Exercise 2.6.4

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I am going to show that the map $f \mapsto \|f\|_1$ defines a norm on $L^1(\Omega)$.

$$\|\lambda f\|_1 = \int_\Omega |\lambda f(x)|\,dx$$
$$= |\lambda| \int_\Omega |f(x)|\,dx$$
$$= |\lambda|\,\|f\|_1$$

Therefore, 1. $\|\lambda \xi\| = |\lambda|\|\xi\|$ for any $\lambda \in \mathbb{C}$, $\xi \in \mathcal{H}$

$$\|f+g\|_1 = \int_\Omega |f(x)+g(x)|\,dx$$
$$\leq \int_\Omega |f(x)|\,dx + \int_\Omega |g(x)|\,dx$$
$$= \|f\|_1 + \|g\|_1$$

Therefore, 2. $\|\xi_1 + \xi_2\| \leq \|\xi_1\| + \|\xi_2\|$ for any $\xi_1, \xi_2 \in \mathcal{H}$

$\|f\| = \int_\Omega |f(x)|\,dx = 0$ if and only if $f = 0$ because the set $L^1(\Omega)$ is defined by $\mathcal{L}(\Omega)/\sim$.

The map $f \mapsto \|f\|_1$ defines a norm on $L^1(\Omega)$.