Reminder V

- Closed box I with volume $v(I)$,
- Covering $S = \{I_j\}_j$ of $\Omega \subset \mathbb{R}^n$ by closed boxes,

  finite or countable

  with $\sigma(S) := \sum_j v(I_j)$,
- Lebesgue outer measure $m^*(\Omega) = \inf_S \sigma(S)$.
- If $\Omega_1 \subset \Omega_2$, $m^*(\Omega_1) \leq m^*(\Omega_2)$

  $m^*(\Omega_1 \cup \Omega_2) \leq m^*(\Omega_1) + m^*(\Omega_2)$

  it is possible

  but even if $\Omega_1 \cap \Omega_2 = \emptyset$, $m^*(\Omega_1 \cup \Omega_2) < m^*(\Omega_1) + m^*(\Omega_2)$
- Lebesgue measurable set if $\forall \varepsilon > 0$, $\exists \Lambda$ open with $\Omega \subset \Lambda$ and $m^*(\Lambda \setminus \Omega) \leq \varepsilon$. Then $m(\Omega) := m^*(\Omega)$.

  Stable for finite or countable unions and intersections, and complement.
- Lebesgue measurable function $f : [a,b] \to \mathbb{R}$ if $f^{-1}((s,\infty))$ is measurable, $\forall s \in \mathbb{R}$.

  Example : simple functions on Lebesgue measurable sets

  stable for addition, multiplication, division (if the denominator is not 0).