Given $\left(\frac{1}{a}\right)^n - \left(\frac{1}{b}\right)^n = 1 - \alpha$, $1 \le a < b, n > 1$ & $n \in \mathbf{N}$ and $0 < \alpha < 1$, we have $Inf(b-a) = \alpha^{-\frac{1}{n}} - 1$

Proof:

$\left(\frac{1}{a}\right)^n - \left(\frac{1}{b}\right)^n = 1 - \alpha \Leftrightarrow \mathrm{b} = (-1 + \alpha + a^{-n})^{-\frac{1}{\mathrm{n}}}$

If $\frac{\mathrm{d}(b-a)}{\mathrm{d}a} > 0 \ \forall a \ge 1$, $a \to 1^+$ will leads the infimun of $b - a$ which equals to $(-1 + \alpha + 1^{-n})^{\frac{1}{\mathrm{n}}} - 1 = \alpha^{-\frac{1}{n}} - 1$

$$\frac{\mathrm{d}(b-a)}{\mathrm{d}a} = -\frac{1}{n}(-1 + \alpha + a^{-n})^{-\frac{1+n}{n}} \cdot (-n)\alpha^{-n-1} - 1$$

$$= (-1 + \alpha + a^{-n})^{-\frac{1+n}{n}} \cdot (a^n)^{-\frac{1+n}{n}} - 1$$

$$= \left((-1 + \alpha)a^n + 1\right)^{-\frac{1+n}{n}} - 1$$

Given $0 < \alpha < 1$, $\mathrm{a} \ge 1$ and $\mathrm{n} > 1$, $(-1 + \alpha)a^n \le 0 \Rightarrow (-1 + \alpha)a^n + 1 \le 1$

For b to be real, one should have $a^{-n} > 1 - \alpha \Leftrightarrow a^n(-1 + \alpha) > -1$

$\Rightarrow (-1 + \alpha)a^n + 1 \in (0,1) \Rightarrow \left((-1 + \alpha)a^n + 1\right)^{-\frac{1+n}{n}} > 1 \Rightarrow \left((-1 + \alpha)a^n + 1\right)^{-\frac{1+n}{n}} - 1 = \frac{\mathrm{d}(b-a)}{\mathrm{d}a} > 0 \ | b \ is \ real.$

$\Rightarrow$ Interval length is minimized when $a \to 1^+$, thus $Inf(b-a) = \alpha^{-\frac{1}{n}} - 1$