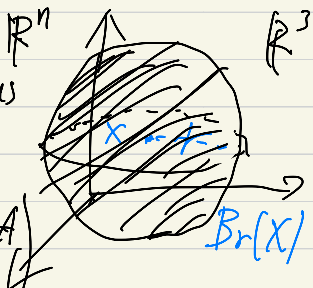
Tarumizu Rintaro 082350588

○ Definition Summary of "Test functions and distributions"

- smooth functions – When we can differentiate "f" for any indeices, this "f" is called "smooth function." The set of all smooth function is denoted by "$C^\infty(\mathbb{R}^n)$".

- $B_r(x)$ – this is the open ball in $\mathbb{R}^n$ centered at "x" and its radius is "r".



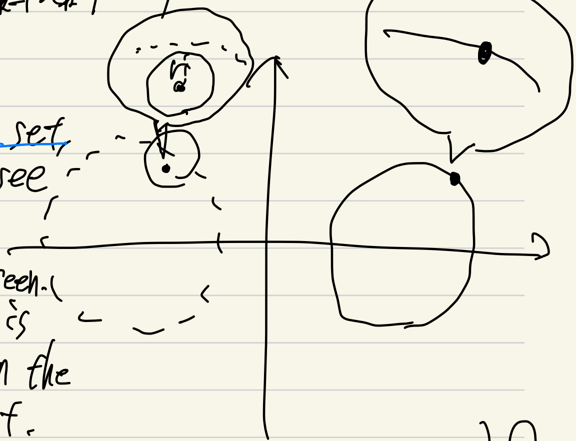
Namely, $B_r(x) := \{y \in \mathbb{R}^n \mid \|x-y\| < r\}$

※ "A := B" means that definition of A is B

$\|X\| := \sqrt{x_1^2 + x_2^2 + \cdots + x_n^2} = \left(\sum_{k=1}^{n} x_k^2\right)^{\frac{1}{2}}$

- open and closed set

If point "X" is in the open set, $B_r(x)$ exists, because, you see, X closer and closer, the gap between X and boundry can be seen. On the other hand, if point "X" is in the closed set and it is on the boundry, $B_r(x)$ doesn't exist.

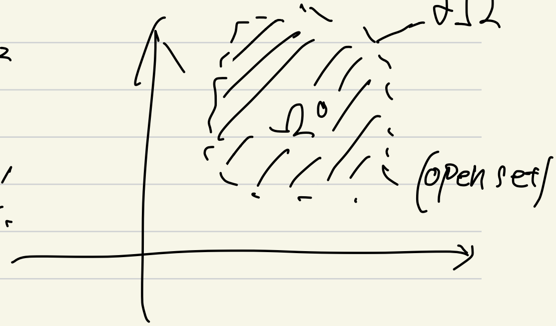


Let Ω be a subset of $\mathbb{R}^2$

$\partial\Omega$ means the boundry of Ω

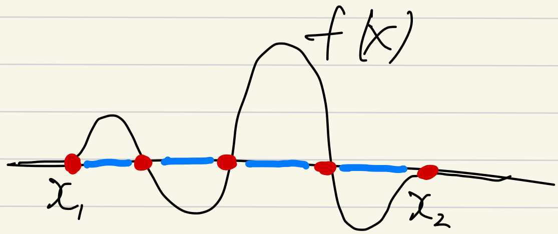
Ω° means the interior of Ω, and doesn't include the boundry.

$\Omega = \Omega^\circ \cup \partial\Omega, \quad \Omega^\circ \cap \partial\Omega = \emptyset$



- support

$\mathrm{supp}(f) := \overline{\{x \in \mathbb{R}^n \mid f(x) \neq 0\}}$



this overline means the closure of the set in $\mathbb{R}^n$.
"closure" makes open sets into closed set.

- Test function - "Test function" = smooth function $\cap$ support
This is denoted by $\mathcal{D}(\mathbb{R}^n)$, so $\mathcal{D}(\mathbb{R}^n) :=$
$\{f \in C^\infty(\mathbb{R}^n)$ with $\mathrm{supp}(f)$ bounded$\}$.

- Convergence in $\mathcal{D}(\mathbb{R}^n)$ -

When $\lim_{j \to \infty} \sup_{x \in \mathbb{R}^n} |\partial^\alpha f_j(x) - \partial^\alpha f_\infty(x)| = 0$ for any $\alpha \in \mathbb{N}^n$ and there exists $r \in \mathbb{R}$ large enough, we write
$f_j \to f_\infty$ in $\mathcal{D}(\mathbb{R}^n)$ as $j \to \infty$

※ sup $\simeq$ max, $\|g\|_\infty := \sup_{x \in \mathbb{R}^n} |g(x)|$

When $\Omega = [a, b]$, $\sup \Omega = \max \Omega = b$,
but when $\Omega = (a, b)$, $\sup \Omega = b$ but $\max \Omega$ does not exist.