Reminder IV

- Convergence in $\mathcal{S}(\mathbb{R}^n)$: $\| x^\beta \partial^\alpha (f_j - f_\infty) \|_\infty \xrightarrow{j \to \infty} 0$

- Tempered distribution : $T : \mathcal{S}(\mathbb{R}^n) \longrightarrow \mathbb{K}$ with

$T(f_j) \longrightarrow T(f_\infty)$ whenever $f_j \longrightarrow f_\infty$ in $\mathcal{S}(\mathbb{R}^n)$.

Set of tempered distributions : $\mathcal{S}'(\mathbb{R}^n)$.

$f \in \mathcal{S}(\mathbb{R}^n)$

- $\forall\, T \in \mathcal{S}'(\mathbb{R}^n)$, set $[\mathcal{F} T](f) := T(\underbrace{\mathcal{F} f}_{\in \mathcal{S}(\mathbb{R}^n)})$

$\mathcal{F} : \mathcal{S}'(\mathbb{R}^n) \longrightarrow \mathcal{S}'(\mathbb{R}^n)$ is bijective.

- Applications : $\mathcal{F} \delta_0 = T_{\frac{1}{(2\pi)^{n/2}} 1}$ ← cst function

constant function

$\mathcal{F} T_1 = (2\pi)^{n/2} \delta_0$

$\mathcal{F} T_h = T_{\hat{h}}$ if $h \in L^2(\mathbb{R}^n)$.

- $\mathcal{P}$ partition of $[a,b]$, $f \in \mathcal{L}^\infty([a,b])$

lower Riemann sum $L(f,\mathcal{P})$, upper Riemann sum $U(f,\mathcal{P})$

- f Riemann integrable if $\sup_{\mathcal{P}} L(f,\mathcal{P}) = \inf_{\mathcal{P}} U(f,\mathcal{P})$.

in this case, $\int_a^b f(x)\,dx$ for

increasing or decreasing

- Continuous functions, monotone functions, characteristique functions are Riemann integrable.