Reminder VIII

equivalence relation, being equal a.e.

- $\underline{L^1(\Omega)} := \mathcal{L}(\Omega)/\sim$, with the norm

$\|f\|_1 := \int_\Omega |f(x)|\, dx$.

3 properties

⇝ Precise definition of $L^1_{loc}(\mathbb{R}^n)$

For $p \geq 1$

- $\underline{L^p(\Omega)} := \{ f : \Omega \to \mathbb{K},\ \mathcal{L}.\text{m.} \mid \int_\Omega |f(x)|^p\, dx < \infty \}/\sim$

with the norm $\|f\|_p := \left(\int_\Omega |f(x)|^p\, dx \right)^{1/p}$.

- Hölder inequality : if $p, q \geq 1$, $\frac{1}{p} + \frac{1}{q} = 1$, and $f \in L^p(\Omega)$

$g \in L^q(\Omega)$, then $fg \in L^1(\Omega)$ and $\|fg\|_1 \leq \|f\|_p \|g\|_q$.

- ess sup, ess inf and $\underline{L^\infty(\Omega)}$ with norm

$\|f\|_\infty := \operatorname{ess\,sup} |f|$.

- Aproximation of elements of $L^p(\Omega)$ by continuous elements of $L^p(\Omega)$.