

Reminder XII

• For $\varphi: (a, b) \rightarrow \mathbb{H}$, a Riemann type integral can be defined, and $\left\| \int_a^b \varphi(x) dx \right\| \leq \int_a^b \|\varphi(x)\| dx$.

• For $\varphi: (a, b) \rightarrow \mathcal{B}(\mathbb{H})$, a Riemann type integral can also be defined, and $\left\| \int_a^b \varphi(x) dx \right\| \leq \int_a^b \|\varphi(x)\| dx$.

⚠ $\varphi_1 \varphi_2$ is not meaningful, but $\varphi_1 \varphi_2$ is meaningful.

• General linear operator: $(A, D(A))$, $D(A) \subset \mathbb{H}$, $A: D(A) \rightarrow \mathbb{H}$.
domain of A subspace linear

• Notions of extension/restriction: $(A, D(A)) \subset (B, D(B))$.

• Example of X on $L^2(\mathbb{R})$, with different domains.

• Closure of a densely defined operator satisfying $\|Bf\| \leq c\|f\|$ for all $f \in D(B)$, and c independent of f . If no such c exists, then the operator is unbounded.

• Closed operator \equiv domain with a good property

• Adjoint $(A^*, D(A^*))$ of a general linear operator $(A, D(A))$.