

Reminder III

- Summable distributions and summability order m
- Continuous extension to $C_b^m(\mathbb{R}^n) \equiv BC^m(\mathbb{R}^n)$

If $g \in \mathcal{D}(\mathbb{R}^n)$, $|g| = 1$, $f \in C_b^m(\mathbb{R}^n)$ and

$f_j(x) = g(\frac{x}{j}) f(x)$, then $T(f) := \lim_{j \rightarrow \infty} T(f_j)$.
bounded convergence property

- Fourier transform $\mathcal{F}f = \hat{f}$ for $f \in L^1(\mathbb{R}^n)$.

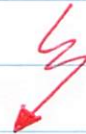
↳ some properties

preserves the norm



- \mathcal{F} extends to a bijective and isometric map on $L^2(\mathbb{R}^n)$.

- $\mathcal{F}\mathcal{D}(\mathbb{R}^n) \not\subset \mathcal{D}(\mathbb{R}^n)$



- Schwartz function $\|X^\beta \partial^\alpha f\|_\infty < \infty \forall \alpha, \beta$
 $\subset L^1(\mathbb{R}^n)$

$\mapsto \mathcal{D}(\mathbb{R}^n) \subset \mathcal{S}'(\mathbb{R}^n) \subset C_b^\infty(\mathbb{R}^n) \subset C^\infty(\mathbb{R}^n) \subset L^1_b(\mathbb{R}^n) \subset \mathcal{D}'(\mathbb{R}^n)$
↑ Schwartz space

- $\mathcal{F}\mathcal{S}'(\mathbb{R}^n) = \mathcal{S}'(\mathbb{R}^n)$. ♥

- $\mathcal{F}(X^\beta (-i\partial)^\alpha f) = (i\partial)^\beta X^\alpha \hat{f}$.