pi\alpha' F_{ij} \frac{r_6^2}{R^2} \right)}$$

$$\tilde{\tilde{G}}_{ij} \equiv \delta_{ij} \quad (i,j = 4,5,6,7)$$

Using a formula for self-dual $F_{ij} = *F_{ij}$

$$\sqrt{\det(g+F)} = \sqrt{\det g} + \frac{1}{4}\sqrt{\det g}\,|F_{ij} * F^{ij}|$$

the D7 action becomes

$$S = -T_{\rm D7} \int d^4x \int d^4\xi \ {\rm tr} \left[ \sqrt{\det \tilde{\tilde{G}}} + \frac{1}{8}(2\pi\alpha')^2 \left( \frac{r^2+y^2}{R^2} \right)^2 \epsilon^{ijkl} F_{ij} F_{kl} \right]$$

This is canceled by the CS term produced by

$$C^{(4)} = \left( \frac{r^2+y^2}{R^2} \right)^2 dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3$$

⇨ No moduli potential

# Our first trial : combining the two

We combine the spike and the instanton for $\mu > m$

"Effective metric" as seen by the instanton is

$$\widetilde{G}_{ij} = g_{ij} + g_{yy}\partial_i y \partial_j y + g^{tt}\partial_i A_t \partial_j A_t (2\pi\alpha')^2$$

$$= \frac{R^2}{r_6^2}\left(\delta_{ij} + \frac{\xi^i \xi^j}{r^2}\left(y'^2 - (2\pi\alpha')^2 A_t'^2\right)\right)$$

This is a conformally flat metric, which allows instanton

→ Calcellation of DBI and CS term, again

→ No potential for the size moduli of the instanton!

→ No favored Higgs phase….. ????

# Inclusion of the Sugra backreaction

On the D7-brane, there is a RR coupling

$$\int C^{(2)} \wedge \mathrm{tr}[F \wedge F \wedge F] \sim \int F^{(3)}_{123} A_0 \mathrm{tr}[F * F]_{4567}$$

→ The background $F^{(3)}_{123}$ generates a CS term,

$$S_{\mathrm{CS}} = \frac{1}{8(2\pi)^4\alpha'} \int d^4x F^{(3)}_{123} \int d^4\xi \ \mathrm{tr}\left[A_0 F_{ij} F_{kl} \epsilon^{ijkl}\right] + \cdots$$

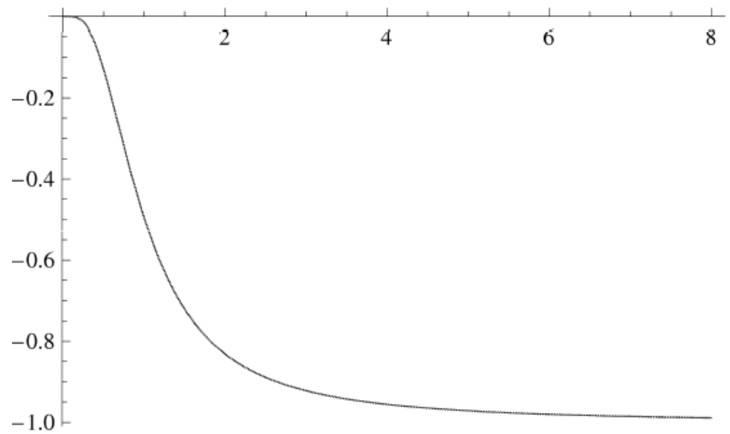
We substitute the BPST instanton, $\mathrm{tr} F_{ij} F_{kl} \epsilon^{ijkl} = \dfrac{192\rho^4}{(\tilde{r}^2 + \rho^2)^4}$

→ Potential for the instanton size moduli !

$$V_B(\rho) = -\frac{2\pi\alpha' \mathbf{d}}{N_c} \int_0^\infty dr \ A_t'(r) \frac{\rho^4(3\tilde{r}^2 + \rho^2)}{(\tilde{r}^2 + \rho^2)^3}$$

# Favoring Color-Flavor Locking

$\rho/r_0$



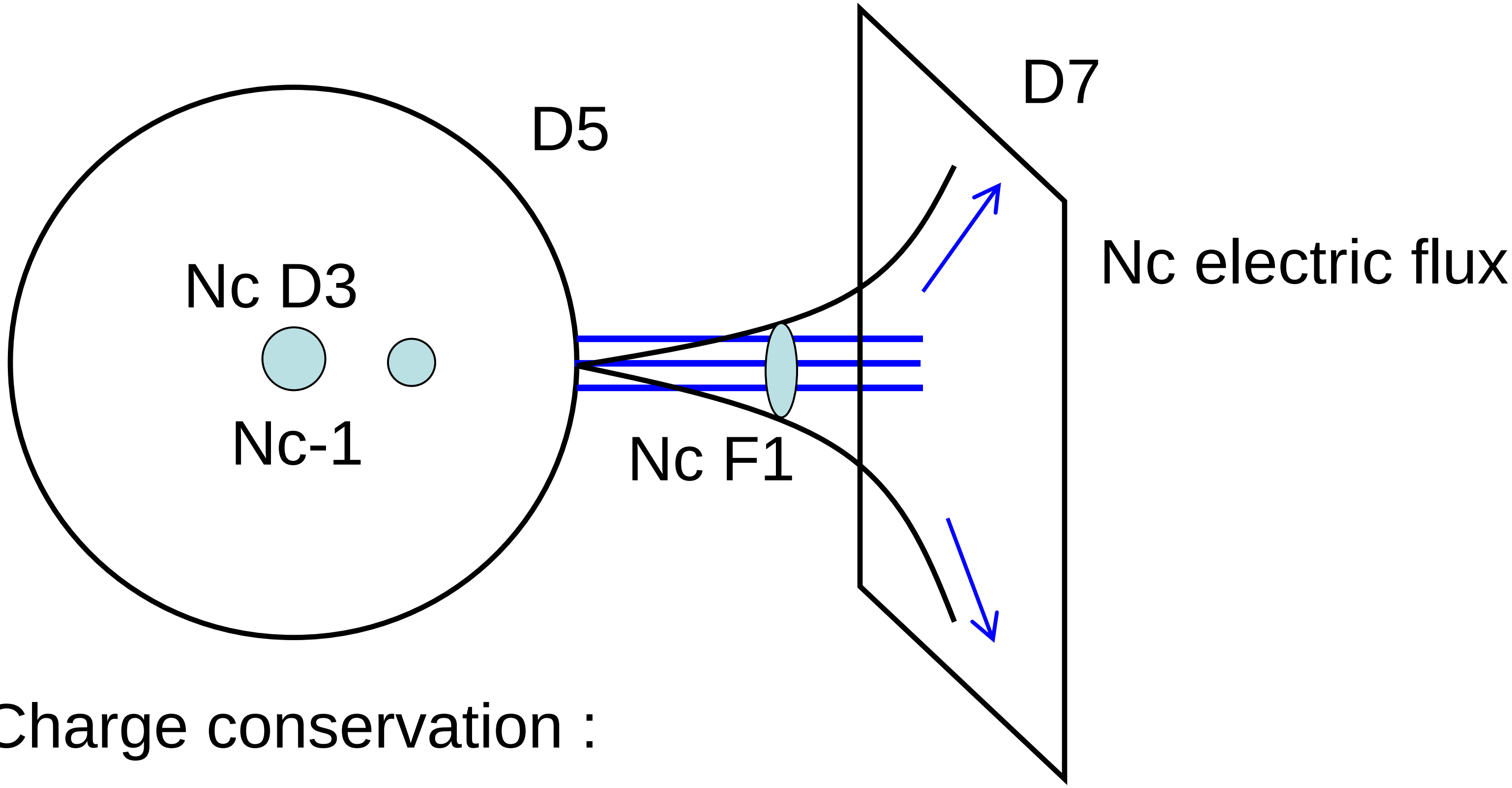

$V_B(\rho)/V_B(\infty)$

Monotonically decreasing function
➔ Instanton is dynamically dissolved!

CFL favored

$$V_B(\rho = 0) - V_B(\rho = \infty) = \frac{2\pi\alpha' \mathbf{d}\mu}{N_c}$$

# Necessity of the RR 3-form flux




Charge conservation :

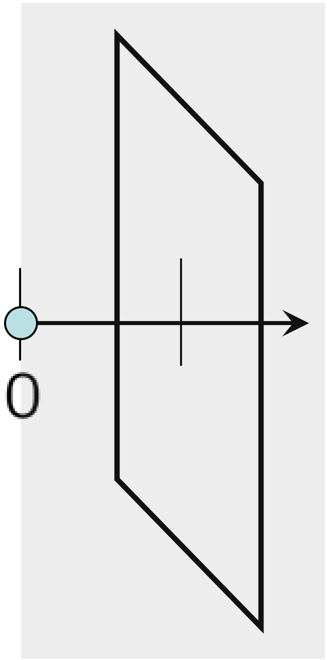
Nc electric charge → (Nc-1) electric charge + D3

The instanton (=D3) should be 1/Nc electrically charged !

Electrically charged instanton is given by a CS term

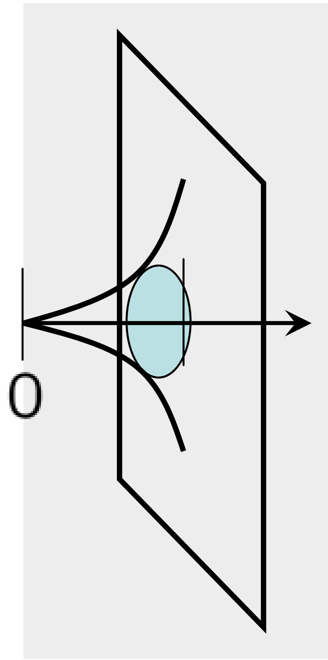
Sugra back reaction should give 1/Nc RR 3-form flux !

# Our result



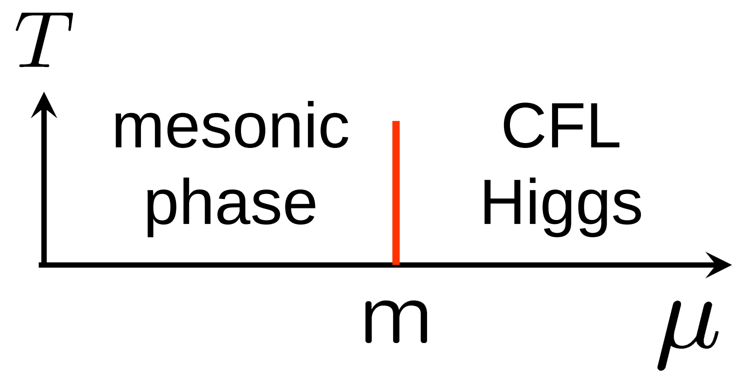

$\mu < m$

“Minkowski”
embedding




$\mu > m$

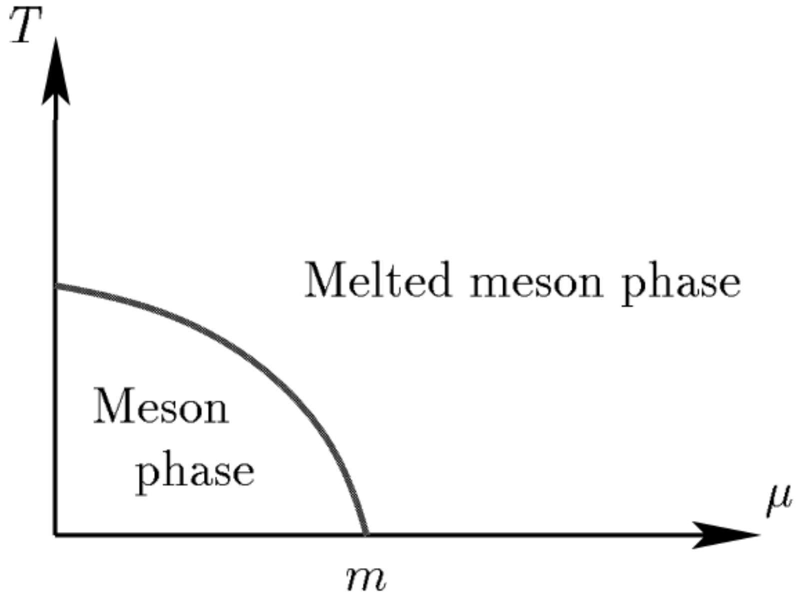
“Black Hole”
embedding
+
D3 dissolved




CFL Higgs phase is favored for $\mu > m$

# Phase transition at finite T

The phase diagram proposed so far is




BH embedding is favored for large $\mu, T$

[Kobayashi,Mateos,Matsuura, Myers,Thomson (06)]
[Nakamura,Seo,Sin,Yogendran (07)]
[Gohroku,Ishihara,Nakamura(07)]
[Karch,O'Bannon(07)]

What will happen if we include the D3-dissolving mechanism?

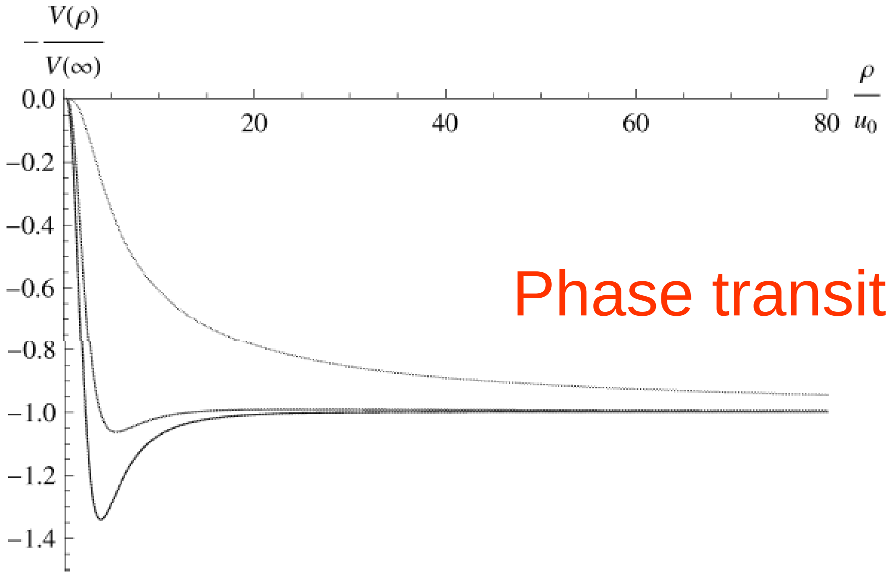
# Thermal potential

For the BH embedding with finite temperature at zero baryon density, it is known that instanton moduli potential is induced and minimized at finite size

[Apreda, Erdmenger, Evans, Guralnik(05)]

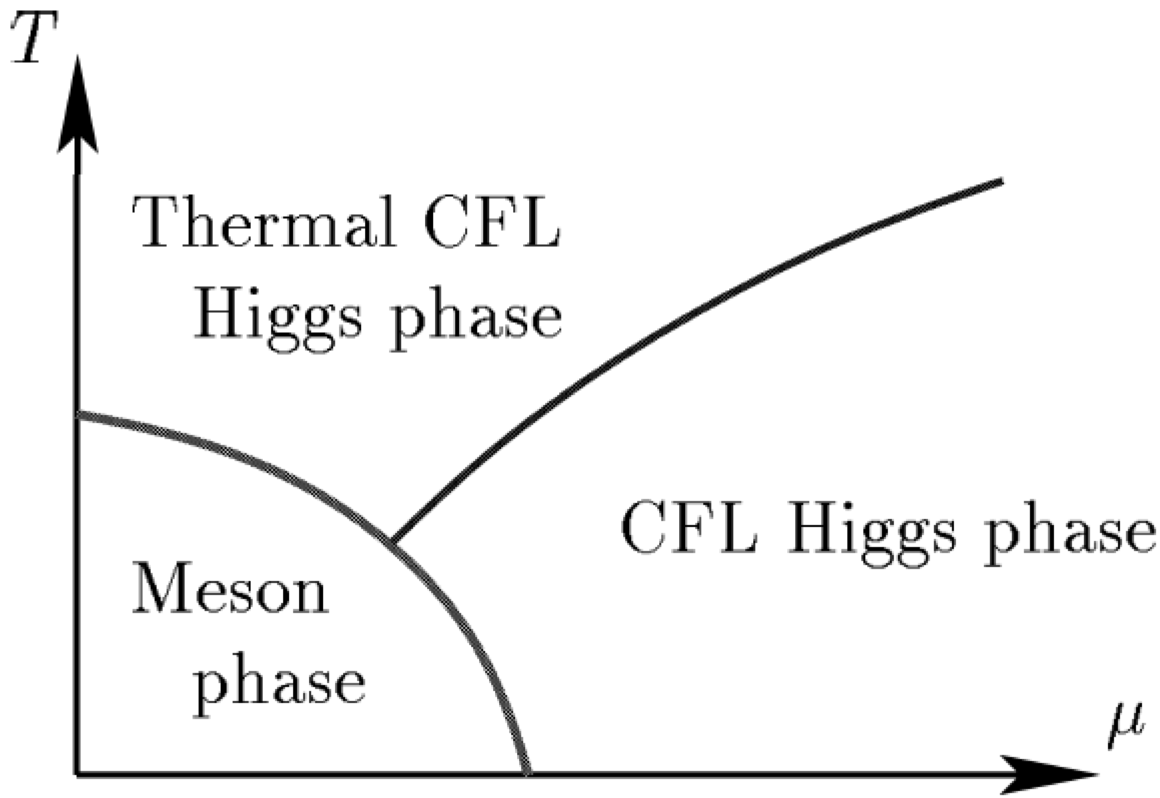
Adding the baryon density induces a potential

⇒ Competition with the thermal potential




Phase transition

# Resultant phase diagram



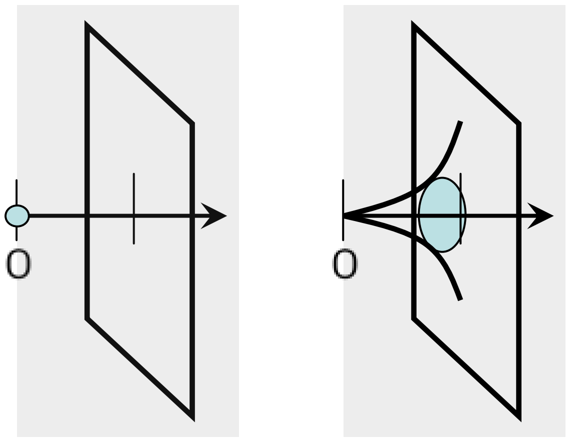
# 6. Conclusion

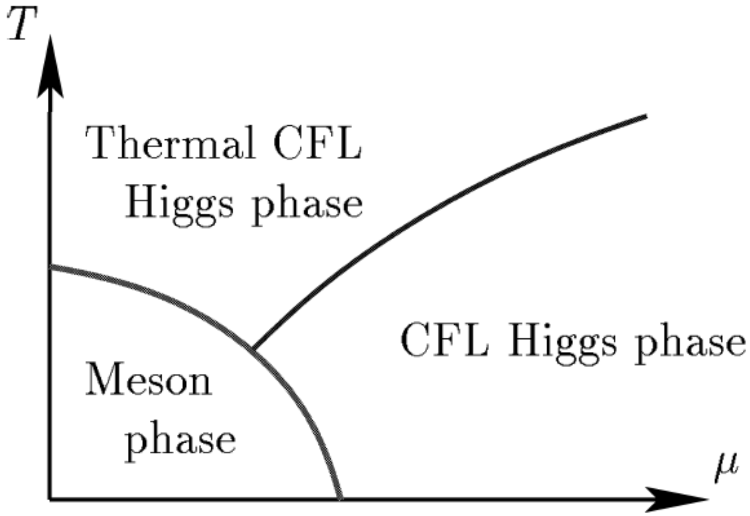
# Summary

In holographic *N*=2 SQCD, we show CFL via dynamical squark condensation for chemical potential > squark mass

Physics involved :

- SUGRA back reaction
- Dissolution of D3 into D7
- Baryon vertex and charge conservation
- Induced CS term and instanton size moduli



# Questions

Problem1) We separate one D3-brane from the other.

← No justification. I am just afraid of trying to solve the full EOM of Sugra with complete backreaction.

Problem2) We computed only RR 3-form / NSNS 2-form backreaction.

← It is just an assumption that the D7 backreaction can be ignored. Again, I am just afraid of …

Necessity of more Sugra analysis!