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Theorem: A connected graph is Eulerian if and only if every vertex has even degree.

Proof

* An Eulerian graph ⇒ Every vertex has even degree.

Let C be an Eulerian tour of connected graph G.

- Each time a traversal passes through a vertex, it contributes 2 to that vertex's degree.
- Since a trail requires no repeated edges, each edge of G is traversed exactly one

⇒ There is an even number of traversed edges incident with each vertex. This number is the degree of that vertex

∴ Each vertex has even degree.

* Every vertex has even degree ⇒ An eulerian graph.
(connected + possess Eulerian tour)

Idea: Using induction on the number of edges in the graph $\|G\|$.

- Base case $\|G\| = 2$, $|G| = 2$

Graph with 2 vertices and 2 edges between them

This graph is obviously Eulerian.

- Suppose $\|G\| > 2$. Suppose less than m edges, every degree is even (induction hypothesis)

- Start with a vertex v, follow edges until return to v.
Call this walk W. This is valid since every time walk W leads to a vertex other than v, it encounters one edge adjacent to it. There are an even number of edges connecting with a vertex, so there is always an available edge to use. This is why the walk always can lead back to v.

- Let E be edges of W. Graph $G' := G - E$ has components $C_1, C_2, \dots, C_k$.

For every $v \in G$, an even number of edges of G at v

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lies in E, so the degree of ~~ed~~ each vertex in G' is even

G is connected $\Rightarrow$ G' has an edge e connecting with a vertex on W

By induction hypothesis, the component C of G' containing e has an Euler tour $E_1 \ldots E_k$.

Assume we follow W by starting at v, follow W until reaching vertex a_1, follow E_1 back at a_1, follow W until reaching a_2, follow E_2 ending back at a_2 ...... After reaching a_k, follow E_k ending at a_k, then finish off W, ending at v. $\square$