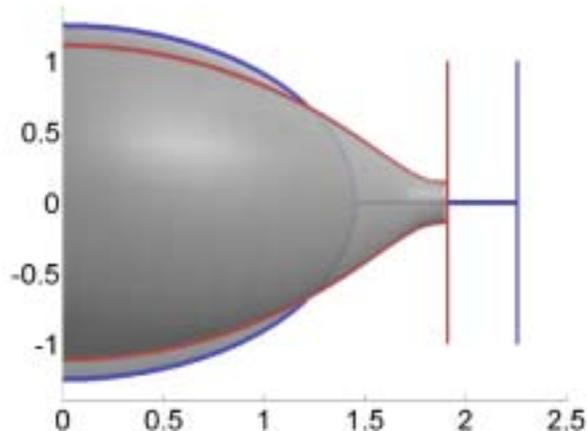


Properties of Kaluza-Klein Black Holes *- From BHs to Black string ? -*

hep-th/0310104

[Hideaki Kudoh \(Kyoto Univ. Japan\)](#)

[Work with Toby Wiseman \(Harvard\)](#)



- ✓ BS & KK BH, phase transition
- ✓ Results (Embedding geometry & thermodynamics)
- ✓ Decompactification
- ✓ Summary

Can we test extra dimensions using strong gravity? Eg. with astrophysical observations of black holes

Black hole phenomenology is much more subtle with extra dimensions.

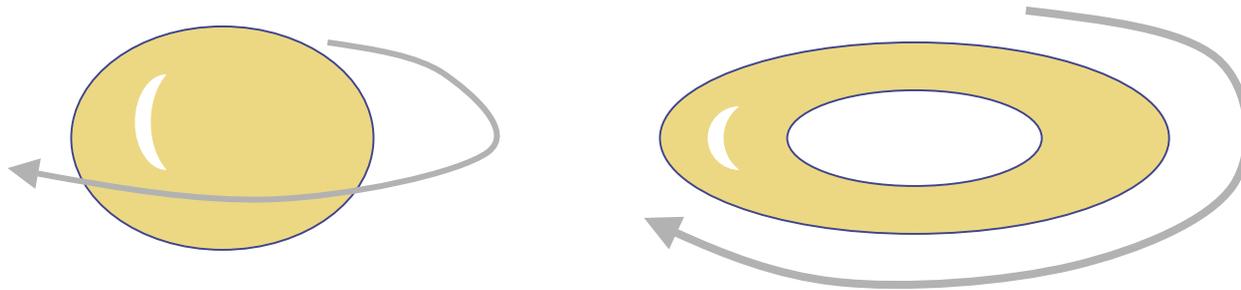
- More dimensions more complex behaviour
 - ◆ Uniqueness theorems, black strings, Gregory-Laflamme instability

- Understanding Kaluza-Klein vacuum solutions
 - ◆ Black holes, black strings ...?

- Do Randall-Sundrum black holes exist?
 - ◆ AdS-CFT  astrophysical black holes are not static!

4-d 5-d ; asymptotically flat case

- In 4-d Schwarzschild and Kerr are unique asymptotically flat, regular vacuum solutions
- Recently Emparan-Reall showed 5-d rotating vacuum black ring solution in addition to Myers-Perry solution



→ Implies lack of uniqueness in higher dimensions

- Now Schwarzschild is proven unique if static *and* stable

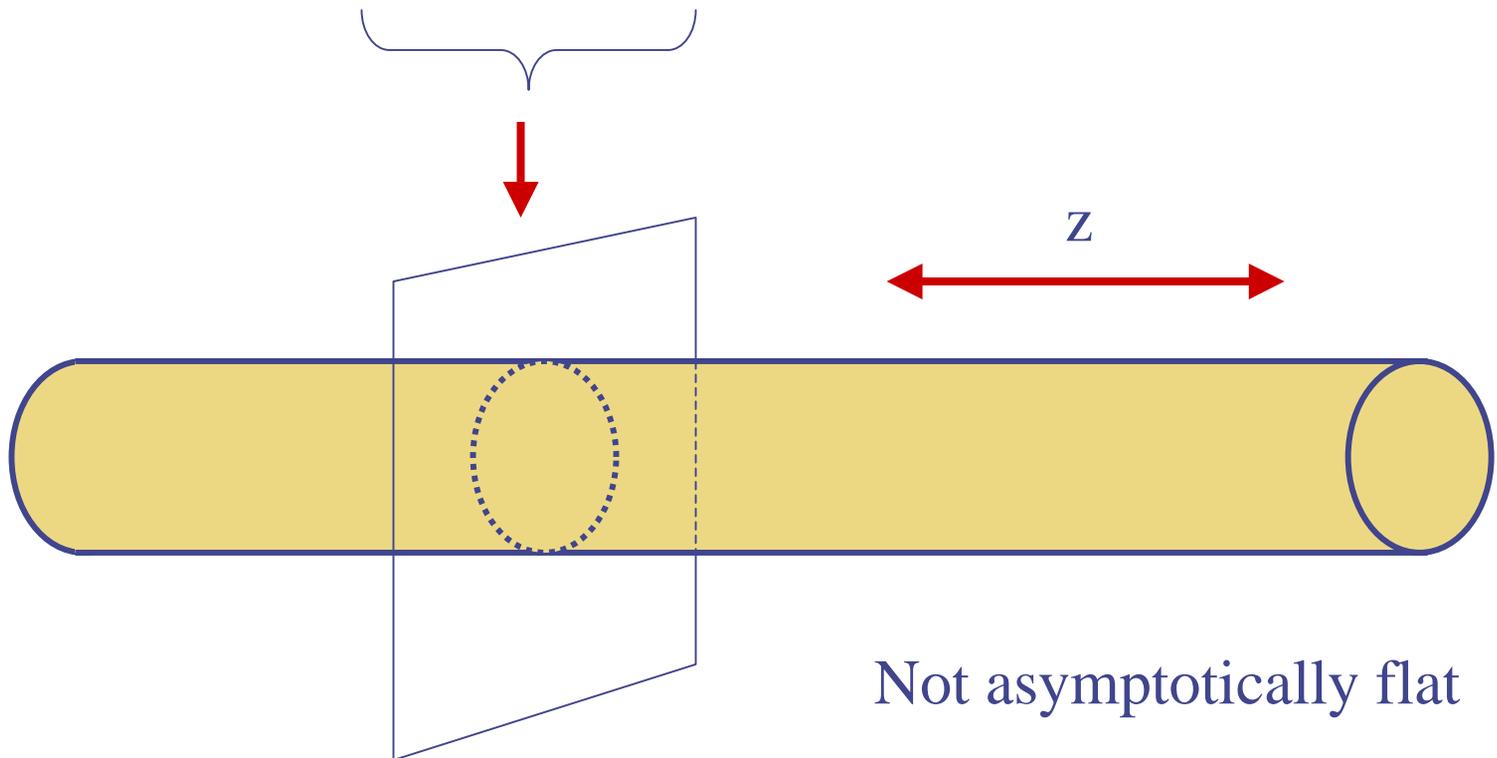
[Gibbons, Ida, Shiromizu '02; Maeda, Ishibashi '03]

- But obviously no uniqueness in general and stability not yet solved for stationary cases

4-d 5-d ; non-asymptotically flat case

- Unlike 4-d, in 5-d or above one has black string solutions
- Uniform vacuum string is just a product geometry
 - ◆ Can have any radius horizon

$$ds^2_{(5d)} = ds^2_{(4d \text{ Schwz})} + dz^2$$



BS vs KK BH

At least two sectors,

Black String Sector ($S^{n-1} \times R$)

■ Gregory-Laflamme instability ('93)

linear pert. redistribution of mass

What is the final state?

an array of BHs ?

new branch of BS solutions ? (No uniqueness)

■ Horowitz-Maeda ('01)

no pinching off in finite affine time

■ Higher order (Gubser '02), and full non-linear solutions (Wiseman '02).

■ Phase Diagram (Kol '02).

■ Other Aspects of BS (and the transition).

Dynamical/Thermo-dynamical stability, Dynamical evolution of B.S. (Choptuik et. Al.), Susskind's argument (Gross-Witten transition ?), A model of BH merger, Wide relevance for higher dim branes and extra dims.

Black Hole Sector (S^n)

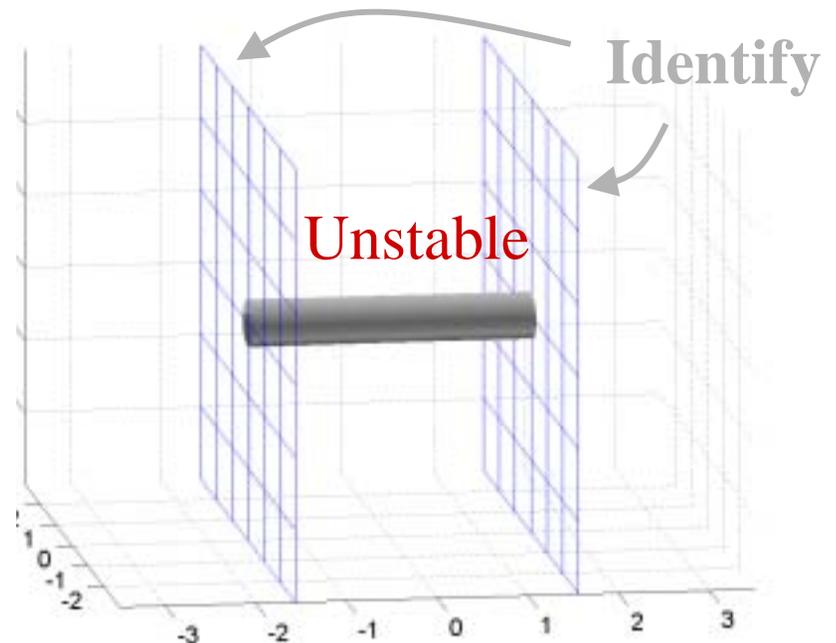
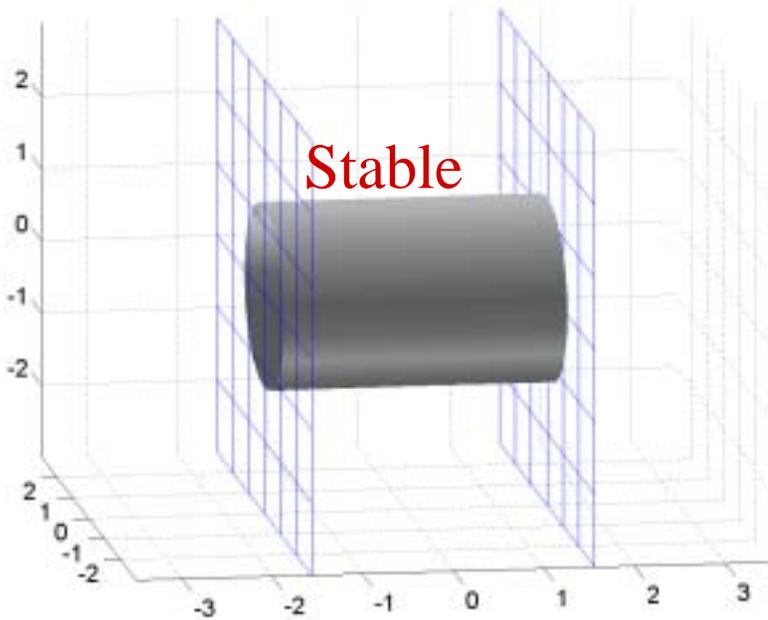
KK BH = BH in KK comp. spacetime.

we have numerically constructed KK BHs and compare it with BS sector.

We want to obtain the whole and complete picture !!

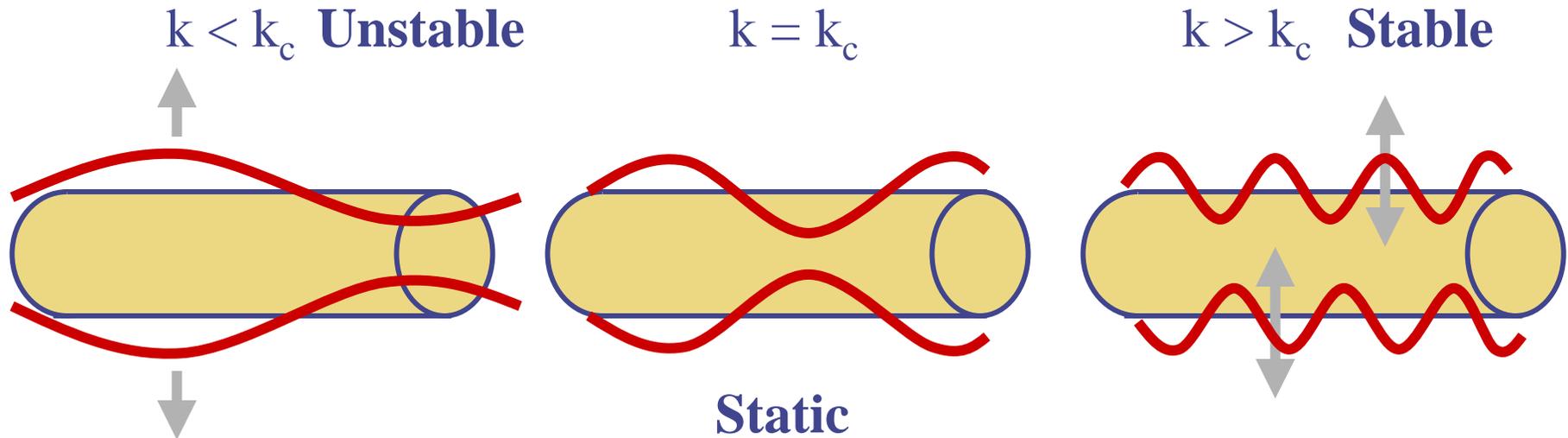
Black String Sector in KK

- Can simply compactify uniform strings
- Choose to fix asymptotic radius of compactification, L , away from symmetry axis
- `Thick' strings are *stable* as infrared GL instability projected out by periodic boundary conditions



- Uniform strings are unstable in infrared
 - s-wave metric perturbation;

$$\delta g_{\mu\nu} = e^{\Omega t} e^{i k z} f_{\mu\nu}(r)$$



End state of decay unknown

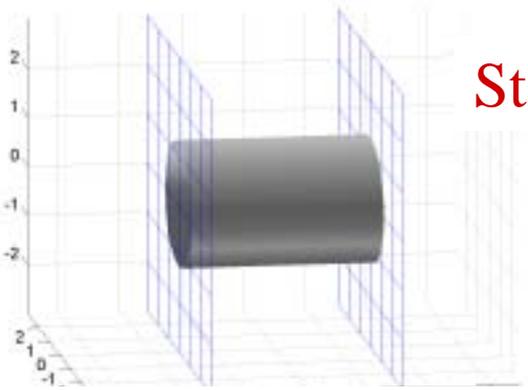
[Horowitz, Maeda '01; Choptuik et al;'03]

New branch of *non-uniform* strings!

- Myers-Perry solution may have GL-like instability

[Empanan, Myers '03]

(6D) Non-Uniform BS sector (Wiseman CQG '03)



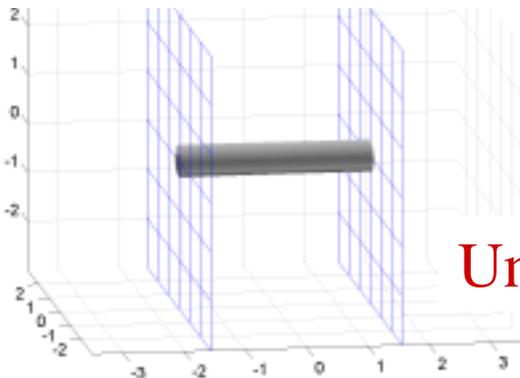
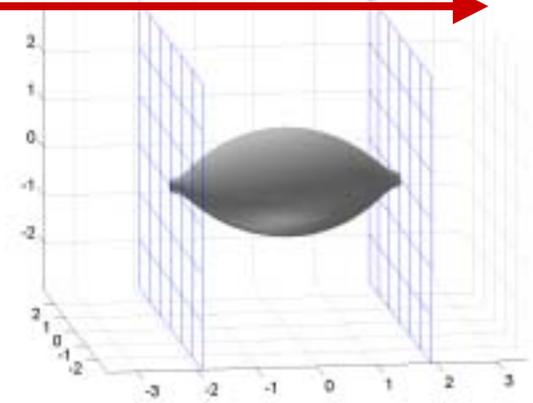
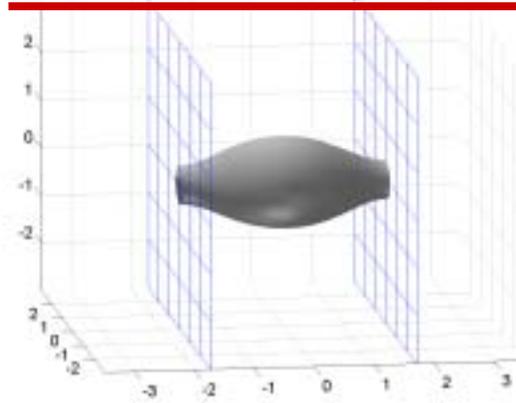
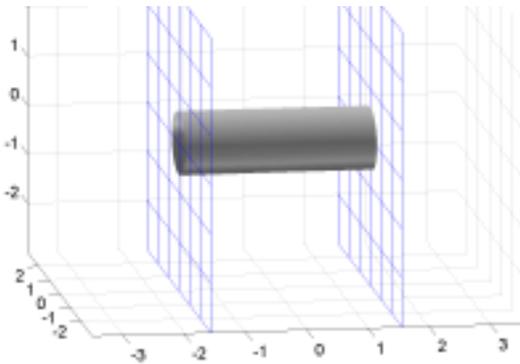
Stable

Question:

Is this the only solution at large masses?

Turn on GL static mode...

Critical uniform string

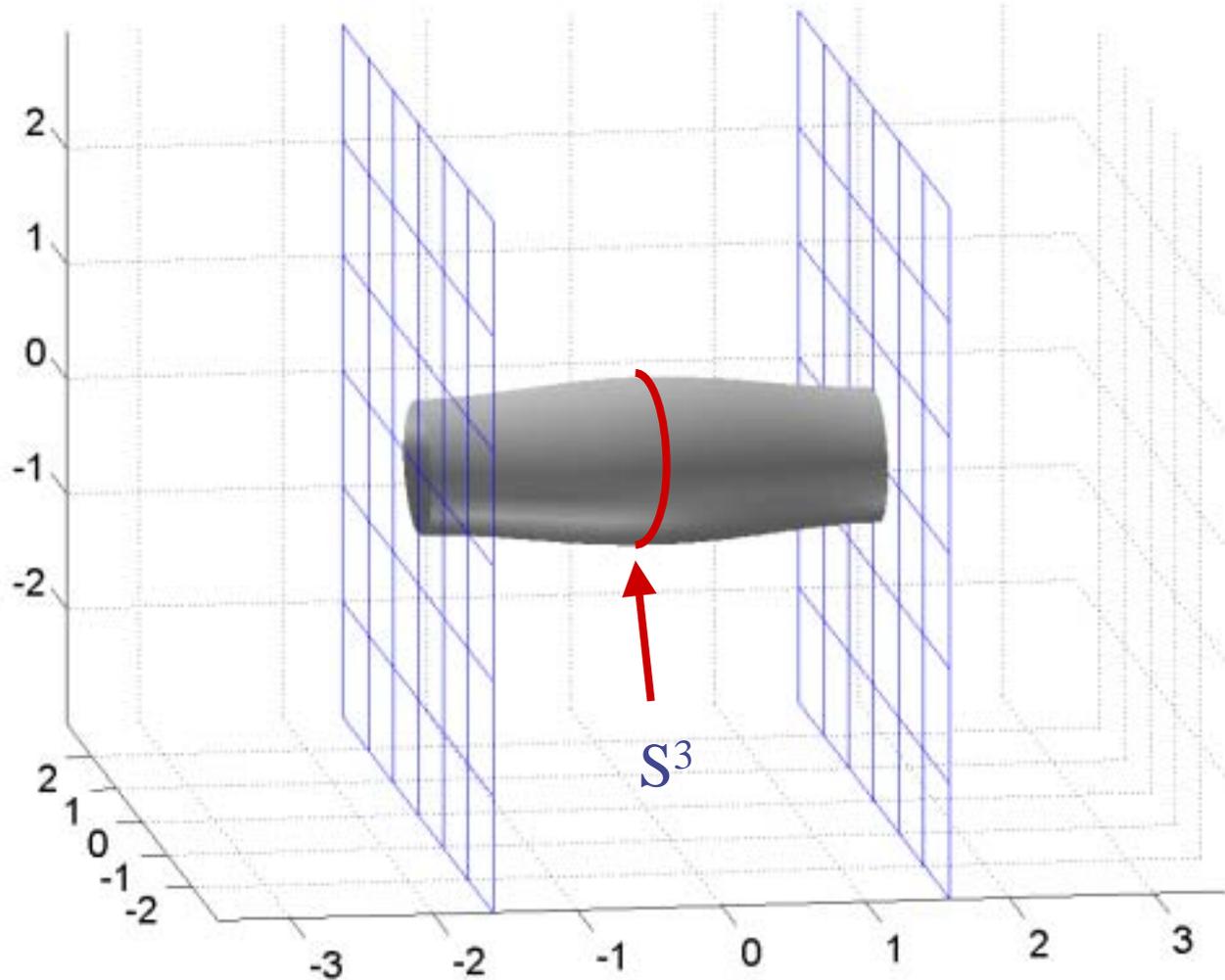


Unstable

- First numerically constructed perturbatively [Gubser '01]
- Then non-pert [Wiseman '02]

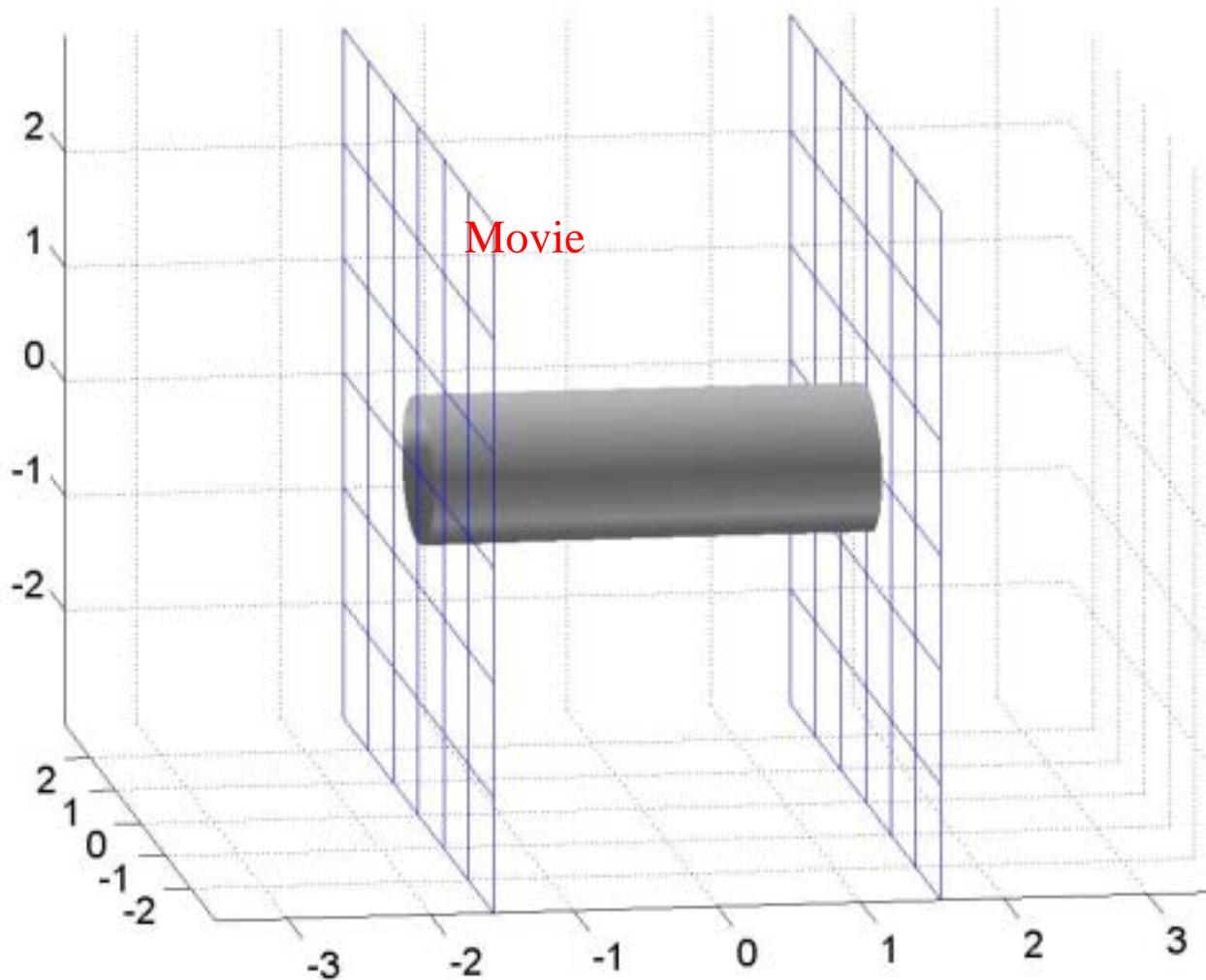
Cont... 6D Non-uniform strings (Wiseman '03 CQG)

Fix asymptotic compactification length, L



Cont... 6D Non-uniform strings (Wiseman '03 CQG)

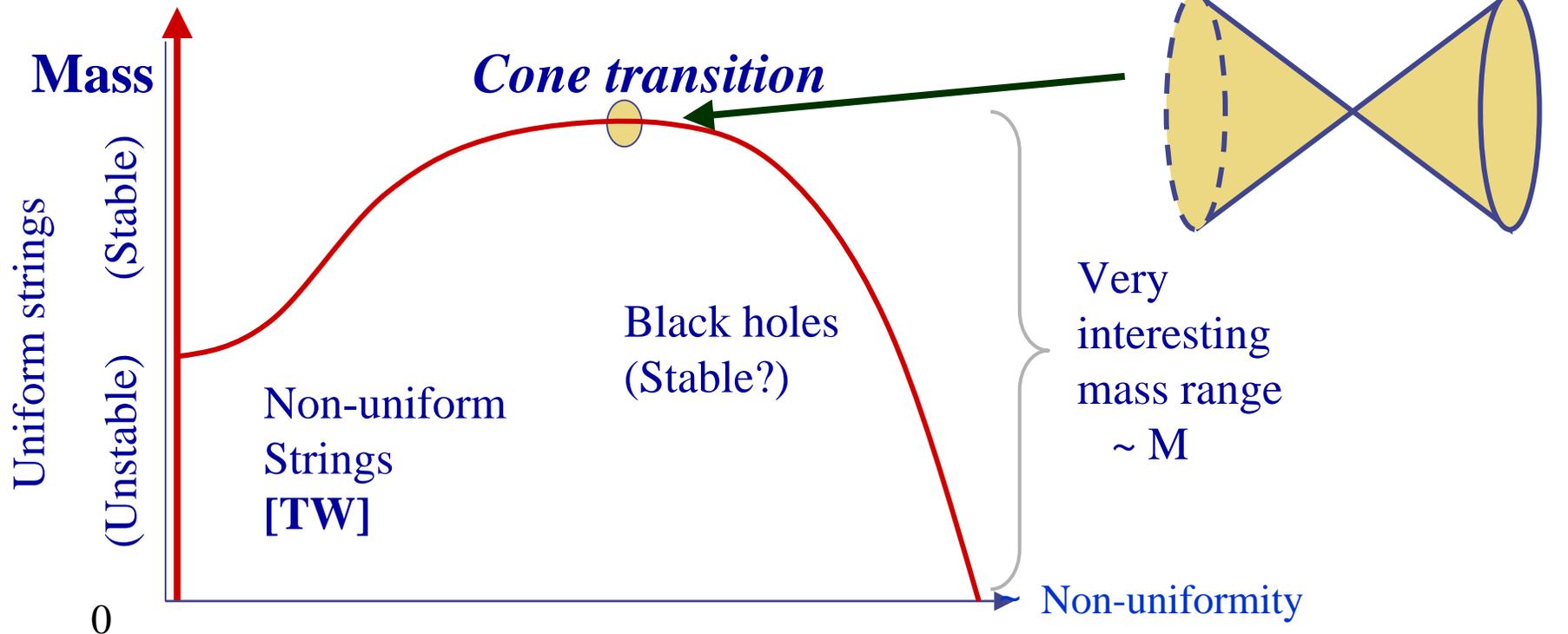
Fix asymptotic compactification length, L



Kol's Conjecture (Kol'02, Gubser'02)

- If transition picture is correct, very likely to be maximum mass for black hole solutions
 - ◆ For 'large' masses, uniform string unique solution (No corrections to 4d)
- Horizon topology change point (naked sing.)
= Phase transition point

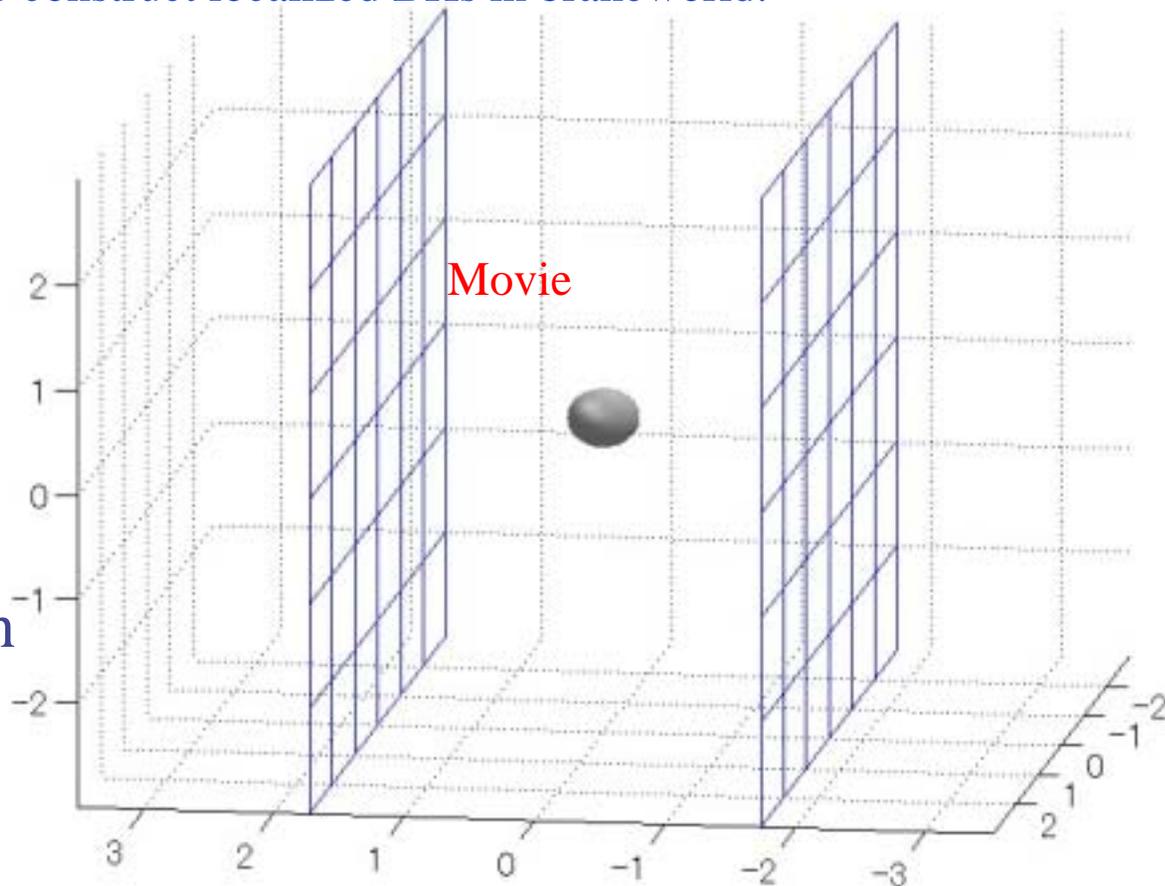
$$\mu = \frac{\beta}{8\pi L} \propto M$$



Cont... 6D Black holes in KK (HK & Wiseman)

- Is Kol's picture correct ?
- We constructed non-wrapping black hole numerically !!
 - ◆ At least BH should exist for $R_{\text{horizon}} \ll L$ (geometry ~ higher dim Schwarzschild)
 - ◆ Analytic construction only in 4d [Myers '87 , A. Frolov & V. Frolov '03]
- Method is the same as that to construct localized BHs in braneworld.

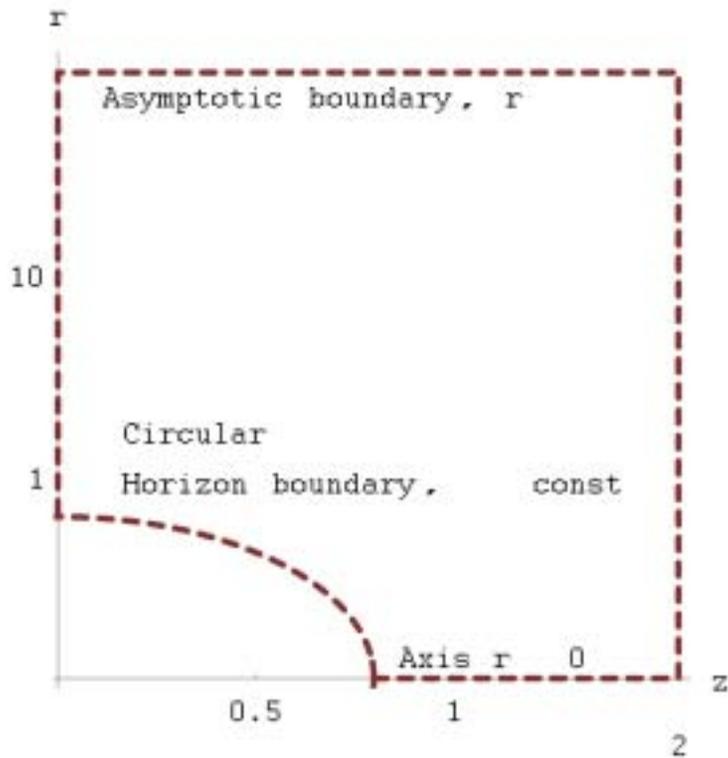
(HK, Tanaka, Nakamura)



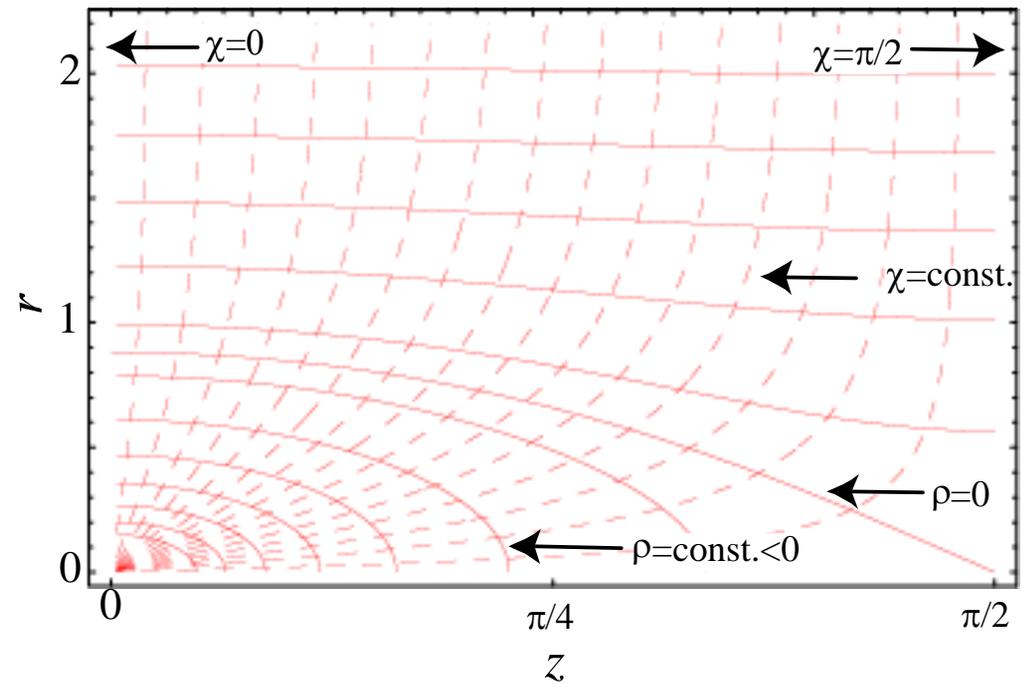
Fix asymptotic
compactification length

Coordinate system

Boundary conditions

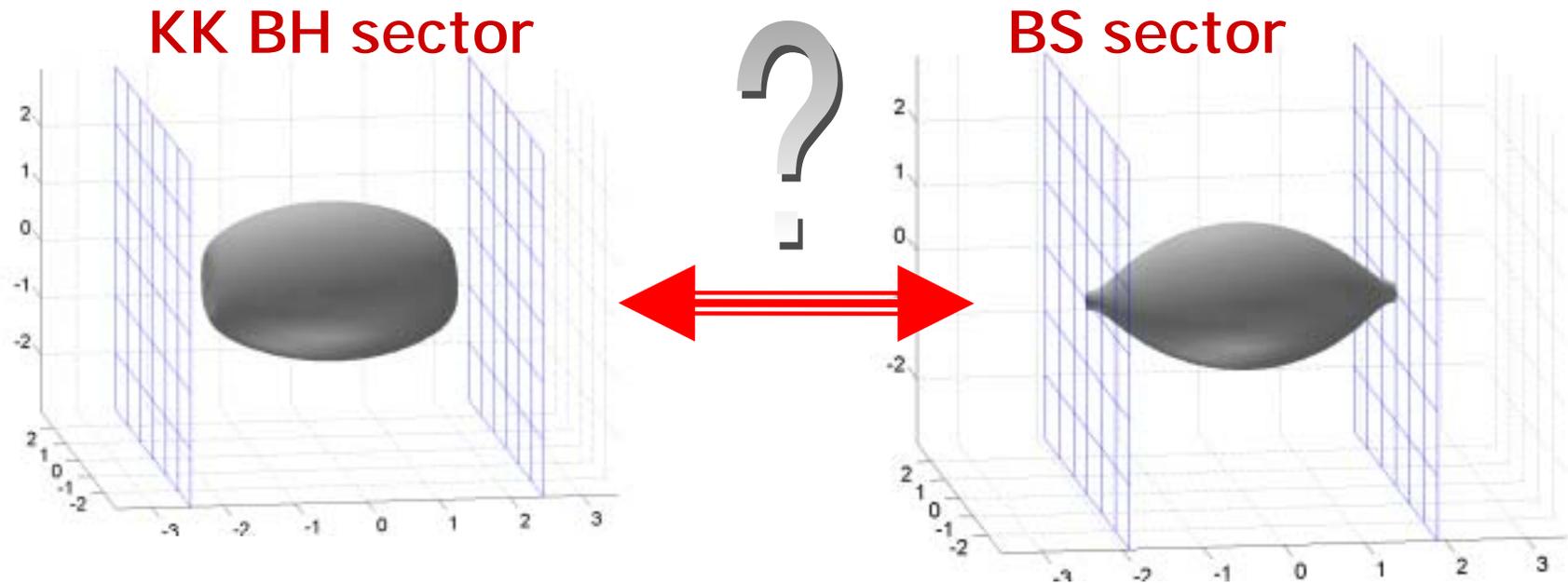


Coordinate system



(KK) Black hole sector (H.K. & Wiseman)

Is the transition between two solutions possible ?



Our conclusion is

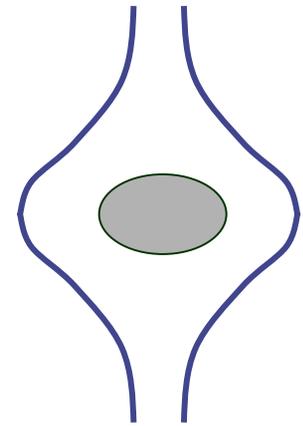
Non-uniform BS cannot be connected to *this* branch of KK BHs in Kol's sense.

(Other unknown branches might be possible...)

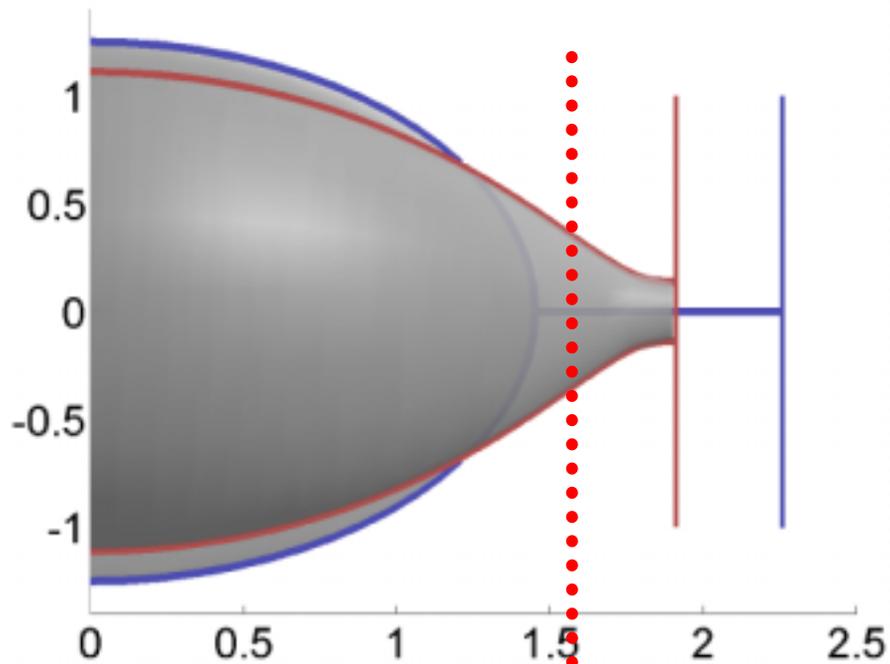
Decompactification

Detailed discussion of embedding geometry

- Asymptotic radius = r_{∞} , & Radius of largest BH $\sim r_{\infty}$
- BH is **bigger** in all aspects compared with BS.
- Axis geometry decompactifies, making 'room' for horizon

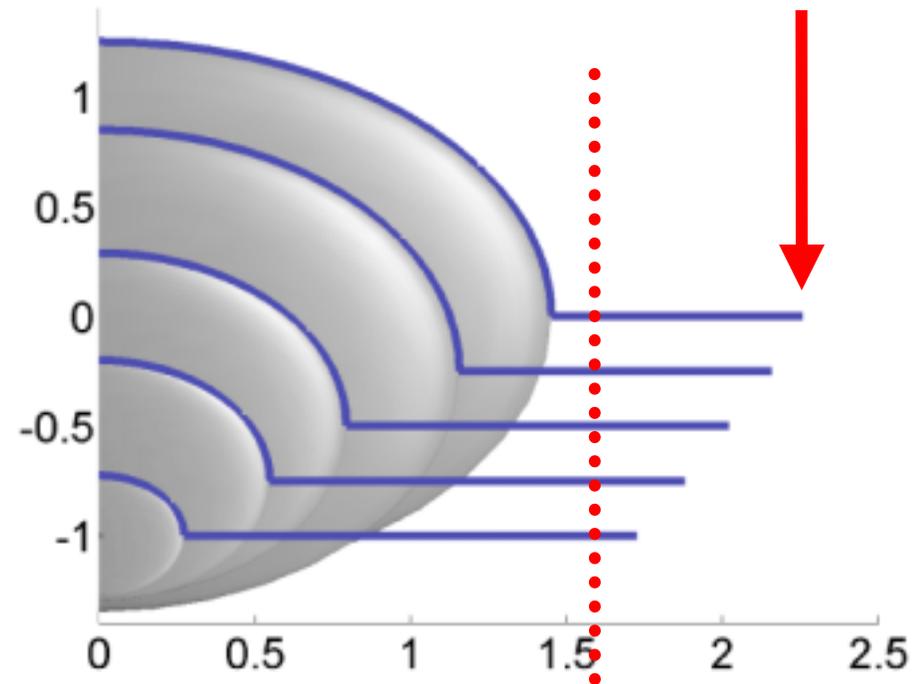


Highly non-uniform BS



Half period of asymptotic radius.

Compactified radius on the axis

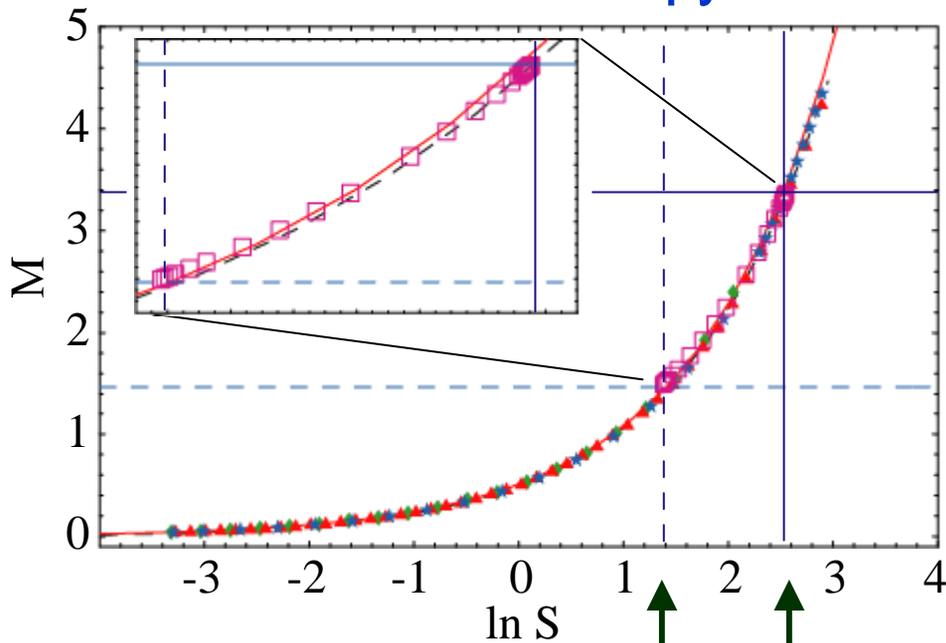


Half period of asymptotic radius.

Thermodynamics

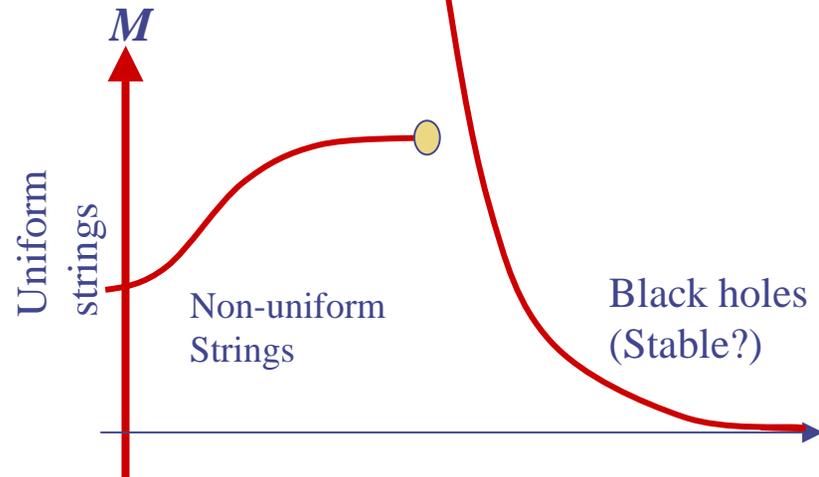
- Mass and Entropy of KK BH are larger than those of BS
- Temperature also shows similar behavior
- No naked singularity appears in our calculations
- Construction of much larger BHs is possible (limited by CPU times/resolutions)
- Deviation from 6D Schwarzschild BH is very small.

Mass vs Entropy



Critical uniform string
Maximally non-uniform BS

Mass does not match with
non-uniform BS



Summary

hep-th/0310104

Black hole phenomenology is much more subtle with extra dimensions.

- ◆ Haven't mentioned bulk matter, stabilization...

■ *BH sector*; 6D KK BH construction

We found

- ◆ **Non-uniform BS cannot be connected** to *this* branch of KK BHs
- ◆ **Dramatic decompactification** occurs.
- ◆ **Three** kind of solutions for the *same mass*

Uniform BS, non-uniform BS, and KK BH

Our numerical calculations are limited by CPU times (numerical resolutions).
But much larger BHs will be possible (in progress).

■ Future work is required to resolve

(i) Whether there is *a maximum mass* for KK BH

→ *BH phenomenology (PBH, induced gravity)*

→ *Two* solutions for very large mass ?

(ii) What is the role of radius stabilization → *generality of decomp. ?*

Summary 2

Related works

- ◆ *5-dimensional KK BH (Numerical approach)*

Kol, Sorkin, and Piran (hep-th/0309190, 0310096)

(i) They could not construct large BHs to discuss BS/BH transition.

(ii) They interpreted numerical instabilities

“as evidence for *an approach to a physical instability*”

However, as we showed, there is no such phenomena. It would be pure numerical instabilities.

- ◆ *Perturbative approach.* Kol et. Al. (in preparation)

Our results show perfect agreements with their results

- ◆ *New phase diagrams.*

Harmark and Obers (hep-th/0309230, 0301206)