Motivation

$\mathbb{C}^*$ - algebra $\mathcal{Z}$

$K_0(\mathcal{Z}) \cong \{\text{Projections}\}/\text{homotopy}$

$K_1(\mathcal{Z}) \cong \{\text{Unitary el.}\}/\text{homotopy}$

2 Abelian groups, attached to $\mathcal{Z}$.

- Classification: not enough information
- Stability: we deal with equivalent claus

If $P$ or $U$ are related to a "physical" system, then all elements of $[P]_0$ or $[U]_1$ share the same property.

$\{p\} \in \mathbb{C} \mapsto \zeta(p) \in C$

$\{u\} \in \mathbb{C} \mapsto \zeta(u) \in C$

Cyclic cohomology computable on certain element.
Suppose $0 \to \mathfrak{g} \to \mathfrak{h} \to \mathfrak{q} \to 0$

then $\exists \phi : K_{1} (\mathfrak{q}) \to K_{0} (\mathfrak{g})$ index map

$\psi : K_{0} (\mathfrak{q}) \to K_{1} (\mathfrak{g})$ exponential map

we can link 2 "topological" invariants, one unitary and one projection

$\to$ bulk - boundary correspondence

* Levinson - theorem