Remark: From $F_{\underline{X}}$ we can obtain $F_{X_1}$ by the formula

$$\underset{\text{marginal cdf}}{F_{X_1}}(x_1) = \lim_{y\to\infty} P(X_1 \le x_1, X_2 \le y, X_3 \le y, \cdots, X_N \le y)$$

And similarly in the continuous case

$$\underset{\text{marginal pdf}}{f_{X_1}}(x_1) = \underbrace{\int_{\mathbb{R}} \cdots \int_{\mathbb{R}}}_{N-1 \text{ integrals}} f_{\underline{X}}(x_1, x_2, \cdots, x_n)\, dx_2 \cdots dx_N$$

As before $g: \mathbb{R}^N \longmapsto \mathbb{R}$ continuous

$$E(g(\underline{X})) = \iiint_{\mathbb{R}^N} g(\underline{x}) f_{\underline{X}}(\underline{x})\, \underset{dx_1 dx_2 \cdots dx_N}{d\underline{x}}$$

Def. Conditional pdf or pmf for $\overbrace{X_{k+1}, \cdots, X_N}^{N-k \text{ random v.}}$ knowing $X_1, \cdots, X_k$ is given by

$$f(x_{k+1}, \cdots, x_N | x_1, \cdots, x_k) = \frac{f_{\underline{X}}(x_1, \cdots, x_N)}{f_{X_1, \cdots, X_k}(x_1, \cdots, x_k)}$$

if the denominator is not 0.

(Example 4.2.2 + 4.2.4 for motivation)

## I.5 Covarience and correlation

Consider X and Y random variables on S,

with $E(X) =: \mu_X$, $E(Y) =: \mu_Y$, $Var(X) := \sigma_X^2$, $Var(Y) := \sigma_Y^2$ We assume they exist.

Def. The covarience of X and Y is defined by

$$Cov(X,Y) = E\big((X-\mu_X)(Y-\mu_Y)\big) = E(XY) - \sigma_X \sigma_Y$$

The corelation of X and Y is defined by

$$\rho_{XY} = \frac{Cov(X,Y)}{\sigma_X \sigma_Y} \in [-1, 1]$$

Lemma: If X and Y are independent then $Cov(X,Y) = \rho_{XY} = 0$ easy exercise

⚠ The converse is not true.

The covarience measures a linear relationship:

$$|\rho_{XY}| = 1 \text{ iff } P(Y = aX + b) = 1 \quad \exists a, b \in \mathbb{R}$$

The pages 3,4 in Appendix 3 are helpful and easy.