

Bernoulli distribution

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pmf of this distribution is

$$P(X=x|p) = p^x (1-p)^{1-x}$$

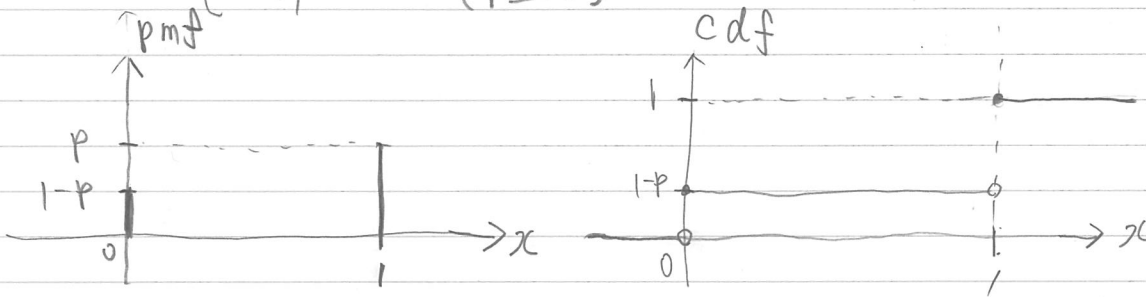
$$x \in \{0, 1\}$$

so $P(X=0|p) = p^0 (1-p)^1 = 1-p$

$$P(X=1|p) = p^1 (1-p)^0 = p$$

and its cumulative distribution function (F) is

$$F(x) = \begin{cases} 0 & (x < 0) \\ 1-p & (0 \leq x < 1) \\ 1 & (1 \leq x) \end{cases}$$



expected value $E(X) = (1-p) \times 0 + p \times 1 = p$

variance $Var(X) = (1-p) \times (0-p)^2 + p \times (1-p)^2 = p(1-p)(p+1-p) = p(1-p)$

moment generating function $M_t(X) = (1-p) \times e^{t \cdot 0} + p \times e^{t \cdot 1} = (1-p) + pe^t$

This distribution shows the probability to get success when one does trial once. $x=1$ means success and $x=0$ means failure. (once) matches this distribution. For example, Coin toss (once) matches this distribution. ($x=1 \Leftrightarrow$ heads and $x=0 \Leftrightarrow$ tails).

Bernoulli(p) distribution is a special form of Binomial(n, p) distribution when $n=1$.