Bivariate discrete distributions.

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Exercise 3.9.

The pair of discrete random variables (X, Y) has joint mass function

$$\mathbb{P}(X=i, Y=j) = \begin{cases} \theta^{i+j+1} & \text{if } i=j=0,1,2 \\ 0 & \text{otherwise.} \end{cases}$$ for some value θ.

- Show that θ satisfies the equation.

$$\theta + 2\theta^2 + 3\theta^3 + 2\theta^4 + \theta^5 = 1.$$

by definition 3.1 the joint mass function is defined by.

$$p_{X,Y}(x,y) = \mathbb{P}(\{\omega \in \Omega : X(\omega) = x \text{ and } Y(\omega) = y\})$$

$$p_{X,Y}(x,y) = \mathbb{P}(X=x, Y=y) \quad , \quad p_{X,Y} : \mathbb{R}^2 \to [0,1]$$

then for $i=j=0,1,2.$ we have:

$\mathbb{P}(X=0, Y=0) = \theta^1.$
$\mathbb{P}(X=0, Y=1) = \theta^2.$
$\mathbb{P}(X=0, Y=2) = \theta^3.$
$\mathbb{P}(X=1, Y=0) = \theta^2.$
$\mathbb{P}(X=1, Y=1) = \theta^3.$
$\mathbb{P}(X=1, Y=2) = \theta^4$
$\mathbb{P}(X=2, Y=0) = \theta^3.$
$\mathbb{P}(X=2, Y=1) = \theta^4$
$\mathbb{P}(X=2, Y=2) = \theta^5.$

$$\sum_{x \in \operatorname{Im} X} \sum_{y \in \operatorname{Im} Y} p_{X,Y}(x,y) = \theta + \theta^2 + \theta^3 + \theta^2 + \theta^3 + \theta^4 + \theta^3 + \theta^4 + \theta^5 =$$

$$= \theta + 2\theta^2 + 3\theta^3 + 2\theta^4 + \theta^5 = 1.$$

- find the marginal mass function of X in terms of θ.

by definition, the marginal mass function of X is defined as $\mathbb{P}(X) = \sum_y p_{X,Y}(x,y) = \mathbb{P}(X=x, Y=0) + \mathbb{P}(X=x, Y=1)$ $+ \mathbb{P}(X=x, Y=2) = \theta^{i+1} + \theta^{i+2} + \theta^{i+3} = \underline{\theta^{i+1}(1+\theta+\theta^2)} = p_X(x)$