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$$\delta^\alpha T_h(f) = (-1)^{|\alpha|} T_h(\delta^\alpha f) \quad (\text{by def. of derivatives of distributions})$$

$$= (-1)^{|\alpha|} \int_{\mathbb{R}^n} h(x) [\delta^\alpha f](x) dx \quad (\text{by def. of } T_h)$$

$$= \int [\delta^\alpha h](x) f(x) dx \quad (\text{by applying the integration by parts several times})$$

$$= T_{\delta^\alpha h}(f) \quad (\text{by def. of the regular distribution defined by } \delta^\alpha h)$$

$$\therefore \delta^\alpha T_h = T_{\delta^\alpha h} \quad \text{for } h \text{ sufficiently differentiable}$$