No.

Date

$2 \to 1$

is [illegible]

$3 \to 4$

Exercise 2.3.4

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From 1, the function f is Lebesgue measurable, so $\{x \in [a,b] \mid f(x) > s\}$ is Lebesgue measurable. From theorem 2.2.10, if Ω is a Lebesgue measurable set, then its complement Ω^c is also a Lebesgue measurable set. Therefore, 3 stands.

Also,

$$\{x \in [a,b] \mid f(x) < s\} = \bigcup_{k=1}^{\infty} \{x \in [a,b] \mid f(x) \leq s - \tfrac{1}{k}\}$$

Hence, if 3 is true, $\{x \in [a,b] \mid f(x) \leq s - \frac{1}{k}\}$ is a measurable set for every k. From 3 after definition 2.2.7, $\{x \in [a,b] \mid f(x) < s\}$ is a measurable set. Consequently, 4 stands.

From 4, the sets $\{x \in [a,b] \mid f(x) < s\}$ are Lebesgue measurable sets. From theorem 2.2.10, 2 stands.

Also,

$$\{x \in [a,b] \mid f(x) > s\} = \bigcup_{k=1}^{\infty} \{x \in [a,b] \mid f(x) \geq s + \tfrac{1}{k}\}$$

If 2 is true, $\{x \in [a,b] \mid f(x) \geq s + \frac{1}{k}\}$ is a measurable set for every k. From 3 after definition 2.2.7, $\{x \in [a,b] \mid f(x) > s\}$ is a measurable set. Consequently, 1 stands.

In conclusion, the 4 statements are equivalent.