Date

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Exercise 3.12

Suppose that (X, Y) has joint mass function

$$\mathbb{P}(X=i, Y=j) = \theta^{i+j+1} \quad \text{for } i,j = 0,1,2.$$

Show that $\mathbb{E}(XY) = \theta^3 + 4\theta^4 + 4\theta^5$

and $\mathbb{E}(X) = \theta^2 + 3\theta^3 + 3\theta^4 + 2\theta^5$

According to the Theorem 3.10, we have that

$$\mathbb{E}(XY) = \sum_{x \in \mathrm{Im}X} \sum_{y \in \mathrm{Im}Y} xy \, \mathbb{P}(x, y)$$

$$= 1 \times 1 \times \theta^{1+1+1} + 1 \times 2 \times \theta^{1+2+1} + 2 \times 1 \times \theta^{2\times1\times1} + 2 \times 2 \times \theta^{2+2+1}$$

$$= \theta^3 + 4\theta^4 + 4\theta^5 \quad \square$$

and

$$\mathbb{E}(X) = \sum_{x \in \mathrm{Im}X} \sum_{y \in \mathrm{Im}Y} x \, P(x, y)$$

$$= 1 \times \theta^{1+0+1} + 1 \times \theta^{1+1+1} + 1 \times \theta^{1+2+1} + 2 \times \theta^{2+0+1} + 2 \times \theta^{2+1+1} + 2 \times \theta^{2+2+1}$$

$$= \theta^2 + 3\theta^3 + 3\theta^4 + 2\theta^5 \quad \square$$

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Exercise 3.9

The pair of discrete random variables (X, Y) has joint mass function

$$\mathbb{P}(X=i, Y=j) = \begin{cases} \theta^{i+j+1} & \text{if } i, j = 0, 1, 2 \\ 0 & \text{otherwise,} \end{cases}$$

for some value of θ. Show that θ satisfies the equation

$$\theta + 2\theta^2 + 3\theta^3 + 2\theta^4 + \theta^5 = 1,$$

and find the marginal mass function of X in terms of θ.

First, we find an equation determining θ.

$$\sum_{i=0}^{2} \sum_{j=0}^{2} \mathbb{P}(i, j) = 1$$

$$\therefore \mathbb{P}(0,0) + \mathbb{P}(0,1) + \mathbb{P}(0,2) + \mathbb{P}(1,0) + \mathbb{P}(1,1) + \mathbb{P}(1,2) + \mathbb{P}(2,0) + \mathbb{P}(2,1) + \mathbb{P}(2,2) = 1$$

$$\therefore \theta^{0+0+1} + \theta^{0+1+1} + \theta^{0+2+1} + \theta^{1+0+1} + \theta^{1+1+1} + \theta^{1+2+1} + \theta^{2+0+1} + \theta^{2+1+1} + \theta^{2+2+1} = 1$$

$$\therefore \theta + 2\theta^2 + 3\theta^3 + 2\theta^4 + \theta^5 = 1 \quad \square$$

Next, we find the marginal mass function of X in terms of θ.

$$\mathbb{P}(x) = \sum_{j \in \mathrm{Im}\, Y} \mathbb{P}(x, j)$$

$$\therefore \mathbb{P}(x) = \mathbb{P}(x,0) + \mathbb{P}(x,1) + \mathbb{P}(x,2)$$

$$= \theta^{x+0+1} + \theta^{x+1+1} + \theta^{x+2+1}$$

$$= \theta^{x+1} + \theta^{x+2} + \theta^{x+3} = \underline{\theta^{x}(\theta + \theta^2 + \theta^3)} \#$$