Probability Report #3

More word problems

1. We have a shuffled 52-card deck & draw a hand of 7 cards

What's the probability that

a) all 4 kings are in your hand
b) 3 kings are in your hand
c) at least 3 kings are in your hand

a) $\mathbb{P}(4\text{ kings}) = \dfrac{\#\text{ possible hands with 4 kings}}{\#\text{ possible hands}}$

To find all possible combinations we can use the formula

$^{n}C_{r} = \dfrac{n!}{r!(n-r)!}$, $n \Rightarrow$ number of cards
$r \Rightarrow$ drawn cards

4 cards of kings only

the other 3 cards can be anything

$\Rightarrow \mathbb{P}(4\text{ kings}) = \dfrac{^{4}C_{4} \cdot {}^{48}C_{3}}{^{52}C_{7}} =$

↳ all possible 7-card hands in a 52-card deck

$$\frac{\overbrace{\frac{4!}{4!\,\underbrace{0!}_{=1}}}^{=1} \times \frac{48!}{3!\,45!}}{\frac{52!}{7!\,\cancel{45!}}} = \frac{48!\,.\,7!}{52!\,.\,3!} = \frac{1}{7735}$$

b) Analogously, for 3 kings

$$\mathbb{P}(3\text{ kings}) = \frac{\overbrace{{}^{4}C_{3}}^{3/4\text{ kings are drawn}} \times \overbrace{{}^{48}C_{4}}^{\text{the other 4 cards can be anything}}}{{}^{52}C_{7}} =$$

$$= \frac{\frac{4!}{3!\,1!} \times \frac{48!}{4!\,44!}}{\frac{52!}{7!\,45!}} = \ldots\ldots = \frac{9}{1547}$$

At least 3 kings means we could have either 3 or 4

$$\Rightarrow \mathbb{P}(\geq 3\text{ kings}) = \mathbb{P}(3\text{ kings}) + \mathbb{P}(4\text{ kings}) =$$

$$= \frac{9}{1547} + \frac{1}{7735} = \frac{46}{7735}$$

2. A relay race has five teams of equal ability

1) What's the probability that teams #1, #2, #3 finish 1^{st}, 2^{nd}, 3^{rd}, respectively

2) What's the probability that #1, #2, #3 finish first (in any order)

No. Date

1) $\mathbb{P}(\text{respectively}) = \frac{(5-3)(5-4)}{5!} = \frac{1}{60}$

2) $\mathbb{P}(\text{any order}) = \frac{3!}{5!} = \frac{1}{10}$

3. If a C. elegans experiment (microinjection) can randomly produce cryophilic (likes cold), thermophilic (likes hot), or isothermal phenotypes, prove that

$$\mathbb{P}(C \cup T \cup I) = \mathbb{P}(C) + \mathbb{P}(T) + \mathbb{P}(I) - \mathbb{P}(C \cap T) - \mathbb{P}(C \cap I) - \mathbb{P}(T \cap I) + \mathbb{P}(C \cap T \cap I)$$

If

$TI = T \cup I$, such that

$$\mathbb{P}(C \cup T \cup I) = \mathbb{P}(C \cup TI) = \mathbb{P}(C) + \mathbb{P}(TI) - \mathbb{P}(C \cap TI)$$

we have

$$\mathbb{P}(TI) = \mathbb{P}(T \cup I) = \mathbb{P}(T) + \mathbb{P}(I) - \mathbb{P}(T \cap I)$$

$$C \cap TI = C \cap (T \cup I) = (C \cap T) \cup (C \cap I)$$

$\Rightarrow$

$$\mathbb{P}(C \cup TI) = \mathbb{P}(C \cap T) + \mathbb{P}(C \cap I) - \mathbb{P}[(C \cap T) \cap (C \cap I)]$$
$$= \mathbb{P}(C \cap T) + \mathbb{P}(C \cap I) - \mathbb{P}[C \cap T \cap I]$$

$$\Rightarrow \mathbb{P}(C \cup T \cup I) = \mathbb{P}(C) + \mathbb{P}(I) + \mathbb{P}(T) - \mathbb{P}(T \cap I) - \mathbb{P}(C \cap T) - \mathbb{P}(C \cap I) + \mathbb{P}(C \cap T \cap I)$$