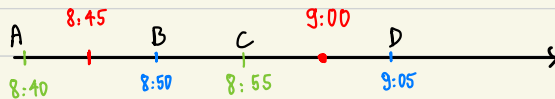


The buses which stop at the end of my road do not keep to the timetable. They should run every quarter hour, at 08.30, 08.45, 09.00, . . . , but in fact each bus is either five minutes early or five minutes late, the two possibilities being equally probable and different buses being independent. Other people arrive at the stop in such a way that, t minutes after the departure of one bus, the probability that no one is waiting for the next one is $e^{-t/5}$. What is the probability that no one is waiting at 09.00? One day, I come to the stop at 09.00 and find no one there; show that the chances are more than four to one that I have missed the nine o'clock bus.



Since the bus is either early or late 5 min than the scheduled time, then:

The 8:45 bus either departs at 8:40 or 8:50.

The 9:00 bus either departs at 8:55 or 9:05.

The probabilities the bus departs at 8:40, 8:50, 8:55 and 9:05 are, respectively

$$P(A) = P(B) = P(C) = P(D) = 0.5$$

Since the two probabilities are equally probable and different buses are independent.

A_t , B_t , C_t are the outcomes that there is no one waiting at the bus stop which the last bus leaves at 8:40, 8:50 and 8:55, respectively.

$$\text{Then } P(A_t) = e^{-\frac{20}{5}} = e^{-4}$$

$$P(B_t) = e^{-\frac{10}{5}} = e^{-2}$$

$$P(C_t) = e^{-\frac{5}{5}} = e^{-1}$$

At 9:00, A_e , B_e , C_e are the outcomes that the last bus leaves at 8:40, 8:50 and 8:55, orderly.

$$\text{Clearly that, } P(C_e) = P(C) = 0.5$$

And A_e or B_e happens if only if D happens.

$$\text{Then } P(A_e) = P(A) \cdot P(D) = 0.5 \times 0.5 = 0.25$$

$$P(B_e) = P(B) \cdot P(D) = 0.5 \times 0.5 = 0.25$$

Let L : there is no one waiting at 9:00

Then, by Partition theorem:

$$P(L) = \sum_i P(L|X_i)P(X_i)$$

$$P(L) = P(A_t)P(A_e) + P(B_t)P(B_e) + P(C_t)P(C_e)$$

$$P(L) = 0.5 e^{-1} + 0.25 \cdot e^{-2} + 0.25 e^{-4}$$

$$P(L) = 0.2223$$

When there is no one waiting,

Probabilities the last bus leaves at 8:55 and no one is waiting:

$$P(C_{t \& e} | L) = \frac{P(C_t)P(C_e)}{P(L)} = \frac{e^{-1} \cdot 0.5}{0.2223} = 0.927 > \frac{4}{5}$$

→ The chances that person missed the 9:00 bus are more than 4 to 1

(4 miss, 1 not miss)