Reminder VII

- $\mathcal{L}([a,b])$ = set of all Lebesgue integrable functions on $[a,b]$
  function can be bounded or unbounded.
- If $f \in \mathcal{L}([a,b])$ and $\Omega \subset [a,b]$, Lebesgue measurable,
  characteristic function on Ω
  then $\int_\Omega f(x)\,dx := \int_a^b (f\chi_\Omega)(x)\,dx$
- For $f : [a,\infty) \to \mathbb{R}_+$, $\int_a^\infty f(x)\,dx := \lim_{b\to\infty} \int_a^b f(x)\,dx$
  if $f\chi_{[a,b]} \in \mathcal{L}([a,b])$ and the limit exists.
  For general f : $\int_a^\infty f(x)\,dx := \int_a^\infty f_+(x)\,dx - \int_a^\infty f_-(x)\,dx$
  if both expressions exist.

{Improper Riemann integrable functions on $[a,\infty)$}

$\neq \mathcal{L}([a,\infty))$. Two limit treated independently.

Same extensions for $\mathcal{L}((-\infty,b])$ and $\mathcal{L}(\mathbb{R})$.

- Pointwise convergence + dominated convergence thm.
  the dominating function is essential.
- Extension to $\mathcal{L}(\Omega)$ with $\Omega \subset \mathbb{R}^n$, closed or open bounded subset. Extension to complex valued functions.
- Fubini's theorem ← exchange of order of integration
- On $\mathcal{L}(\Omega)$, $f \sim g$ if $f = g$ a.e.