

Reminder XI

• Invertibility, with $B^{-1} \in \mathcal{B}(\mathcal{H})$ if $\text{Ran}(B) = \mathcal{H}$.

• Neumann series $(1-B)^{-1} = \sum_{j=0}^{\infty} B^j$ if $\|B\| < 1$.

• Special classes of operators:

Self-adjoint, projection, unitary, isometry,

partial isometry, finite rank:

↙ similar to matrices

↙ bra-ket notation

$$Af = \sum_{j=1}^n \langle g_j, f \rangle h_j = \sum_{j=1}^n |h_j\rangle \langle g_j| f$$

⚠ $g_j, h_j \in \mathcal{H}, \forall j$.

↙ improve convergence

compact operator $\mathcal{K}(\mathcal{H})$.

• Vector valued functions $\varphi: (a,b) \rightarrow \mathcal{H}$,

2 notions of continuity or differentiability.

• Operator valued functions $\varphi: (a,b) \rightarrow \mathcal{B}(\mathcal{H})$,

3 notions of continuity or differentiability.