Reminder III

- Summable distributions and summability order m
- Continuous extension to $C_b^m(\mathbb{R}^n) \equiv BC^m(\mathbb{R}^n)$

If $g \in \mathcal{D}(\mathbb{R}^n)$, $g(0) = 1$, $f \in C_b^m(\mathbb{R}^n)$ and

$f_j(x) = g\left(\frac{x}{j}\right) f(x)$, then $T(f) := \lim_{j \to \infty} T(f_j)$.

bounded convergence property

- Fourier transform $\mathcal{F} f = \hat{f}$ for $f \in L^1(\mathbb{R}^n)$.

↳ some properties

- $\mathcal{F}$ extends to a bijective and isometric map on $L^2(\mathbb{R}^n)$.

preserves the norm

- $\mathcal{F}\,\mathcal{D}(\mathbb{R}^n) \not\subset \mathcal{D}(\mathbb{R}^n)$ ↯

- Schwartz functions $\|x^\beta \partial^\alpha f\|_\infty < \infty \ \forall \alpha, \beta$

$\subset L^1(\mathbb{R}^n)$

$\rightsquigarrow \mathcal{D}(\mathbb{R}^n) \subset \mathcal{S}(\mathbb{R}^n) \subset C_b^\infty(\mathbb{R}^n) \subset C^\infty(\mathbb{R}^n) \subset L^1_{loc}(\mathbb{R}^n) \subset \mathcal{D}'(\mathbb{R}^n)$

Schwartz space

- $\mathcal{F}\,\mathcal{S}(\mathbb{R}^n) = \mathcal{S}(\mathbb{R}^n)$.

- $\mathcal{F}\left(x^\beta(-i\partial)^\alpha f\right) = (i\partial)^\beta x^\alpha \hat{f}$.