Theorem: Law of subconscious statistician

X is a discrete random variable X on $(\Omega, \mathcal{F}, \mathbb{P})$.

$g: \mathbb{R} \to \mathbb{R}$ then $\mathbb{E}(g(X)) = \sum_{x \in X(\Omega)} g(x) P_X(x)$.

whenever this sum converges absolutely, that is $\sum_{x \in X(\Omega)} |g(x)| P_X(x) < \infty$

Proof: Suppose $Y = g(X) = g \circ X$.

$$\mathbb{E}(Y) = \sum_{y \in g(X(\Omega))} y P_Y(y)$$

$$= \sum_{y \in g(X(\Omega))} y \sum_{x \in X(\Omega):\, g(x) = y} P_X(x).$$

$$= \sum_{x \in X(\Omega)} g(x) P_X(x) \quad \square$$

(Provided that the sum on RHS converges absolutely)

$\boxed{ex}$: Suppose X is the Poisson distribution with parameter $\lambda > 0$

? Find the expectation value of $Y = e^X$

$$\mathbb{E}(Y) = \mathbb{E}(e^X) = \sum_{k=0}^{\infty} e^k \mathbb{P}(X = k) \quad (\leftarrow \text{apply the theorem here})$$

$$= \sum_{k=0}^{\infty} e^k \frac{1}{k!} \lambda^k e^{-\lambda}$$

$$= e^{-\lambda} \underbrace{\sum_{k=0}^{\infty} \frac{1}{k!} (e\lambda)^k}_{e^{e\lambda}}$$

$$= e^{\lambda(e-1)}$$

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