DA lecture 2022-07-20

Control Simulation Experiments with the Lorenz-96 Model

Qiwen SUN[1,2], Takemasa MIYOSHI[1,3,4], Serge RICHARD[1,2]

[1]Data Assimilation Research Team, RIKEN Center for Computational Science (R-CCS)

[2]Graduate School of Mathematics, Nagoya University (NU)

[3]Prediction Science Laboratory, RIKEN Cluster for Pioneering Research (CPR)

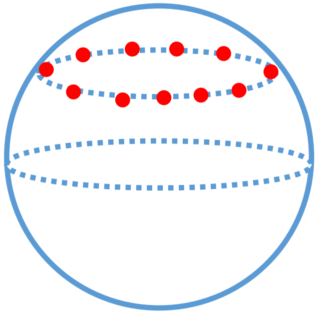
[4]RIKEN interdisciplinary Theoretical and Mathematical Sciences Program (iTHEMS)

Outline

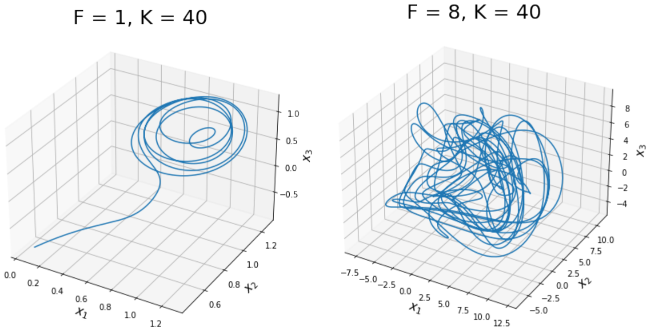
- Lorenz-96 model
- Extreme events
- Control Simulation Experiments
- Full control
- Partial control
- Partial observations
- Summary

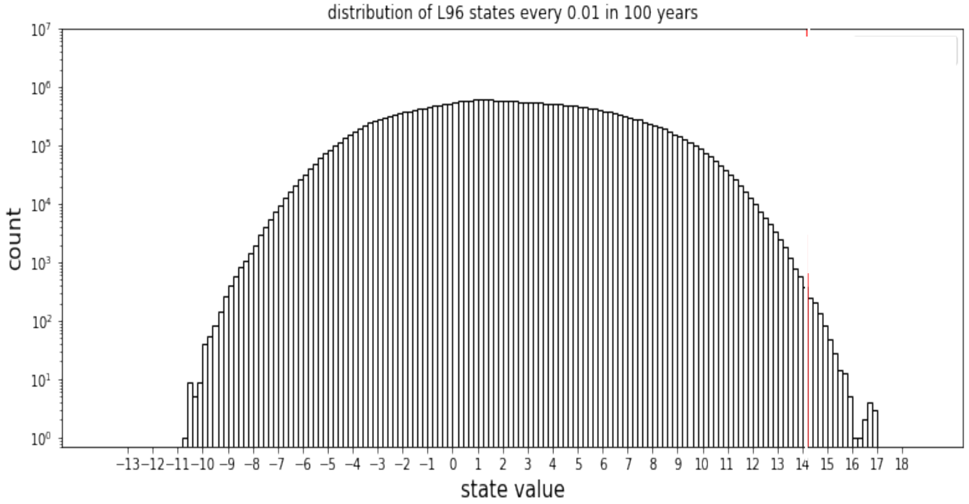
Lorenz-96 Model

- A dynamical system defined by
$$\frac{dx_k}{dt} = (x_{k+1} - x_{k-2})x_{k-1} - x_k + F$$
where $k = 1, \cdots, K$, and $x_{k-K} = x_{k+K} = x_k, K > 3$[1].



- x_k: values of some atmospheric quantity in K sectors of a latitude circle.
- F and the linear terms simulate the external forcing and internal dissipation.
- Quadratic terms simulate the advection.
- Total energy defined as: $(\sum_1^K (x_k)^2)/2$
- For the following experiments: $F = 8,\ K = 40$.
- **1 time unit = 5 days** of atmospheric time.





Mean of x_k lies in [0, F],
Standard deviation lies in [0, F/2]

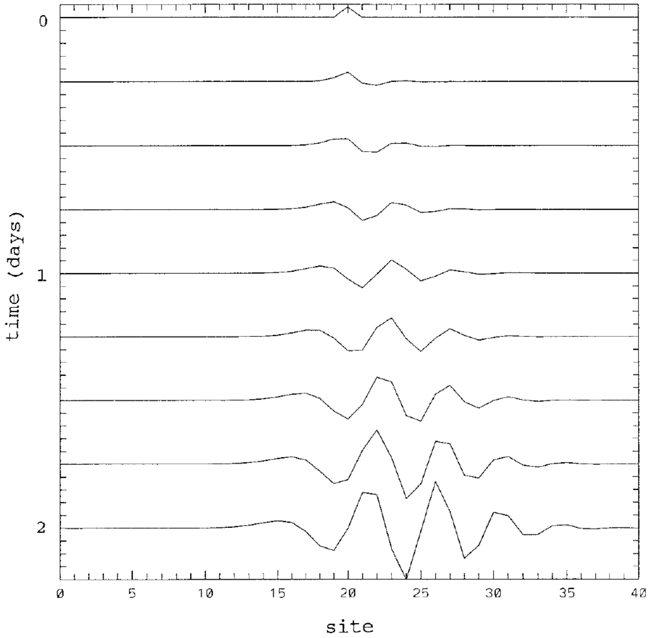
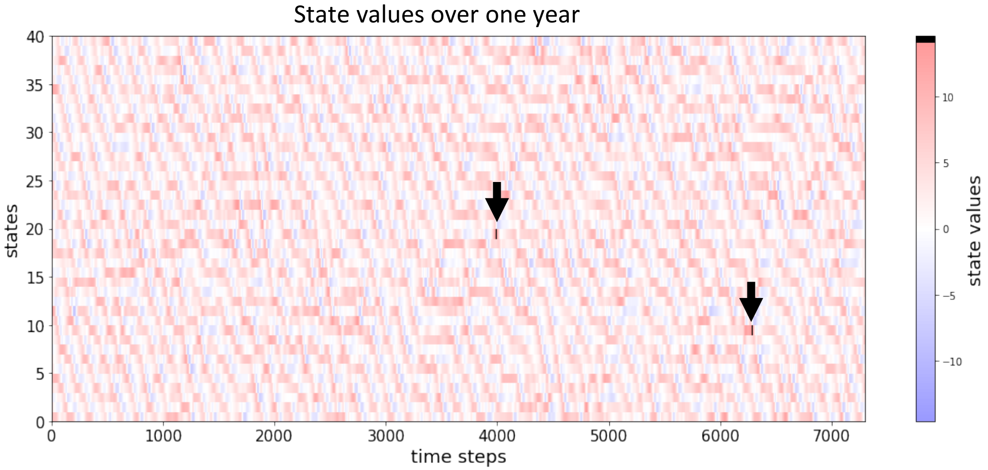


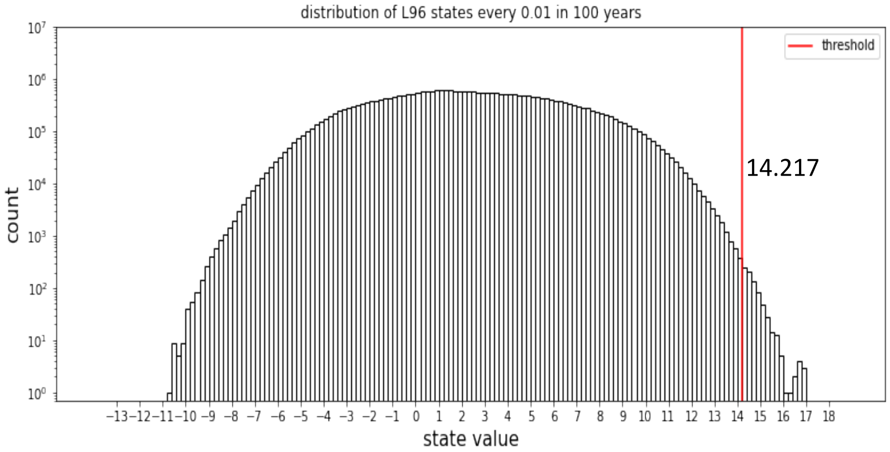
FIG. 1. Longitudinal profiles of X_j at 6-h intervals, as determined by Eq. (1) with $N = 40$ and $F = 8.0$, when initially $X_{20} = F + 0.008$ and $X_j = F$ when $j \neq 20$. On horizontal portion of each curve, $X_j = F$. Interval between successive short marks at left and right is 0.01 units.

Lorenz & Emanuel 1998

CSE - extreme events

- Aim: to avoid extreme values.
- Integrate L96 over 110 years, keep the record of every dt=0.01 for the last 100 years. Record the maximum value over each 6h period.





- The first 200 maximum values are extreme values (on average 2 times / year), the threshold for extreme events is 14.217.

Forecast ability

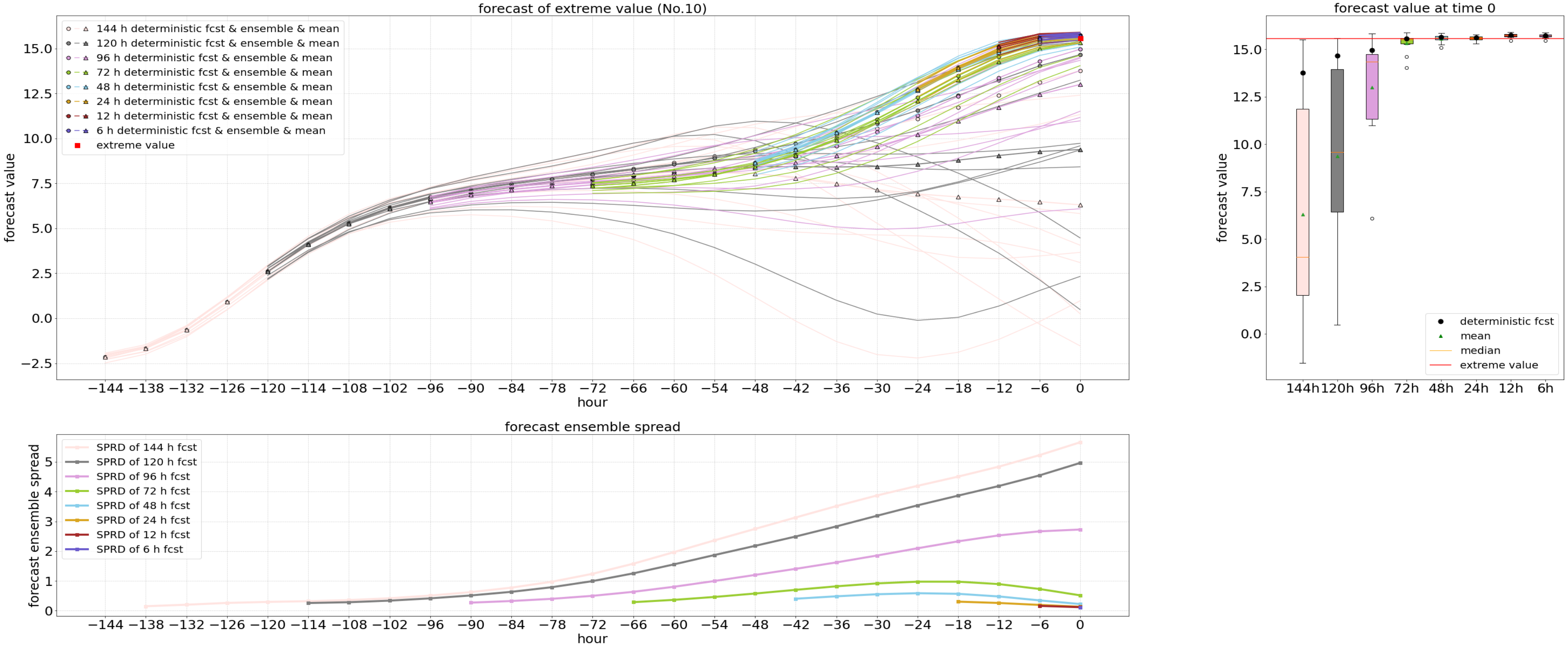
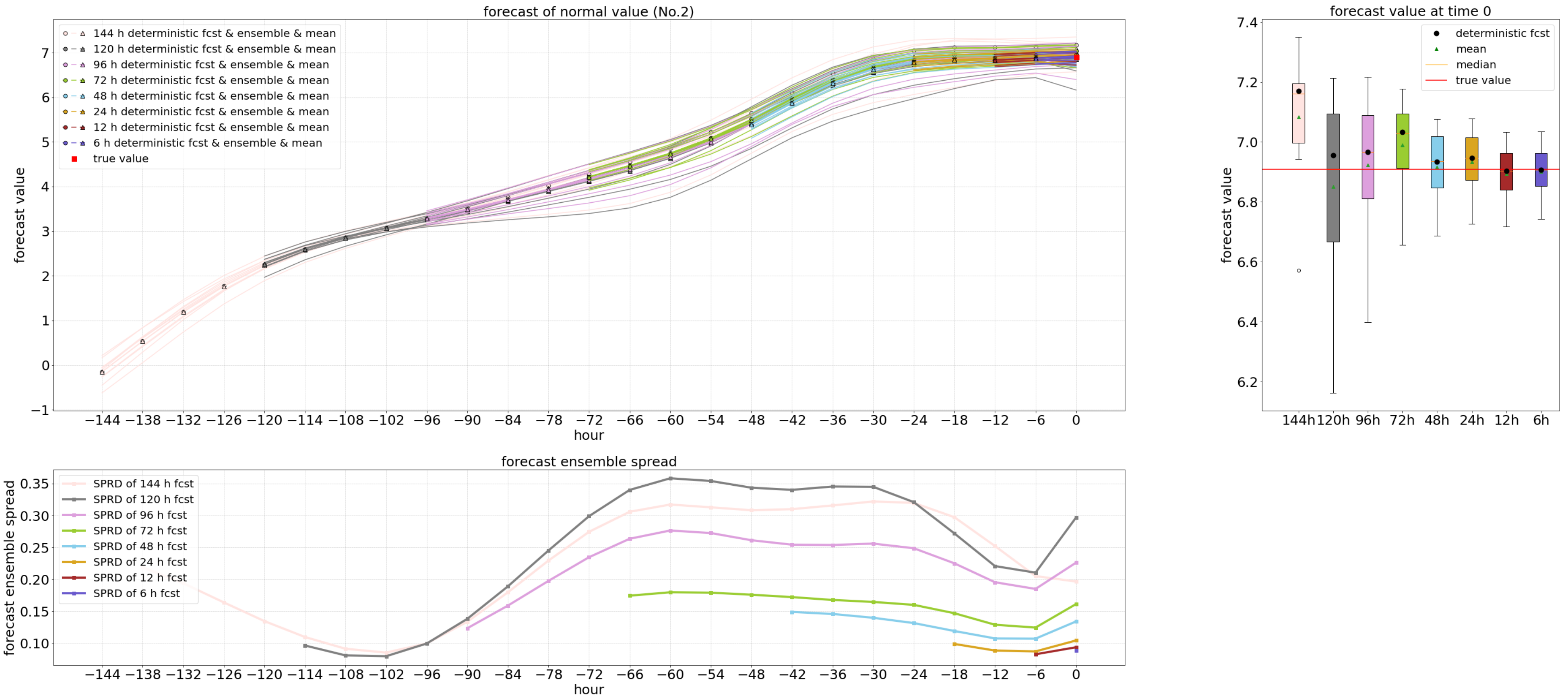


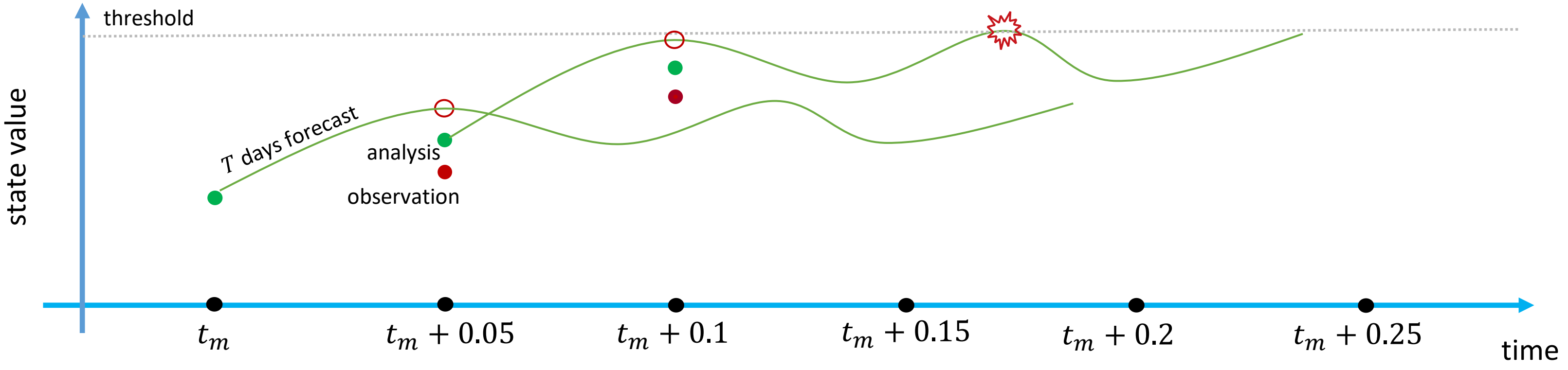
Figure 4. Forecast of maximum in 100 years run. (1) The 144h, 120h, 96h, 72h, 48h, 24h 12h and 6h forecast ensembles. 120h and 96h forecast ensemble fail to predict the extreme value. (2) Forecast ensemble spread. (3) The boxplot of forecast ensemble values at time 0.

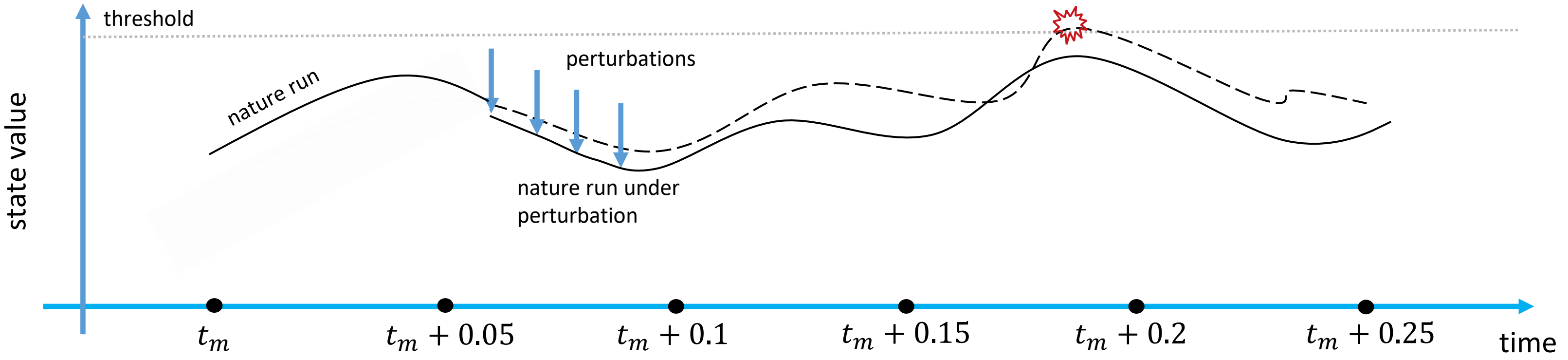
Forecast ability



CSE - procedure

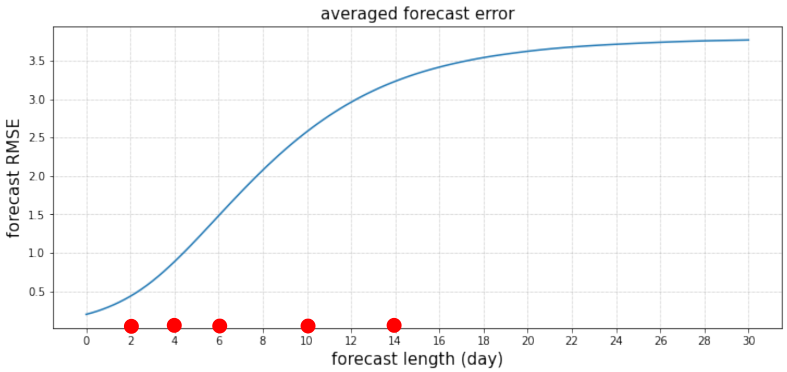
- The observations are noised (nature run + Normal(0, 1)).

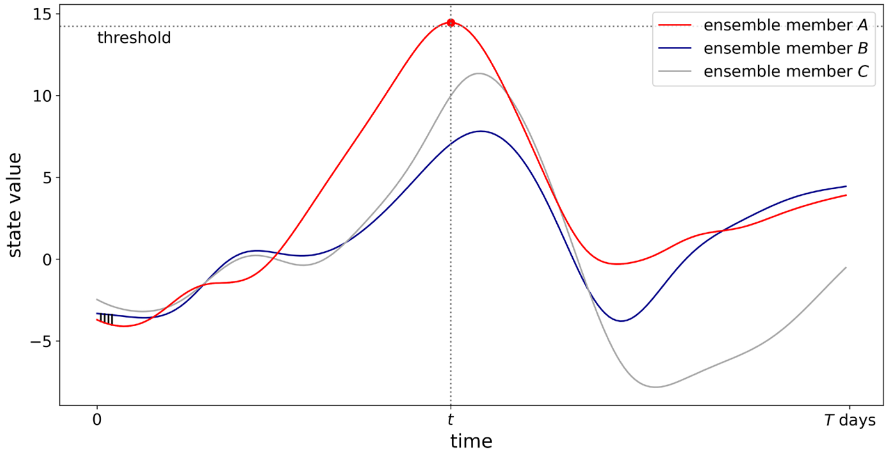




CSE - LETKF

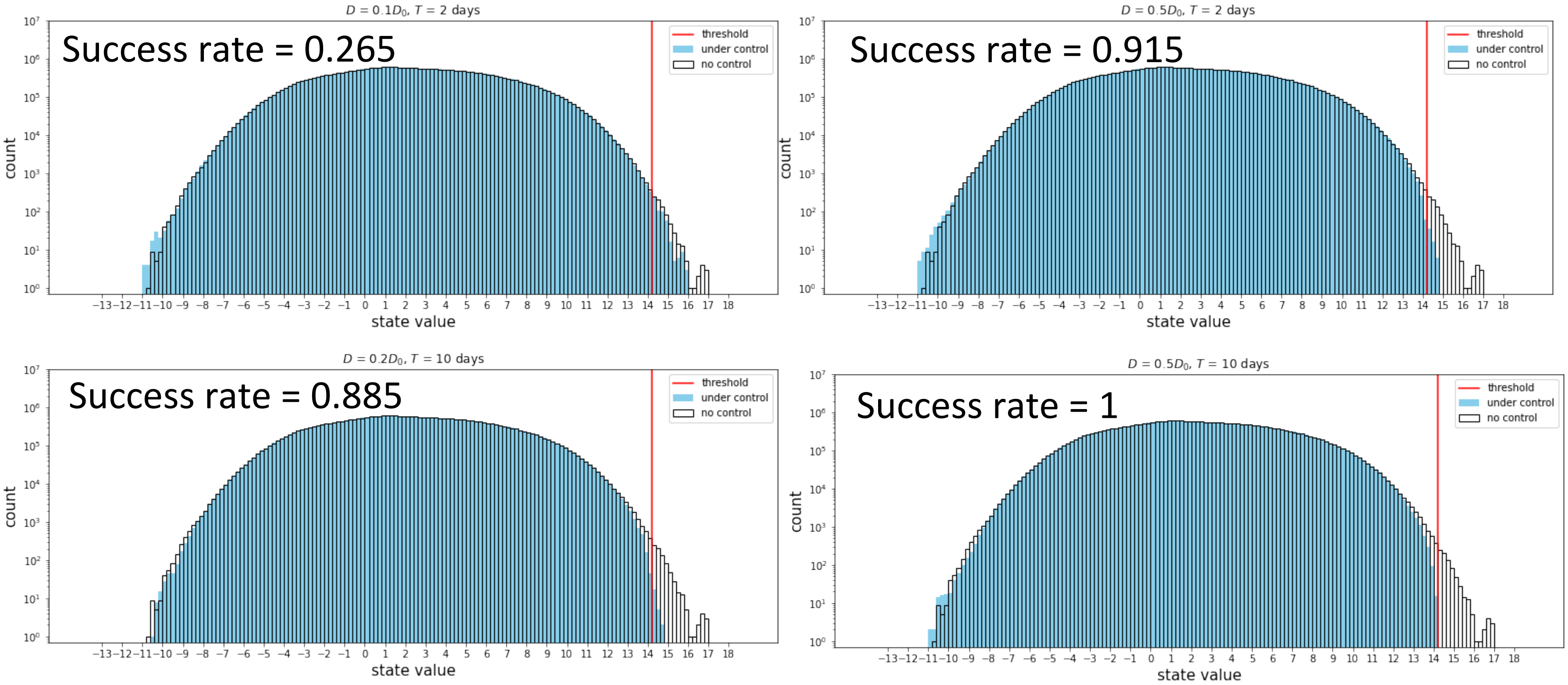
- We use LETKF with 10 ensemble members, $\rho = 1.06$, R-Localization (cut-off radius $2\sqrt{\frac{10}{3}} \times 5.45$) : analysis RMSE$\approx 0.19890 \ldots$
- Forecast length T.





- Perturbation vector: the difference between two proper ensemble members at appropriate time points rescaled to a fixed norm.
- Norm of perturbation vectors : $D = \alpha D_0$ where D_0 is equal to the analysis RMSE.

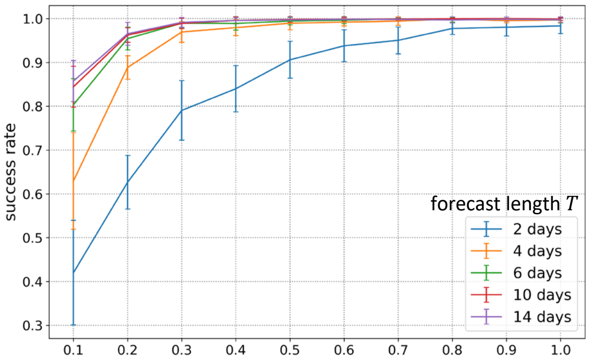
Full control: results for 100 years



Success rate := 1- (#extreme events in 100 years) / 200

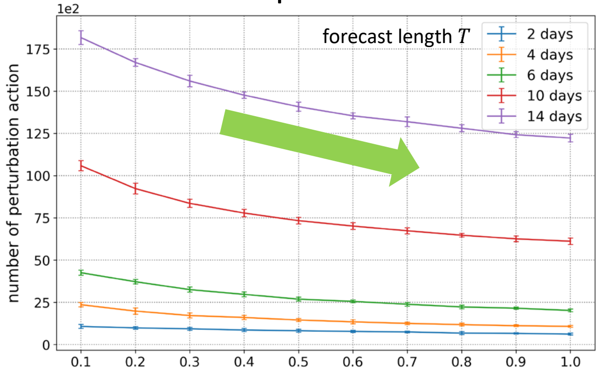
Full control – efficiency and perturbation energy

success rate



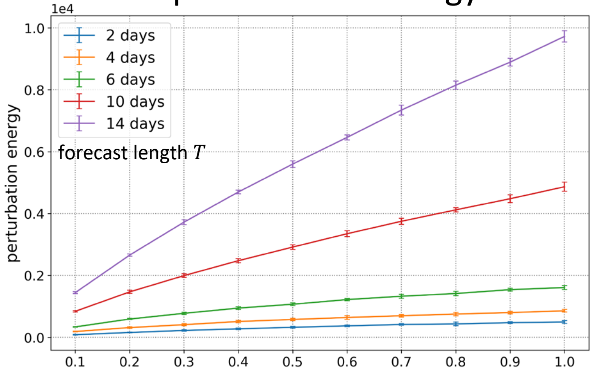
α

number of perturbation action



α

perturbation energy



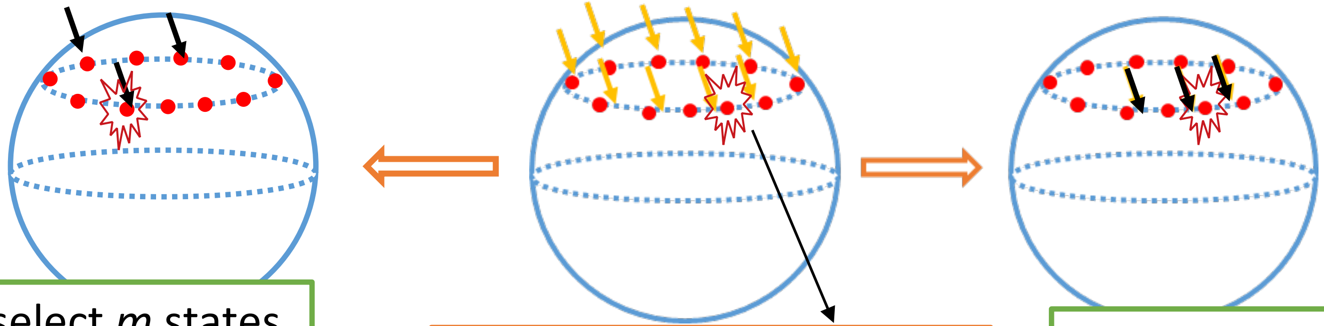
α

❖ norm of perturbation vectors $D = \alpha D_0$, D_0 = analysis RMSE

$$\text{perturbation energy} := 4\alpha D_0 \times \#(\text{perturbation actions})$$

- When the norm of perturbation vectors is small and the forecast length is short, the perturbation vectors are less efficient in terms of avoiding extreme events.
- In contrast, big norm and longer forecast require more energy of perturbations.

Partial control



Randomly select *m* states. Perturb these states when necessary.

State shows the most extreme value in forecast

Perturb the corresponding state (and its neighbors)

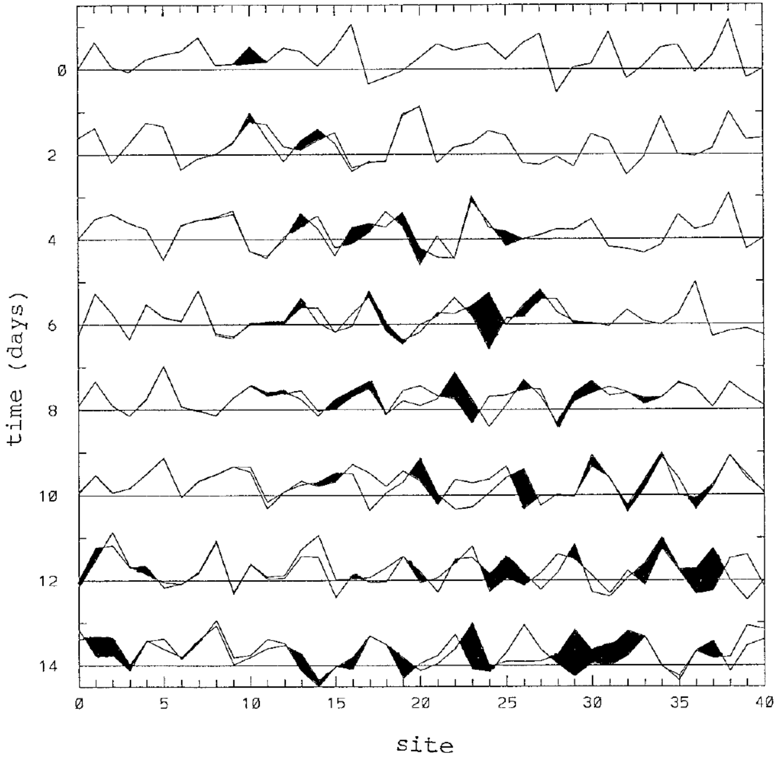
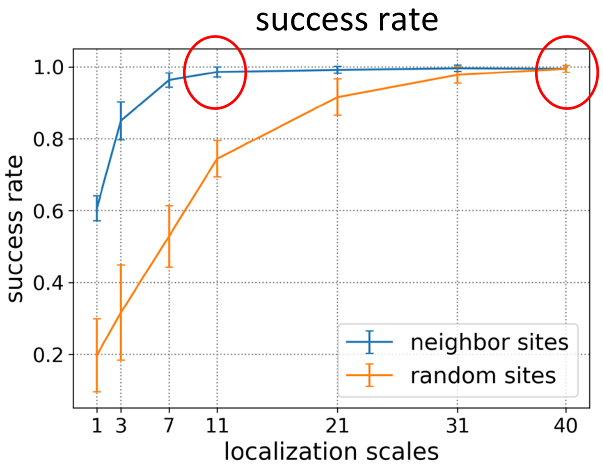
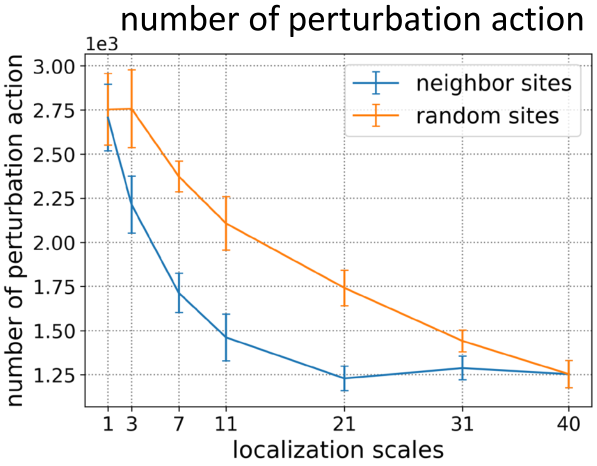


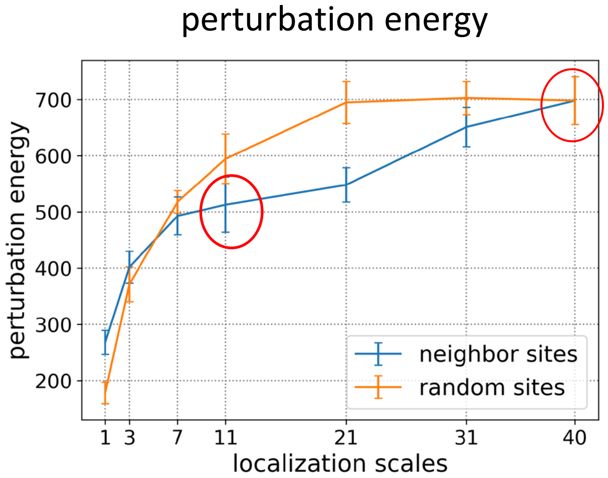
FIG. 3. Longitudinal profiles of X_j as in Fig. 2, but at 2-day intervals, with the initial profile of Fig. 2, and with a second set of profiles superposed. The superposed initial profile is formed by adding 4.0 units to X_{10}. Where the second profile lies above the original one, the area between the profiles is shaded.

Lorenz & Emanuel 1998

Partial control (forecast length $T = 4$ days, $\alpha = 0.7$)



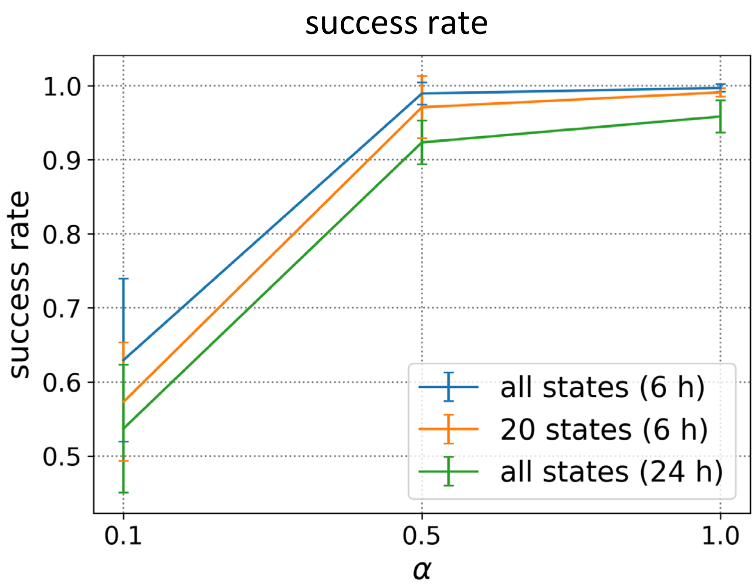


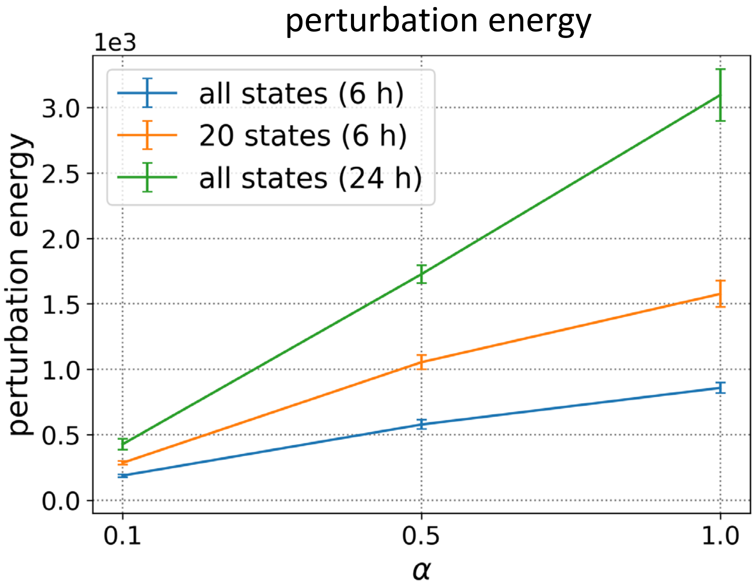


- Perturb random positions generates more actions and cost more energy.
- Perturb random positions is less efficient.

Partial observations (forecast length $T = 4$ days, $\alpha = 0.7$)

- Observe 20 states in every 6 hours.
- Observe all states in every 24 hours.





- Partial observations are less efficient compared with observing all states in every 6 hours.

Summary

- The CSE results show effective control to avoid extreme values.
- Less effective control with

  1) perturbations with small norm

  2) short forecast length

  3) fewer observations, less accurate analysis
- Partial perturbation around the locations of the extreme event is effective.

References

[1] Lorenz, E., 1996: Seminar on Predictability, Vol. I, ECMWF.
[2] Flossmann, A. I., et al., 2019: Bull. Amer. Meteorol. Soc., 100, 1465-1480.
[3] Miyoshi, T., Sun, Q., 2022: Nonlin. Processes Geophys., 29, 133–139.
[4] Houtekamer, P. L., and Zhang, F.: Review of the ensemble Kalman filter for atmospheric data assimilation, Mon. Wea. Rev., 144, 4489-4532, 2016.
[5] Evensen, G.: Advanced data assimilation for strongly nonlinear dynamics, Mon. Wea. Rev., 125, 1342–1354, 1997.
[6] Brian R. Hunt, Eric J. Kostelich, Istvan Szunyogh, Efficient data assimilation for spatiotemporal chaos: A local ensemble transform Kalman filter, Physica D: Nonlinear Phenomena, Volume 230, Issues 1–2, 2007, 112-126, ISSN 0167-2789, 2007.

Thank you.