
Homework 10

Exercise 1 *Consider the vector field* $f : \mathbb{R}^2 \ni (x, y) \mapsto (2x^3y^4 + x, 2x^4y^3 + y) \in \mathbb{R}^2$. *Compute the curve integral along the curve defined by* $c(t) := \big(t\cos(\pi t) - 1, \sin(\pi t/2)\big)$ *for* $t \in [0, 1]$.

Exercise 2 *Compute the following integrals:*

$$\iint_\Omega x^2 y \,\mathrm{d}x\,\mathrm{d}y \quad \textit{with} \quad \Omega = [1, 2] \times [-3, 4],$$
$$\iiint_\Omega \sin(x) yz \,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z \quad \textit{with} \quad \Omega = [0, \pi] \times [0, 1] \times [0, 2].$$

Exercise 3 *1) Compute the integral* $\iint_\Omega (x - y)\,\mathrm{d}x\,\mathrm{d}y$ *with* Ω *the subset of* $\mathbb{R}^2$ *defined by the three lines of equation* $x = 0$, $y = x + 2$, *and* $y = -x$,

2) Compute the integral $\iint_\Omega \mathrm{e}^{x+y}\,\mathrm{d}x\,\mathrm{d}y$ *with* Ω *the subset of* $\mathbb{R}^2$ *defined by* $\{(x, y) \in \mathbb{R}^2 \mid |x| + |y| \leq 1\}$,

3) Compute the integral $\iint_\Omega xy\,\mathrm{d}x\,\mathrm{d}y$ *with* Ω *the subset of* $\mathbb{R}_+ \times \mathbb{R}_+$ *defined by the two functions of equation* $y = x^2$ *and* $y = x^4$.

Exercise 4 *Compute the integral* $\iiint_\Omega (x + y + z)^2\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$ *with* Ω *the subset of* $\mathbb{R}^3$ *defined by the four planes of equation* $x = 0$, $y = 0$, $z = 0$, *and* $x + y + z = 1$.