
Homework 11

Exercise 1 Find the integral of the function $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ defined by $f(x, y) = \frac{1}{(x^2 + y^2 + 1)^{3/2}}$ on the disc of radius R and centered at the origin of \mathbb{R}^2 .

Exercise 2 Find the mass of a spherical ball of radius R if the density of the ball at any point is equal to a constant k times the distance of that point to the center of the ball.

Exercise 3 Find the integral of the function $f : \mathbb{R}^3 \rightarrow \mathbb{R}$ given by $f(x, y, z) = x^2$ over the portion of the cylinder defined by $x^2 + y^2 = a^2$ and lying between the planes defined by $z = 0$ and $z = b$, with $a > 0$ and $b > 0$.

Exercise 4 Compute the area enclosed by the curve given in polar coordinate by $r^2 = \cos(\theta)$. Sketch this area.

Exercise 5 Compute the area of the domain defined in $\mathbb{R}_+ \times \mathbb{R}_+$ by the four curves of equation

$$y = ax, \quad y = x/a, \quad y = b/x, \quad y = 1/bx \quad \text{for } a > 1, b > 1.$$

Exercise 6 Let X, Y be two vectors in \mathbb{R}^2 . Check that the area of the parallelogram spanned by X and Y is equal to the absolute value of the determinant of the matrix $(X \ Y) \in M_2(\mathbb{R})$. More generally, if X_1, \dots, X_n are n vectors of \mathbb{R}^n , one writes $\text{Vol}(X_1, \dots, X_n)$ for the volume of the n -dimensional box spanned by X_1, \dots, X_n . Why is it natural to have

$$\text{Vol}(X_1, \dots, X_n) = |\text{Det}(X_1 \dots X_n)| \ ?$$