About Def 3.1.1 If H is normal subgroup, then G/H is also a locally compact group.

pf We define the topology of G/H by quotient map $\pi : G \to G/H$.

That is, $O \subset G/H$ is open $\overset{\text{def}}{\Longleftrightarrow}$ $\pi^{-1}(O)$ is open in G.

We will check G/H is locally compact group in this topology.

- locally compact
  - For any $p \in G/H$, there exists a $g \in G$ s.t. $\pi(g) = p$ since π is surjective.

    Since G is locally compact, there exists a compact set $K \subset G$ s.t. $g \in K^i$ (K^i is a interior of K)

    Moreover, $p = \pi(g) \in \pi(K^i) \subset \pi(K)^i$ since π is open map, and $\pi(K)$ is compact since π is continuous.

    Thus, $\pi(K)$ is a compact neighborhood of p.

- G/H is a topological group

  We show the map
  $$\begin{array}{ccc} G/H \times G/H & \to & G/H \\ \rotatebox{90}{$\in$} & & \rotatebox{90}{$\in$} \\ (aH, bH) & \longmapsto & (aH)(bH)^{-1} \end{array}$$
  is continuous.

  We take any open set U s.t. $(aH)(bH)^{-1} \in U$

  By $(aH)(bH)^{-1} = ab^{-1}H$, then $ab^{-1} \in \pi^{-1}(U)$ and $\pi^{-1}(U)$ is open since π is continuous.

  Since G is a topological group, there exist open sets V and W s.t. $a \in V$, $b \in W$, $ab^{-1} \in VW^{-1} \subset \pi^{-1}(U)$

  Thus, we get $aH = \pi(a) \in \pi(V)$, $bH = \pi(b) \in \pi(W)$.

  $(aH)(bH)^{-1} = ab^{-1}H = \pi(ab^{-1}) \in \pi(VW^{-1}) = \pi(V)\pi(W)^{-1} \subset U$ since π is surjective homorphism.

  Moreover, $\pi(V)$ and $\pi(W)$ are open sets since π is a open map.

  This implies that
  $$\begin{array}{ccc} G/H \times G/H & \to & G/H \\ \rotatebox{90}{$\in$} & & \rotatebox{90}{$\in$} \\ (aH, bH) & \longmapsto & (aH)(bH)^{-1} \end{array}$$
  is continuous.