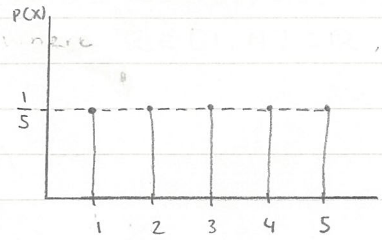
Pre & Nadia

DISCRETE UNIFORM DISTRIBUTION

DEFINITION

For a discrete, finite set of N events, $P(X = x \mid N) = \frac{1}{N}$, where $x = 1, 2, \dots, N$



PROPERTIES

> EXPECTED VALUE

$$E(X) = \sum_{x=1}^{N} x f(x)$$

$$= \sum_{x=1}^{N} x \cdot \frac{1}{N}$$

$$= \frac{1}{N}\sum_{x=1}^{N} x$$

$$= \frac{1}{N} \cdot \frac{N(N+1)}{2}$$

$$= \frac{N+1}{2}$$

> MOMENT GENERATING FUNCTION

$$M_x(t) = E(e^{tx})$$

$$= \sum_{x=1}^{N} e^{tx} f(x)$$

$$= \frac{1}{N}\sum_{x=1}^{N} e^{tx}$$

> VARIANCE

$$Var(X) = E((X - E(X))^2)$$

$$= E(X^2) - (E(X))^2$$

$$E(X^2) = \sum_{x=1}^{N} x^2 f(x)$$

$$= \sum_{x=1}^{N} x^2 \cdot \frac{1}{N}$$

$$= \frac{1}{N}\sum_{x=1}^{N} x^2$$

$$= \frac{1}{N}\frac{N(N+1)(2N+1)}{6}$$

$$= \frac{(N+1)(2N+1)}{6}$$

$$(E(X))^2 = \left(\frac{N+1}{2}\right)^2 = \frac{(N+1)^2}{4}$$

$$Var(X) = \frac{(N+1)(2N+1)}{6} - \frac{(N+1)^2}{4}$$

$$= \frac{2(N+1)(2N+1) - 3(N+1)^2}{12}$$

$$= \frac{(N+1)(N-1)}{12}$$

17 33

APPLICATION

4m

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

In biology, especially the study of ecology, quadrant sampling is used to reduce the amount of data collected to find results that are representative of the population. For example, when surveying the population of a certain plant species in a 4×4m field, rather than surveying the entire $16m^2$, a person can divide the field into 4 sections along the x and y axis, and survey only 6 (6 is arbitrary; you can choose to survey any number) of the resulting quadrants, which sums to $6m^2$. The average population in each quadrant can then be calculated, then multiplied by the total number of quadrants to estimate the total population. In order for the data to be representative, the probability that each quadrant is selected for sampling must be equal. In this case, $N = 16$, and each quadrant x is assigned a value from $1, 2, \dots, 16$. This gives:

$$P(X) = \frac{1}{16} \quad E(X) = \frac{17}{2} \quad Var(X) = \frac{255}{12} \quad M_x(t) = \frac{1}{16}\sum_{x=1}^{16} e^{tx}$$