

Alberto-John Thornton

Katayama Marin

Ex: $h(x) = |x|$ on \mathbb{R}

What is δT_h ?

$$[\delta T_h](f) = -T_h(f') \quad \text{by definition}$$

$$= -\int_{-\infty}^{\infty} |x| f'(x) dx$$

$$= -\int_0^{\infty} x f'(x) dx + \int_{-\infty}^0 x f'(x) dx$$

$$= -x f(x) \Big|_0^{\infty} + \int_0^{\infty} f(x) dx + x f(x) \Big|_{-\infty}^0 - \int_{-\infty}^0 f(x) dx$$

$$T_h(f) = \int_0^{\infty} f(x) dx - \int_{-\infty}^0 f(x) dx$$

$$\therefore [\delta T_h](f) = T_{\tilde{h}}(f)$$

$$\text{where } \tilde{h}(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ -1 & \text{if } x < 0 \end{cases}$$