Negative binomial distribution

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1 Summary

First, one has to introduce Bernoulli trial which is used in the negative binomial distribution. Bernoulli trial (or binomial trial) is a random experiment with exactly two possible outcomes, “success” and “failure”, in which the probability of success is the same every time the experiment is conducted.

The negative binomial distribution is a discrete probability distribution of the number of successes in a sequence of Bernoulli trials before a specified number of failures, denoted “r” occurs.

Let a random variable X denote the number of successes at which the r^{th} failure occurs, and let p be the probability of failure. The probability mass function, derived from Bernoulli trials, of this particular distribution is

$$P(X=x|r,p) = \binom{r+x-1}{x} p^r (1-p)^x, x = 0, 1, ... \tag{1}$$

where the quantity in parentheses corresponds to the binomial coefficient. The negative binomial distribution gets its name from the relationship

$$\binom{r+x-1}{x} = \frac{(r+x-1)(r+x-2)...r}{x!} = (-1)^x \frac{(-r)(-r-1)...(-r-x+1)}{x!} = (-1)^x \binom{-r}{x}$$

$\binom{-r}{x}$ is called the negative binomial coefficient.

Mean and variance are,

$$E(X) = \frac{r(1-p)}{p}, Var(X) = \frac{r(1-p)}{p^2}$$

Let a random variable Y be $X + r$, then one gets an alternate form of the pmf

$$P(Y=y|r,p) = \binom{y-1}{r-1} p^r (1-p)^{y-r}, y = r, r+1, ... \tag{2}$$

2 An example of application

A technique known as an inverse binomial sampling is useful in sampling biological populations. If the proportion of individuals possessing a certain characteristic is p and we sample until we see r such individuals, then the number of sampled individuals who don’t possess that characteristic is a negative binomial random variable (1).

3 A graph of a specific example of the probability mass function (1)

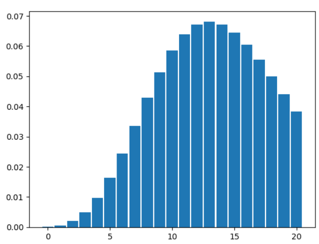

Figure 1: The probability mass function (1) at $r = 10$ and $p = 0.4$. A random variable X ranges from 0 to 20.

4 Reference

http://www.math.ntu.edu.tw/ hchen/teaching/StatInference/notes/lecture16.pdf(accessed 15:30 May 6, 2019)