Date

Exercise 3.1.10

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There is f_1, f_2 such that they satisfy

$$\langle f_1, g \rangle = 0 \qquad \langle f_2, g \rangle = 0$$

Then, from the second property after definition 3.1.1,

$$\langle f_1 + \lambda f_2, g \rangle = \langle f_1, g \rangle + \bar{\lambda} \langle f_2, g \rangle = 0$$

stands , so $M^\perp$ is linear.

Let $(f_j)_{j \in \mathbb{N}} \subset M^\perp$ be a sequence converging strongly to $f_\infty \in \mathcal{H}$. From the linearity property for the inner product,

$$\lim_{j \to \infty} \langle f_j - f_\infty, g \rangle = \lim_{j \to \infty} (\langle f_j, g \rangle - \langle f_\infty, g \rangle)$$

In this equation $\lim_{j \to \infty} \langle f_j - f_\infty, g \rangle = 0$ because f_j and g is in Hilbert space and satisfies the definition of weak convergence. Also, $\lim_{j \to \infty} \langle f_j, g \rangle = 0$ from property 1 of example 3.1.9. Therefore, $\langle f_\infty, g \rangle = 0$ Hence, inner product $= 0$ and the sequence converges in $M^\perp$.

In conclusion, $M^\perp$ is a closed subspace because it is linear and the sequence converges in $M^\perp$.